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Re-examining $\sin 2\beta$ and Δm_d from evolution of B_d^0 mesons with decoherence

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ABSTRACT

In the time evolution of neutral meson systems, a perfect quantum coherence is usually assumed. The important quantities of the B_d^0 system, such as $\sin 2\beta$ and Δm_d, are determined under this assumption. However, the meson system interacts with its environment. This interaction can lead to decoherence in the mesons even before they decay. In our formalism this decoherence is modelled by a single parameter λ. It is desirable to re-examine the procedures of determination of $\sin 2\beta$ and Δm_d in meson systems with decoherence. We find that the present values of these two quantities are modulated by λ. Re-analysis of B_d^0 data from B-factories and LHCb can lead to a clean determination of λ, $\sin 2\beta$ and Δm_d.

1. Introduction

In neutral meson systems, quantum coherence plays a crucial role in the determination of many observables. However, any real system interacts with its environment and this interaction can lead to a loss of quantum coherence. The environmental effects may arise at a fundamental level, such as the fluctuations in a quantum gravity space–time background [1,2]. They may also arise due to the detector environment itself. Irrespective of the origin of the environment, its effect on the neutral meson systems can be taken into account by using the ideas of open quantum systems [3–5]. This formalism enables the inclusion of effects such as decoherence and dissipation in a systematic manner [6]. Such an inclusion is in accordance with the general principle of fluctuation–dissipation theorem which states that dissipation is balanced by fluctuations.

The time evolution of neutral mesons, which are coherently produced in meson factories, is used to measure a number of parameters of the standard model of particle physics and also to search for physics beyond the standard model. However, decoherence is an unavoidable phenomenon as any physical system is inherently open due to its inescapable interactions with a pervasive environment. With the inclusion of the decoherence effects, the measured values of some of these parameters can get masked. As the source of decoherence in the case of mesons could be expected to be coming from a much finer scale, it may happen that the numerical value of some of the masked observables is not greatly affected. This should however be verified experimentally.

In this work, we study the effect of decoherence on the important observables in the B_d^0 meson system, such as the CP violating parameter $\sin 2\beta$ and the B_d^0–$\bar{B}_d^0$ mixing parameter Δm_d. We show that these parameters are affected by decoherence. So far only one attempt has been made to determine decoherence in B_d meson system [7]. The bounds on the decoherence parameter were obtained from the data on R_d, the ratio of the total same-sign to opposite sign dilepton rates in the decays of coherent B_d–$\bar{B}_d$ coming from the $\Upsilon(4S)$ decays. The data on R_d has not been updated in the last two decades [8], whereas the B-factories have provided direct and precise information on the B_d–$\bar{B}_d$ mixing parameters. In this work, we also suggest a number of methods which will enable clean determination of the decoherence parameter along with the other observables quite easily at the LHCb or B-factories. We also attempt determination of the decoherence parameter and Δm_d using Belle data on the time dependent flavor asymmetry of semi-leptonic B_d^0 decays as given in Ref. [9].

The evolution of the B_d^0 system is built up from first principles. The effect of the environment forces the evolution to be a semi-group rather than a unitary one [10,11,6]. We use the density matrix formalism to represent the time evolution of the B_d^0 system. This ensures the complete positivity of the state of the

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system and hence its physical validity. In this formalism, the decoherence is modelled by a single parameter λ. By construction, the density matrices are trace preserving.

The work presented here, we hope, would lead to the inclusion of the effects of decoherence in the analysis of data from the B_d^0 systems. It may be worthwhile to re-analyse the data from the B factories and LHCb to verify if a signature of decoherence is already inherent in it. Thus a detailed study of B_d^0 observables can lead to tests of physics at scales much higher than those typical of flavour physics.

We first study the parameter $\sin 2\beta$, whose measurement is the first signal for CP violation outside the neutral kaon system. The precision measurement of its value is the corner stone in establishing the CKM mechanism for CP violation. With the inclusion of the decoherence effects, it turns out that the experimentally measured CP asymmetry depends both on the decoherence parameter λ and the angle β of the unitarity triangle. Next we study Δm_d, which denotes the mixing in the B_d^0 system and is an important input in extracting $\sin 2\beta$ from the measured time dependent CP asymmetry. We find that Δm_d is also affected by the decoherence effects. Finally, we suggest a method of analysis by which the three quantities, (a) λ, (b) Δm_d and (c) $\sin 2\beta$ can all be measured.

2. Determination of $\sin 2\beta$

In the following, we develop the formalism which is applicable to B_d^0 as well as B_s^0 mesons. We are interested in the decays of B^0 and $\bar{B}^0$ mesons as well as $B^0 \leftrightarrow \bar{B}^0$ oscillations. To describe the time evolution of all these transitions, we need a basis of three states: $|B^0\rangle$, $|\bar{B}^0\rangle$ and $|0\rangle$, where $|0\rangle$ represents a state with no B meson and is required for describing the decays. In this basis, we can define $\rho_{B^0(\bar{B}^0)}(0)$, the initial density matrix for the state which starts out as $B^0(\bar{B}^0)$. The time evolution of these matrices is governed by the Kraus operators $K_i(t)$ as $\rho(t) = \sum_i K_i(t)\rho(0)K_i^\dagger(t)$ [12]. The Kraus operators are constructed taking into account the decoherence in the system which occurs due to the evolution under the influence of the environment [13,14]. The time dependent density matrices are

$$\frac{\rho_{B^0}(t)}{\frac{1}{2}e^{-\Gamma t}} = \begin{pmatrix} a_{ch} + e^{-\lambda t}a_c & -a_{sh} - ie^{-\lambda t}a_s & 0 \\ -a_{sh} + ie^{-\lambda t}a_s & a_{ch} - e^{-\lambda t}a_c & 0 \\ 0 & 0 & 2(e^{\Gamma t} - a_{ch}) \end{pmatrix},$$
$$\frac{\rho_{\bar{B}^0}(t)}{\frac{1}{2}e^{-\Gamma t}} = \begin{pmatrix} a_{ch} - e^{-\lambda t}a_c & -a_{sh} + ie^{-\lambda t}a_s & 0 \\ -a_{sh} - ie^{-\lambda t}a_s & a_{ch} + e^{-\lambda t}a_c & 0 \\ 0 & 0 & 2(e^{\Gamma t} - a_{ch}) \end{pmatrix}, \tag{1}$$

for B^0 and $\bar{B}^0$, respectively. In the above equation, $a_{ch} = \cosh\left(\frac{\Delta\Gamma t}{2}\right)$, $a_{sh} = \sinh\left(\frac{\Delta\Gamma t}{2}\right)$, $a_c = \cos(\Delta m t)$, $a_s = \sin(\Delta m t)$, $\Gamma = (\Gamma_L + \Gamma_H)/2$, $\Delta\Gamma = \Gamma_L - \Gamma_H$, where Γ_L and Γ_H are the respective decay widths of the decay eigenstates B_L^0 and B_H^0. Also λ is the decoherence parameter, due to the interaction between one-particle system and its environment. As our main motivation is to bring out the fact that fundamental parameters of B–$\bar{B}$ mixing and B sector CP violation are affected by decoherence, here, and from here on, we will neglect the small mixing induced CP violation to keep our formulae simple.

We define the decay amplitudes $A_f \equiv A(B^0 \to f)$ and $\bar{A}_f \equiv A(\bar{B}^0 \to f)$. The hermitian operator describing the decays of the B^0 and $\bar{B}^0$ mesons into f is

$$\mathcal{O}_f = \begin{pmatrix} |A_f|^2 & A_f{}^*\bar{A}_f & 0 \\ A_f\bar{A}_f^* & |\bar{A}_f|^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{2}$$

The probability, $P_f(B^0/\bar{B}^0; t)$, of an initial $B^0/\bar{B}^0$ decaying into the state f at time t is given by $\mathrm{Tr}\left[\mathcal{O}_f\, \rho_{B^0(\bar{B}^0)(t)}\right]$.

Let us now consider $B_d^0 \to J/\psi K_S$ decay. One can define a CP violating observable

$$\mathcal{A}_{J/\psi K_S}(t) = \frac{P_{J/\psi K_S}(\bar{B}_d^0; t) - P_{J/\psi K_S}(B_d^0; t)}{P_{J/\psi K_S}(\bar{B}_d^0; t) + P_{J/\psi K_S}(B_d^0; t)}. \tag{3}$$

Calculating the probabilities using Eqs. (1) and (2) we get

$$\mathcal{A}_{J/\psi K_S}(t) = \frac{\left(|\lambda_f|^2 - 1\right)\cos(\Delta m_d t) + 2\,\mathrm{Im}(\lambda_f)\sin(\Delta m_d t)}{\left(1 + |\lambda_f|^2\right)\cosh\left(\frac{\Delta\Gamma_d t}{2}\right) - 2\,\mathrm{Re}(\lambda_f)\sinh\left(\frac{\Delta\Gamma_d t}{2}\right)} e^{-\lambda t}, \tag{4}$$

where $\lambda_f = A(\bar{B}_d^0 \to J/\psi K_S))/A(B_d^0 \to J/\psi K_S)$. Putting $\lambda = 0$ in the above equation, we get the usual expression for CP asymmetry in the interference of mixing and decay. Thus the presence of decoherence modifies the expression for CP asymmetry in the interference of mixing and decay.

In order to determine $\sin 2\beta$ from asymmetry defined in Eq. (4), it is usually assumed that, $\Delta\Gamma_d \approx 0$, $|\lambda_f| = 1$, i.e., no direct CP asymmetry and $\mathrm{Im}(\lambda_f) \approx \sin 2\beta$. With these approximations, the above expression simplifies to

$$\mathcal{A}_{J/\psi K_S}(t) = \sin 2\beta\, e^{-\lambda t} \sin(\Delta m_d t)\,. \tag{5}$$

Therefore we see that the coefficient of $\sin(\Delta m_d t)$ in the CP asymmetry is $\sin 2\beta\, e^{-\lambda t}$ and not $\sin 2\beta$! The measurement of $\sin 2\beta$ is masked by the presence of decoherence. Thus in order to have a clean determination of $\sin 2\beta$, an understanding of λ is imperative.

Decoherence is expected to come from a scale much finer than that of flavor physics and is likely to be small. Therefore, in the actual comparison to the data, one should include all the known effects, which are usually neglected in the extraction of $\sin 2\beta$ and then do a fit for clean determination of $\sin 2\beta$ and λ. The full fledged formula, of course, will include the CP violation in mixing and decay width $\Delta\Gamma_d$. Apart from these effects, one should also take into account the penguin contributions. The theoretical precision for the extraction of CP violating phase $\sin 2\beta$ from the CP asymmetry of $B_d^0 \to J/\psi K_S$ decay, defined in Eq. (4), is limited by contributions from doubly Cabibbo-suppressed penguin topologies [15,16]. This involves computation of non-perturbative hadronic parameters which, at present, cannot be achieved reliably using QCD. However, a way to control the penguin effects is offered by the U-spin symmetry of strong interactions which relates $B_s^0 \to J/\psi K_S$ to $B_d^0 \to J/\psi K_S$ [17]. Ref. [16] discusses the constraining of the relevant penguin parameters by making use of this symmetry as well as plausible assumptions for various modes of similar decay dynamics.

3. Determination of Δm_d

It is obvious that in order to determine $\sin 2\beta$, we need to know Δm_d and λ. If Δm_d is measured using observables which are independent of λ, then we only need to determine λ for the clean extraction of $\sin 2\beta$. If the determination of Δm_d is also masked by the presence of decoherence then we need to have a clean determination of Δm_d.

The present world average of Δm_d quoted in PDG is (0.510 ± 0.003) ps^{-1} [18] which is an average of measurements of Δm_d

from OPAL [19], ALEPH [20], DELPHI [21], L3 [22], CDF [23], BaBar [24], Belle [25], D0 [26] and LHCb [27] experiments. There are several ways in which Δm_d can be determined experimentally. LHCb, CDF and D0 experiments determine Δm_d by measuring rates that a state that is pure B_d^0 at time $t=0$, decays as either as B_d^0 or $\bar{B}_d^0$ as function of proper decay time. In the presence of decoherence, the survival (oscillation) probability of initial B_d^0 meson to decay as $B_d^0(\bar{B}_d^0)$ at a proper decay time t is given by

$$P_{\pm}(t,\lambda) = \frac{e^{-\Gamma t}}{2}\left[\cosh(\Delta\Gamma_d t/2) \pm e^{-\lambda t}\cos(\Delta m_d t)\right] . \qquad (6)$$

The positive sign applies when the B_d^0 meson decays with the same flavor as its production and the negative sign when the particle decays with opposite flavor to its production. We see that the survival (oscillation) probability of B_d^0 is λ dependent! The time dependent mixing asymmetry, used to determine Δm_d, is then given by

$$A_{\rm mix}(t,\lambda) = \frac{P_+(t,\lambda) - P_-(t,\lambda)}{P_+(t,\lambda) + P_-(t,\lambda)} = e^{-\lambda t}\frac{\cos(\Delta m_d t)}{\cosh(\Delta\Gamma_d t/2)} . \qquad (7)$$

Thus we see that the in the limit of neglecting $\Delta\Gamma_d$, the otherwise pure cosine dependence of mixing asymmetry is modulated by $e^{-\lambda t}$. Belle and BaBar experiments determine Δm_d by measuring time dependent probability $P_+(t)$ of observing unoscillated $B_d^0\bar{B}_d^0$ events and $P_-(t)$ of observing oscillated $B_d^0B_d^0/\bar{B}_d^0\bar{B}_d^0$ events for two neutral B_d mesons produced in an entangled state in the decay of the $\Upsilon(4S)$ resonance. The expressions for $P_{\pm}(t)$, in the presence of decoherence, are the same as those given in Eq. (6), except that the proper time t is replaced by the proper decay-time difference Δt between the decays of the two neutral B_d mesons. Therefore, we see that the determination of Δm_d at LHCb, CDF, D0, Belle and BaBar experiments is masked by the presence of λ. The true value of Δm_d, along with $\Delta\Gamma_d$, can be determined by a three parameter (Δm_d, $\Delta\Gamma_d$, λ) fit to the time dependent mixing asymmetry $A_{\rm mix}(t,\lambda)$ defined in Eq. (7). This in turn will enable a determination of true value of $\sin 2\beta$ using Eq. (4).

Determination of Δm_d in the LEP experiments is mainly based on time independent measurements, i.e., from the ratio of the total same-sign to opposite-sign semi-leptonic rates (R_d) or the total B_d^0–$\bar{B}_d^0$ mixing probability (χ_d). We shall now see that these observables are also λ dependent. Therefore all the methods used to determine Δm_d depend upon λ.

4. Correlated B_d^0 meson semi-leptonic decays

The entangled B_d^0–$\bar{B}_d^0$ mesons, produced in the decay of the $\Upsilon(4S)$ resonance, can both decay semi-leptonically. The effects of decoherence on the resulting dilepton signal was studied in [7]. Here we calculate these effects using the formalism described in the previous section. The entangled B_d^0–$\bar{B}_d^0$ state can be written as

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(\left|B_d\bar{B}_d^0\right\rangle - \left|\bar{B}_d^0 B_d\right\rangle\right) . \qquad (8)$$

The time evolution of the above state is described by the following density matrix [28–30]:

$$\rho(t_1,t_2) = \frac{1}{2}\Big(\rho_1(t_1)\otimes\rho_2(t_2) + \rho_2(t_1)\otimes\rho_1(t_2) - \rho_3(t_1)\otimes\rho_4(t_2) - \rho_4(t_1)\otimes\rho_3(t_2)\Big) , \qquad (9)$$

where $\rho_1(t) = \rho_{B^0}(t)$, $\rho_2(t) = \rho_{\bar{B}^0}(t)$ which are given in Eq. (1), while $\rho_{3/4}(t) = \sum_i K_i\rho_{3/4}(0)K_i^{\dagger}$, where $\rho_{3/4}(0) = \left|B^0(\bar{B}^0)\right\rangle\left\langle\bar{B}^0(B^0)\right|$ and are given by

$$\frac{\rho_3(t)}{\frac{1}{2}e^{-\Gamma t}} = \begin{pmatrix} -a_{sh} - ie^{-\lambda t}a_s & a_{ch} + e^{-\lambda t}a_c & 0 \\ a_{ch} - e^{-\lambda t}a_c & -a_{sh} + ie^{-\lambda t}a_s & 0 \\ 0 & 0 & 2a_{sh} \end{pmatrix} ,$$

$$\frac{\rho_4(t)}{\frac{1}{2}e^{-\Gamma t}} = \begin{pmatrix} -a_{sh} + ie^{-\lambda t}a_s & a_{ch} - e^{-\lambda t}a_c & 0 \\ a_{ch} + e^{-\lambda t}a_c & -a_{sh} - ie^{-\lambda t}a_s & 0 \\ 0 & 0 & 2a_{sh} \end{pmatrix} . \qquad (10)$$

Here the parameters are as in Eq. (1). The double decay rate, $G(f,t_1;g,t_2)$, that the left-moving meson decays at proper time t_1 into a final state f, while the right-moving meson decays at proper time t_2 into the final state g, is then given by ${\rm Tr}[(\mathcal{O}_f\otimes\mathcal{O}_g)\,\rho(t_1,t_2)]$. From this a very useful quantity called the single time distribution, $\Gamma(f,g;t)$, can be defined as $\Gamma(f,g;t) = \int_0^{\infty} d\tau\, G(f,\tau+t;g,\tau)$, where $t = t_1 - t_2$ is taken to be positive.

We now consider the decays of B_d^0 mesons into semi-leptonic states $h\,l\,\nu$, where h stands for any allowed charged hadronic state. Under the assumption of CPT conservation and no violation of $\Delta B = \Delta Q$ rule, the amplitudes for $B_d^0/\bar{B}_d^0$ into $h^-l^+\nu$ can be written as

$$A\left(B_d^0 \to h^-l^+\nu\right) = M_h , \qquad A\left(\bar{B}_d^0 \to h^-l^+\nu\right) = 0 , \qquad (11)$$

whereas the amplitudes for $B_d^0/\bar{B}_d^0$ into $h^+l^-\bar{\nu}$ are

$$A\left(B_d^0 \to h^+l^-\bar{\nu}\right) = 0 , \qquad A\left(\bar{B}_d^0 \to h^+l^-\bar{\nu}\right) = M_h^* . \qquad (12)$$

There are two important observables which can be affected by interaction with the environment. One is the ratio of the total same-sign to opposite-sign semi-leptonic rates

$$R_d = \frac{\Gamma(h^+,h^+) + \Gamma(h^-,h^-)}{\Gamma(h^+,h^-) + \Gamma(h^-,h^+)} , \qquad (13)$$

and the other is the total B_d^0–$\bar{B}_d^0$ mixing probability

$$\chi_d = \frac{\Gamma(h^+,h^+) + \Gamma(h^-,h^-)}{\Gamma(h^+,h^+) + \Gamma(h^-,h^-) + \Gamma(h^+,h^-) + \Gamma(h^-,h^+)} . \qquad (14)$$

Time independent probabilities, $\Gamma(f,g)$, can be obtained by integrating the distribution $\Gamma(f,g;t)$ over time.

The expressions for R_d and χ_d are obtained to be

$$R_d = \left[1 - (1-y^2)\left((1+\lambda')^2 + x^2\right)^{-1}\right] \times \left[1 + (1-y^2)\left((1+\lambda')^2 + x^2\right)^{-1}\right]^{-1} , \qquad (15)$$

$$\chi_d = \frac{1}{2}\left[1 - (1-y^2)\left((1+\lambda')^2 + x^2\right)^{-1}\right] , \qquad (16)$$

where we $x = \Delta m/\Gamma$, $y = \Delta\Gamma/2\Gamma$ and $\lambda' = \lambda/\Gamma$. We see that R_d and χ_d are both functions of $(1-y^2)$ and $(1+\lambda')^2$. It is interesting to note that in the limit of small λ' and y, these combinations have a linear term in λ' but only a quadratic term in y. Thus we see that along with Δm_d and $\Delta\Gamma_d$, these observables also depend upon the decoherence parameter λ.

For the observable R_d, the last experimental update was given about two decades ago [8]. This value was used in Ref. [7] to estimate the value of λ to be (-0.072 ± 0.118) ps^{-1}. It is important to re-analyse the BaBar and Belle data on the time dependent mixing asymmetry in terms of the three parameters (λ, Δm_d, $\Delta\Gamma_d$) using the expression given in Eq. (7). One should also obtain the value of χ_d from CDF, DO and LHCb. Then the expression in Eq. (16) can be verified using the values obtained from the fit to the time dependent mixing asymmetry. This will provide an additional consistency check on assumptions made regarding decoherence. Finally, the values of λ, Δm_d and $\Delta\Gamma_d$ from the $A_{\text{mix}}(t,\lambda)$ fit can be used in Eq. (4) to obtain a clean measurement of $\sin 2\beta$.

The present analysis can easily be extended to the B_s^0 system as well. The expression for the time dependent CP asymmetry in the mode $B_s^0 \to J/\psi\phi$ will be a function of four parameters: λ, $\sin 2\beta_s$, Δm_s and $\Delta\Gamma_s$. The time dependent mixing asymmetry defined in Eq. (7) will determine λ, Δm_s and $\Delta\Gamma_s$. These two time-dependent asymmetries should be re-analysed using a four parameter fit for a clean determination of $\sin 2\beta_s$, Δm_s, $\Delta\Gamma_s$ and λ. Also, like $\sin 2\beta_d$, the extraction of $\sin 2\beta_s$ from time dependent CP asymmetry in the mode $B_s^0 \to J/\psi\phi$ is restricted due to penguin pollution. In this case, the analysis of CP violation is more involved in comparison to $B_d^0 \to J/\psi K_S$. This is due to the fact that the final state involves two vector mesons. The admixture of different CP eigenstates can be disentangled through a time-dependent angular analysis of the decay products of the vector mesons [31,32]. The penguin contribution to $B_s^0 \to J/\psi\phi$ can be estimated using decays $B_d^0 \to J/\psi\rho$ and $B_s^0 \to J/\psi\bar{K}^*$ [15,33].

5. Estimation of λ: an example

Here we make an attempt of a clean determination of λ, Δm_d and $\Delta\Gamma_d$ using the experimental data of the time dependent flavor asymmetry of semi-leptonic B_d^0 decays as given in Ref. [9]. We perform a χ^2 fit to $A_{\text{mix}}(\Delta t,\lambda)$, using the efficiency corrected distributions given in Table I of Ref. [9]. First, the fit is done by assuming no decoherence, i.e., $\lambda = 0$. In this case, we find $\Delta m_d = (0.489 \pm 0.010)$ ps^{-1} and $\Delta\Gamma_d = (0.087 \pm 0.054)$ ps^{-1} with $\chi^2/d.o.f = 8.42/9$. We then redo the fit including decoherence. This gives $\lambda = (-0.012 \pm 0.019)$ ps^{-1} along with $\Delta m_d = (0.490 \pm 0.010)$ ps^{-1} and $\Delta\Gamma_d = (0.144 \pm 0.088)$ ps^{-1} with $\chi^2/d.o.f = 8.02/8$. Thus we see that the decoherence parameter λ is very loosely bounded. The upper limit on λ is 0.03 ps^{-1} at 95% C.L. We also find in this example that Δm_d is numerically unaffected where as $\Delta\Gamma_d$ can be affected by inclusion of decoherence. Given the wealth of data coming from LHCb and expected from the KEK Super B factory, a clear picture is expected to emerge.

6. Conclusions

In this work, we have studied the effect of decoherence on two important observables $\sin 2\beta$ and Δm_d in a neutral meson system. We find that the asymmetries which determine these quantities are also functions of the decoherence parameter λ. Hence it is imperative to measure λ for a clean determination of these quantities. We suggest a re-analysis of the data on the above asymmetries for an accurate measurement of all the three quantities λ, $\sin 2\beta$ and Δm_d. The present analysis can easily be extended to the B_s^0 system as well.

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