An observer-based tracker for hybrid interval chaotic systems with saturating inputs: The chaos-evolutionary-programming approach

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Abstract

This paper presents a novel chaos-evolutionary-programming algorithm (CEPA), which merges a modified chaotic optimization algorithm (COA) with a modified evolutionary-programming algorithm (EPA). Due to the nature of chaotic variable, i.e. pseudo-randomness, ergodicity and irregularity, the CEPA can effectively and quickly search many local minimum or maximum in parallel thereby enhancing the probability of finding the global one. The CEPA is then successfully applied to solve challenging non-convex optimization problems and to obtain the best nominal dual-rate observer-based digital tracker for robust tracking a periodic solution embedded into a hybrid interval chaotic system with saturating inputs and not to track the strange attractor itself. An illustrative example is presented to demonstrate the effectiveness of the proposed algorithm.

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1. Introduction

A general real system always encounters parameter variations, nonlinear distortions, physical constraints, etc. The analysis and design of such nonlinear uncertain systems, subject to many complex, conflicting, mathematically difficult and highly constrained multi-objective problems, belongs to the class of problems referred to as nondeterministic polynomial (NP) problems. With a suitable mapping or transformation of the NP problem, the intractable NP problem becomes a tractable NP-hard problem. The obtained solution may be less than optimal but often good enough with reasonable computational burden. It is well-known that evolutionary computation is the most effective way to solve NP-hard problems. There are two major types of evolutionary computation: Genetic Algorithms (GAs) \cite{1} and Evolutionary-Programming Algorithms (EPAs) \cite{2}. The applications of GAs and EPAs to aerospace, robotics, signal processing, control systems, etc. can be found in \cite{3,4}. Both algorithms guarantee a high chance of reaching a global optimum by starting with multiple random search points, and by considering several candidate solutions, simultaneously. However, their population size must be large in order to avoid premature convergence. A large population size requires more time to converge. As a result, the rate of convergence is, generally, slow, and for a

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fixed convergence time, the obtained solution may be less than optimal. Recently, an emerging strategy called Chaotic Optimization Algorithm (COA) has been proposed [5] and successfully applied to the tracking control of a nonlinear system [6].

The philosophy of this COA is very simple, which is based on two main steps: first, it carries out a transform from the chaotic space to the solution space, and then it executes the global optimal searching based on the chaotic dynamics itself rather than guiding its searching at random. There are many good properties to execute the chaotic search, such as ergodicity, stochastic properties, and regularity [5]. A chaotic movement can go through every state in a certain area according to its own regularity, and every state is obtained only once. This algorithm can easily escape from local minimum or maximum value and easily find the global value. By taking advantages of both EPA and COA, the EPA [7] with a modified mutation mechanism (11) together with the COA [5] with a modified mutation mechanism (14), called the Chaos-Evolutionary-Programming Algorithm (CEPA), is developed in this paper and applied to solve non-convex optimization problems and to design an observer-based tracker for hybrid interval chaotic systems with saturating inputs.

A controller often generates a signal, which is larger than the operating range of the actuator. Thus, it behaves as a nonlinear saturation element at the inputs of the system. To reduce the effects of the input saturation, linear conditioning techniques [8,9] are commonly utilized. Generally, linear conditioning means augmenting a system with a linear transfer function to modify the system’s behavior during saturation, and thus to quickly escape from saturation. However, most of the existing linear conditioning techniques are developed in continuous-time setting. Recently, a digital dual-rate conditioning transfer scheme has been developed in [10], which produces a digitally redesigned fast-rate inner-state compensator for systematically reducing the windup effects and a digitally redesigned slow-rate observer-based predictive tracker for effectively tracking the orbit of a chaotic system with saturating actuators. The objective of this paper is to extend the methodology developed in [10] for a nominal Chen’s chaotic system [11,12] to an interval Chen’s chaotic system with bounded interval parameters. Some of the methods [10] have been used before by us but without the chaos-evolutionary-programming algorithm (CEPA) which appears to be novel in this kind of applications.

Most practical dynamical systems and industrial control processes are often formulated in a continuous-time (analog) framework for which the well-established control theories are available for analysis and design. The resulting analog controller is often desired to be implemented using a digital controller for better reliability, lower cost and more flexibility due to rapid advances in digital technology and computers. The process of converting a continuous-time (analog) controller to its equivalent discrete-time (digital) controller, so that the states of the digitally controlled sampled-data system will closely match those of the analogously controlled system, is the so-called digital redesign. The digital redesign methods can be found in [13,14]. The digital redesign method [14] is able to convert a theoretically pre-designed high-gain analog tracker into a practically implementable low-gain digital tracker for chaotic orbit tracking.

By taking advantage of the digital redesign methodology, a low-gain digital observer, instead of a high-gain analog observer, can also be developed for the implementation of the developed digitally redesigned low-gain digital tracker to effectively carry out digital control of the hybrid chaotic systems.

This paper is organized as follows. The development of the CEPA is shown in Section 2. Then, the optimal linear modeling of a nonlinear system is developed in Section 3. Next, the designs of analog optimal tracker and observer are discussed in Section 4. In addition, the development of digitally redesigned tracker and observer for sampled-data systems is presented in Section 5. The developed digital redesign method for linear systems is then extended to nonlinear systems in Section 6. Moreover, the utilization of the developed CEPA to find the digitally redesigned observer-based tracker for uncertain nonlinear systems with saturating inputs is shown in Section 7. An example is illustrated in Section 8. Finally, the conclusion is drawn in Section 9.

2. The chaos-evolutionary-programming algorithm

2.1. The chaotic optimization algorithm

The chaotic equation for COA can be selected as the logistic mapping [5], namely

$$
t_{k+1} = f(\mu_, t_k) = \mu t_k (1 - t_k) \quad k = 1, 2, \ldots, N,
$$

(1)
Table 1
Simulation results for Eq. (2)

<table>
<thead>
<tr>
<th>Iterative number</th>
<th>Values of $t$ when $t_0 = 0.2$</th>
<th>Values of $t$ when $t_0 = 0.2001$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.2001</td>
</tr>
<tr>
<td>2</td>
<td>0.64</td>
<td>0.6402</td>
</tr>
<tr>
<td>3</td>
<td>0.9216</td>
<td>0.9213</td>
</tr>
<tr>
<td>4</td>
<td>0.28901</td>
<td>0.2899</td>
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<tr>
<td>5</td>
<td>0.82194</td>
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<tr>
<td>6</td>
<td>0.58542</td>
<td>0.5815</td>
</tr>
<tr>
<td>7</td>
<td>0.97081</td>
<td>0.9734</td>
</tr>
<tr>
<td>8</td>
<td>0.11334</td>
<td>0.1034</td>
</tr>
<tr>
<td>9</td>
<td>0.40197</td>
<td>0.3708</td>
</tr>
<tr>
<td>10</td>
<td>0.96156</td>
<td>0.9332</td>
</tr>
<tr>
<td>11</td>
<td>0.14784</td>
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<tr>
<td>12</td>
<td>0.50392</td>
<td>0.7484</td>
</tr>
<tr>
<td>13</td>
<td>0.99994</td>
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</tr>
<tr>
<td>14</td>
<td>0.0002463</td>
<td>0.7434</td>
</tr>
<tr>
<td>15</td>
<td>0.00098498</td>
<td>0.7630</td>
</tr>
<tr>
<td>16</td>
<td>0.003936</td>
<td>0.7232</td>
</tr>
<tr>
<td>17</td>
<td>0.015682</td>
<td>0.8007</td>
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<tr>
<td>18</td>
<td>0.061745</td>
<td>0.6384</td>
</tr>
<tr>
<td>19</td>
<td>0.23173</td>
<td>0.9234</td>
</tr>
</tbody>
</table>

where, $0 \leq t_0 \leq 1$ and $0 \leq \mu \leq 4$. It is easy to find that Eq. (1) is a deterministic dynamic system without any stochastic disturbance. The chaotic equation has the chaotic properties such as ergodicity, stochastic properties and regularity [5]. The chaotic motion can go non-repeatedly through every state in a certain domain and it tends to amplify small errors until they become very large. When $\mu = 4$, Eq. (1) becomes chaotic and is represented as

$$t_{k+1} = 4t_k (1 - t_k).$$

The simulation results of the chaotic system (2) with its chaotic space $[0, 1]$, $t_0 = 0.2$ and $t_0 = 0.2001$ are shown in Table 1.

2.2. The chaos-evolutionary-programming algorithm

The joint algorithms of the EPA [7] and COA [5] are developed as follows.

If the optimization problems are continuous problems rather than discrete problems and the constraints of the variables are known, the optimization problems can be described as

$$\min f(x_i) \quad \text{or} \quad \max f(x_i), \quad i = 1, 2, \ldots, n$$
$$a_i \leq x_i \leq b_i \quad a_i \leq x_i \leq b_i.$$ \hspace{1cm} (3)

Suppose that the natural numbers are represented in the scale of notation with radix $R$, then

$$n = a_0 + a_1 R + a_2 R^2 + \cdots + a_m R^m, \quad 0 \leq a_i \leq R.$$ \hspace{1cm} (4)

Write the digits of these numbers in the reverse order, preceded by a decimal point. This gives the number

$$\phi_R(n) = a_0 R^{-1} + a_1 R^{-2} + \cdots + a_m R^{-m-1}.$$ \hspace{1cm} (5)

Halton [15] extended the two-dimensional result of Van Der Corput [16,17] to $\rho$-dimensions, when $R_1, R_2, \ldots, R_\rho$ are mutually coprime. We show a binary scale and an illustration in Tables 2 and 3.

Since $\phi_R(n) < 1$, to satisfy this range, scaling any varying parameter (e.g., a real number $\eta$ from its range $[\underline{\eta} \overline{\eta}]$ to $[0 1]$) is required. Let the interval real ($\mathcal{R}$) matrix $X \in \mathcal{R}^{n \times m}$ be a set of degenerate real matrices defined by

$$X = [L, U] = \{[x_{ij}] | l_{ij} \leq x_{ij} \leq u_{ij}; 1 \leq i \leq n, 1 \leq j \leq m\},$$ \hspace{1cm} (6)

where $L$ and $U$ are constant real matrices. We introduce the variable $\eta_{ij}, 0 \leq \eta_{ij} \leq 1$ such that
Table 2
Natural numbers in binary scale

<table>
<thead>
<tr>
<th>n (decimal)</th>
<th>(Binary)</th>
<th>(\varphi_2(n)) (binary)</th>
<th>(Decimal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>0.01</td>
<td>0.25</td>
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<tr>
<td>3</td>
<td>11</td>
<td>0.11</td>
<td>0.75</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>0.001</td>
<td>0.125</td>
</tr>
<tr>
<td>5</td>
<td>101</td>
<td>0.101</td>
<td>0.625</td>
</tr>
<tr>
<td>6</td>
<td>110</td>
<td>0.011</td>
<td>0.375</td>
</tr>
<tr>
<td>7</td>
<td>111</td>
<td>0.111</td>
<td>0.875</td>
</tr>
<tr>
<td>8</td>
<td>1000</td>
<td>0.0001</td>
<td>0.0625</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Table 3
Quasi-random sequences

<table>
<thead>
<tr>
<th>(\phi_R(n))</th>
<th>(R)</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>11</th>
<th>13</th>
<th>17</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N)</td>
<td></td>
<td>1</td>
<td>0.5000</td>
<td>0.333</td>
<td>0.2000</td>
<td>0.1429</td>
<td>0.0909</td>
<td>0.0769</td>
<td>0.0588</td>
</tr>
<tr>
<td>2</td>
<td>0.2500</td>
<td>0.6667</td>
<td>0.4000</td>
<td>0.2857</td>
<td>0.1818</td>
<td>0.1538</td>
<td>0.1176</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.7500</td>
<td>0.1111</td>
<td>0.6000</td>
<td>0.4286</td>
<td>0.2727</td>
<td>0.2308</td>
<td>0.1765</td>
<td>...</td>
<td></td>
</tr>
<tr>
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<td>...</td>
<td>...</td>
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<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>48</td>
<td>0.0469</td>
<td>0.1975</td>
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<td>0.9796</td>
<td>0.3967</td>
<td>0.7101</td>
<td>0.8304</td>
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</tr>
<tr>
<td>49</td>
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<td>0.9680</td>
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<td>0.4876</td>
<td>0.7870</td>
<td>0.8893</td>
<td>...</td>
<td></td>
</tr>
<tr>
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<td>0.5785</td>
<td>0.8639</td>
<td>0.9481</td>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

\[ x_{ij} = l_{ij} + \eta_{ij}(u_{ij} - l_{ij}), \quad (7) \]

and use the notation \(\eta = [\eta_{11}, \ldots, \eta_{1m}, \eta_{21}, \ldots, \eta_{2m}, \eta_{n1}, \ldots, \eta_{nm}]\). Then the interval matrix \(X\) can be denoted as \(X(\eta)\). Let \(\eta_{11} = \phi_2(n), \eta_{12} = \phi_3(n), \eta_{13} = \phi_5(n), \ldots\) and so on, to construct the desired initial population of size \(N\) (e.g., \(N = 50\)).

The merging of the EPA [7] and COA [5] for the proposed CEPA is described as follows:

1. **Individual population:** Choose individual population based on the quasi-random sequence (QRS) [18] to form an initial population \(P_{ini} = [P_1, P_2, \ldots, P_N]\) of size \(N\) by initializing each \(\rho\)-dimensional solution vector \(P_i\) in \(S\).
2. **Objective function:** Assign each \(P_i, i = 1, \ldots, N\), an objective function score. Arrange \(P_i, i = 1, \ldots, N\), in the descending order, starting from the best one generated from the objective function score.
3. **Fitness function:** Assign each sorted \(P_i, i = 1, \ldots, N\), a fitness function (FF) score to weigh those high-quality individuals in the pool of individuals based on the obtained objective function scores. Search some \(P^*\) in the solution \(P_i, i = 1, \ldots, N\), so that the objective function (OF) value \(OF(P_i)\) is minimal, using

\[ FF(OF(P_i)) = \left( \frac{\beta - \bar{\beta}}{\bar{OF}(P_i) - OF(P_i)} \right) (OF(P_i) - \bar{OF}(P_i)) + \beta. \quad (8) \]

On the other hand, we search some \(P^*\) in the solution \(P_i, i = 1, \ldots, N\), so that the objective function value \(OF(P_i)\) is maximal, using

\[ FF(OF(P_i)) = \left[ \left( \frac{\beta - \bar{\beta}}{\bar{OF}(P_i) - OF(P_i)} \right) (OF(P_i) - \bar{OF}(P_i)) + \beta \right]^{-1}. \quad (9) \]

This function linearly maps the real-valued space \([\bar{OF}(P_i), \bar{OF}(P_i)]\) to any appropriate specified space, \([\beta, \bar{\beta}]\) (e.g., \([\beta, \bar{\beta}] = [1, 10]\)), where \(\beta > 0\), for weighting the objective function scores. Hence, the better an individual is, the higher the objective function score that it will have.
(4) Probability function: Calculate the probability function (PF) score of each $P_i$, $i = 1, \ldots, N$, using the fitness function score:

$$PF(FF(P_i)) := PF(P_i) = \frac{FF(P_i)}{\sum_{i=1}^{N} FF(P_i)}. \quad (10)$$

This equation guarantees that each individual has a non-zero selection probability. It is an essential factor to determine the preservation or extinction of the individual.

(5) Mutation: In order to make sure that the better individuals have minor variations and huge ones have more variations in this generation, use the following approach to double the population size. First, mutate each $P_i$, $i = 1, \ldots, N$, based on statistics to increase the population size from $N$ to $1.6N$; assign $P_{i+N}$ the following value

$$P_{i+N,j} := P_{i,j}(1 + \text{sgn}(N(0, 1))\gamma(1 - FP(P_i))) \quad \text{for } i = 1, 2, \ldots, 0.6N,$$

and values of $P_{1.6N+1,j}$ to $P_{2N,j}$ are produced from the best $P_{\text{best},j}$ as follows:

$$P_{1.6N+k,j} := P_{\text{best},j}(1 + k\% \times \alpha_1) \quad \text{for } k = 1, 2, \ldots, 0.2N,$$

$$P_{1.8N+k,j} := P_{\text{best},j}(1 - k\% \times \alpha_1) \quad \text{for } k = 1, 2, \ldots, 0.2N,$$

where $\alpha_1$ is a weighting factor, $P_{i,j}$ is the $j$th element in the $i$th individual, $N(\mu, \sigma^2)$ is the Gaussian random variable with mean $\mu$ and variance $\sigma^2$, $\gamma$ is a weighting factor for the percentage change of $P_{i,j}$, and $\text{sgn}(\cdot)$ is the standard sign function, such that the better and poor individuals in $P_{i,j}$, $i = 1, \ldots, N$ will yield a minor and a huge variations to form the mutated individuals, $P_{N+k,j}$ for $k = 1, 2, \ldots, 0.6N$, respectively. Then, the rest individuals are produced from the best individual $P^*_{i,j}$. Whenever $P_{i+N,j} \notin [P_j, \overline{P}_j]$, some modification is required

$$P_{i+N,j} := \begin{cases} P_j & \text{if } P_{i+N,j} < P_j \\ \overline{P}_j & \text{if } P_{i+N,j} > \overline{P}_j. \end{cases} \quad (12)$$

Properly adjusting the weighting factor $\gamma$ can possibly avoid the undesired situation $P_{i+N,j} \notin [P_j, \overline{P}_j]$. It is notable that $\gamma$ heavily dominates the convergence rate of the EP.

(6) Selection: Calculate the objective function score of each $P_{i+N}$, $i = 1, \ldots, N$. Rank the objective function scores of $P_i$, $i = 1, \ldots, 2N$. Record $P_i$, $i = 1, \ldots, 2N$, in a descending order, starting from the best individual in the pool of the population (proportional selection). The first $N$ individuals are selected for the next generation, in which the top one of each generation (elitist model), denoted $P^*_{g,i}$, always survives and is selected for the next generation. Whenever $P^*_{g,i}$ is no longer the best during the evolutionary process, update it by the newly generated best one.

(7) Penalty: Tune $\gamma$ in the following way, to further avoid the search from being trapped into a local extreme

$$\gamma := \begin{cases} \gamma & \text{if } |\text{OF}(P^*_{g-1,i}) - \text{OF}(P^*_{g,i})| > \varepsilon \\ 1.5\gamma & \text{if } |\text{OF}(P^*_{g-2,i}) - \text{OF}(P^*_{g,i})| \leq \varepsilon \\ 0.5\gamma & \text{if } |\text{OF}(P^*_{g-2,i}) - \text{OF}(P^*_{g,i})| \leq \varepsilon \quad \text{and } |\text{OF}(P^*_{g-1,i}) - \text{OF}(P^*_{g,i})| \leq \varepsilon, \end{cases} \quad (13)$$

where $\varepsilon$ is some tolerable error bound and $g$ is the generation index. Then, go to Step (2) and continue until the desired extreme value $\text{OF}(P^*_{g,i})$ cannot be further improved and/or the allowable generation is obtained.

(8) EPA termination condition: After some generations without improvement, we stop the EPA.

(9) Carry out the proposed chaotic search:

(a) Generate $N - 1$ chaotic variables by (2).

(b) Change $N - 1$ chaotic variables to real variables: First, appropriately specify the search field $[P^*_{g,i} - \theta, P^*_{g,i} + \theta]$. Then, amplify the ergodic areas of the $N - 1$ chaotic variables to the variance ranges of real variables by (14). Set $\overline{P}_{1,j} = P^*_{g,j}$, which is the best population.

$$\overline{P}_{k+1,j} = (P^*_{g,j} - \theta) + 2 \times \theta \times t_{k,j} \quad \text{for } k = 1, 2, \ldots, N - 1,$$

$$\overline{P}_{N+k,j} := P^*_{g,j}(1 + k\% \times \alpha_2) \quad \text{for } k = 1, 2, \ldots, 0.2N,$$

$$\overline{P}_{1.2N+k,j} := P^*_{g,j}(1 - k\% \times \alpha_2) \quad \text{for } k = 1, 2, \ldots, 0.2N,$$

where $\theta$ is the confidence factor and $\alpha_2$ is a modification factor.
where $\theta = \xi P^*_{s,j}$, $\xi$ denotes some percentage of $P^*_{s,j}$ and $\alpha_2$ is a weighting factor.

(c) Calculate the objective function score $OF(\bar{P}_k)$ of each $\bar{P}_k$, $k = 1, 2, \ldots, 1.4N$.

(d) Sort the minimum or maximum value of $OF(\bar{P}_k)$. Then update the $\bar{P}_{1,j}$ from $OF(\bar{P}_k)$.

(e) Calculate $t_{k+1,j} = \mu t_{k,j} (1 - t_{k,j})$ for $k := k + N - 1$, then perform (14) and repeat $n_1$ times.

(f) Go to Step (b) and repeat $n_2$ times. When $OF(\bar{P}_k)$ holds, stop it.

2.3. Application of the CEPA to solve non-convex optimization problems

Four complex functions shown in (15)–(18) [19,20] have been selected to test the CEPA and EPA. It is desired to find minimum values of four complex functions. The results obtained by CEPA and EP, respectively, are summarized in Table 4 in which $N = 50$, $\beta = [1, 10]$, $\gamma = 0.3$, $\alpha_1 = 1$, $\varepsilon = 0.0001$, $\xi = 0.1$, $\alpha_2 = 1$, $n_1 = 5$, $n_2 = 2$. The convergent values of $F_1$ to $F_4$ are shown in Figs. 1–4.

\[
F_1 = 100(x_1^2 - x_2)^2 + (1 - x_1)^2, \quad -2.048 \leq x_i \leq 2.048, \quad (15)
\]

\[
F_2 = 4 + 4.5x_1 - 4x_2 + x_1^2 + 2x_2^2 - 2x_1x_2 + x_1^4 - 2x_1^2x_2, \quad -8 \leq x_i \leq 8, \quad (16)
\]

\[
F_3 = (x_1^2 + x_2^2)^{0.25} [\sin^2(50(x_1^2 + x_2^2)^{0.1}) + 1], \quad -100 \leq x_i \leq 100, \quad (17)
\]

\[
F_4 = \left(4 - 2.1x_1^2 + \frac{x_1^4}{3}\right)x_1^2 + x_1x_2 + (-4 + 4x_2^2)x_2^2, \quad -100 \leq x_i \leq 100. \quad (18)
\]

The comparison result shows that CEPA is more powerful than EP algorithm. So, we use the CEPA to find a dual-rate observer-based tracker for a hybrid interval chaotic system with saturating actuators in the next section.

The interval system of interest in this paper contains uncertain parameters. The parameter variations do not follow any of the probability distributions and are quantified in terms of amplitude bounds. The CEPA is utilized in this paper to solve challenging non-convex optimization problems and to find a linear digital tracker for a hybrid interval chaotic system with saturating inputs.
Fig. 2. The convergent values of $F_2$.

Fig. 3. The convergent values of $F_3$.

Fig. 4. The convergent values of $F_4$. 
For the tracker scheme to function properly, the innovation error [21] is defined as

$$e(k) = y(k) - r(k),$$

(19)

which represents a linear combination of lower-bound and upper-bound percentage changes of measurable output signals. A chaos-evolutionary-programming approach is proposed in this paper to minimize the objective function (OF) score

$$\text{OF} := E[e(k)^T e(k)] \approx \frac{1}{k_f} \sum_{i=1}^{n} \sum_{k=1}^{k_f} e^2(k).$$

(20)

3. Optimal linearization of nonlinear systems

Some nonlinear systems usually have complex dynamical behaviors such as chaos. One common approach to solve the nonlinear problems is to find a linearized model via the gradient methods so that the well-established linear control theory and design methods can be applied for finding local analog controllers to improve performance. However, the gradient-based linearization model cannot correctly represent the exact local linear model at any operation state along the trajectory. Recently, a least-squares linearization model of a nonlinear system has been developed by Teixeira and Zak [22] and successfully applied to design a digital tracker for a hybrid nominal chaotic system without saturating inputs [14]. The optimal linearization method is briefly described as follows.

First, consider a nonlinear model

$$\dot{x}(t) = f(x(t)) + G(x(t)) u(t),$$

(21)

where $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $G: \mathbb{R}^n \rightarrow \mathbb{R}^m$ are nonlinear, $x(t) \in \mathbb{R}^n$ is the state vector, and $u(t) \in \mathbb{R}^m$ is the control input. Suppose that it is desired to have a local “linear model” $(A_k, B_k)$ at an interested operation state, $x_k(t) \in \mathbb{R}^n$, in the form

$$\dot{x}(t) = A_k x(t) + B_k u(t),$$

(22)

where $A_k$ and $B_k$ are constant matrices with appropriate dimensions. We use a common approach, Taylor expansion, to solve this problem. However, the truncated Taylor expansion usually results in an affine rather than a linear model. Even though the operating point is a system equilibrium, Taylor series linearization often does not yield a local model that is linear in $x(t)$ and $u(t)$. Assume that the operating points $x_k(t)$ and $u_k(t)$ are system equilibriums, that is,

$$f(x_k(t)) + G(x_k(t)) u_k(t) = 0,$$

(23)

where $x_k(t) \in \mathbb{R}^n$ and $u_k(t) \in \mathbb{R}^m$. In this case, the resulting linear model is

$$\frac{d}{dt}(x - x_k) = f(x_k) + G(x_k) u_k + A_k (x - x_k) + B_k (u - u_k)$$

$$= A_k x - x_k + B_k (u - u_k)$$

$$= A_k x + B_k u - (A_k x_k + B_k u_k).$$

(24)

Obviously, this is an affine rather than linear model as a result of the non-vanishing constant term. To avoid the difficulty, assume that we are given an operating state, $x_k \neq 0$, which is not necessarily an equilibrium of the given system (21). The goal is to construct a local model due to the generally non-vanishing constant term, linear in $x$ and also linear in $u$, that can well-approximate the dynamical behaviors of (21), in the vicinity of the operating point. That is, we wish to find two constant matrices $A_k$ and $B_k$, such that in a neighborhood of $x_k$ we have

$$f(x) + G(x) u \approx A_k x + B_k u \quad \text{for any } u,$$

(25)

and

$$f(x_k) + G(x_k) u = A_k x_k + B_k u \quad \text{for any } u.$$  

(26)

Since the control input $u$ is an arbitrary function to be designed, we must have
\[ G(x_k) = B_k. \]  

Therefore, then (25) and (26) can be reduced to brief forms as

\[ f(x) \approx A_k x, \]  

and

\[ f(x_k) = A_k x_k. \]  

In order to satisfy these, we denote \( a_i^T \) as the \( i \)th row of the matrix \( A_k \) so that (28) and (29) can be represented as

\[ f_i(x) \approx a_i^T x \quad i = 1, 2, \ldots, n, \]  

and

\[ f_i(x_k) = a_i^T x_k \quad i = 1, 2, \ldots, n, \]  

where \( f_i : \mathbb{R}^n \rightarrow \mathbb{R}^n \) is the \( i \)th row of the matrix \( f \). To expand the left-hand side of (30) about \( x_j \) and to neglect the second and higher order term, we can get

\[ f_i(x_k) + [\nabla f_i(x_k)]^T (x - x_k) \approx a_i^T x, \]  

where \( \nabla f_i(x_k) : \mathbb{R}^n \rightarrow \mathbb{R}^n \) is the gradient column vector of \( f_i \) evaluated at \( x_k \). By (31), we can rewrite (32) as

\[ [\nabla f_i(x_k)]^T (x - x_k) \approx a_i^T (x - x_k), \]  

in which \( x \) is arbitrary and close to \( x_k \) so that the approximation is good. Our objective is to determine a constant vector, \( a_i^T \), such that it is as close as possible to \([\nabla f_i(x_k)]^T\) and that it also satisfies \( a_i^T x_k = f_i(x_k) \). Then we can formulate our objective as a constrained optimization problem to minimize

\[ E := \frac{1}{2} \| \nabla f_i(x_k) - a_i \|^2 \quad \text{subject to } a_i^T x_k = f_i(x_k), \]  

where \( a_i^T \) is the \( i \)th row of the matrix \( A_k \). Let us consider the case for all components of \( x_k \neq 0 \), i.e., \( x_k,j \neq 0 \), for \( j = 1, 2, \ldots, n \). Thus, using the Lagrange multiplier method, the optimal solution is

\[ a_i = \nabla f_i(x_k) + \frac{f_i(x_k) - x_k^T \nabla f_i(x_k)}{\| x_k \|^2} x_k \quad \text{for } x_k,j \neq 0, j = 1, 2, \ldots, n, \]  

where \( \| x_k \|^2 = x_k^T x_k \) is the square magnitude of the point \( x_k \).

The controllability matrix for the nonlinear system (21) at the operating state \( x_k \) is derived from the linearized model \( A_k, B_k \) (25), resulting in

\[ \bar{C} = \begin{bmatrix} \bar{B}_k \\ \bar{A}_1 \bar{B}_k \\ \bar{A}_2 \bar{B}_k \\ \vdots \\ \bar{A}_n^{n-1} \bar{B}_k \end{bmatrix}, \]  

where \( \bar{A}_k \) and \( \bar{B}_k \) are constructed via the following rule: the \( j \)th columns of \( A_k \) and \( B_k \) are set to be zero whenever the \( j \)th component of \( x_k \) is zero [14]. Consequently, the constrain on all components of \( x_k \), i.e., \( x_k,j \neq 0 \) for \( 1 \leq j \leq n \), can be relaxed provided that \((A_k, B_k)\) in (22) are replaced by \((\bar{A}_k, \bar{B}_k)\) for some control purpose.

4. Analog linear quadratic tracker and observer design

In order to derive the dual-rate conditioning-transfer tracker, we introduce an optimal state-feedback control law [23] that forces the plant output to track a desired reference trajectory \( r(t) \). Consider a controllable and observable
optimally linearized model of a nonlinear system, which is described as

\[
\begin{align*}
\dot{x}(t) &= \bar{A}_k x(t) + \bar{B}_k u_{c,k}(t), \\
y(t) &= \bar{C} x(t) + \bar{D} u_{c,k}(t), \\
x(t_0) &= x_0,
\end{align*}
\]

(36)

where \( x(t) \in \mathbb{R}^n \) is the state vector, \( u_{c,k}(t) \in \mathbb{R}^m \) is the control input, \( y(t) \in \mathbb{R}^p \) is the measurable output and \((\bar{A}_k, \bar{B}_k, \bar{C}, \bar{D})\) are system matrices with appropriate dimensions. The optimal state-feedback control law is desired to minimize the following performance index:

\[
J = \int_0^\infty \left[ (\bar{C} x(t) - r(t))^T Q (\bar{C} x(t) - r(t)) + u_{c,k}^T(t) R u_{c,k}(t) \right] dt,
\]

(37)

where \( Q = Q^T \geq 0, R = R^T > 0 \) and \( r(t) \) is the given reference input. This optimal controller with \( \bar{D} = 0 \) is

\[
u_{c,k}(t) = -K_{c,k} x(t) + E_{c,k} r(t),
\]

(38)

which results in the closed-loop system

\[
\dot{x}(t) = (\bar{A}_k - \bar{B}_k K_{c,k}) x(t) + \bar{B}_k E_{c,k} r(t),
\]

(39)

where the analog state-feedback gain \( K_{c,k} \in \mathbb{R}^{m \times n} \), the forward gain \( E_{c,k} \in \mathbb{R}^{m \times m} \) are given by

\[
K_{c,k} = R^{-1} \bar{B}_k^T S, \\
E_{c,k} = -R^{-1} \bar{B}_k^T [(\bar{A}_k - \bar{B}_k K_{c,k})^{-1}]^T \bar{C}^T Q,
\]

(40)

and \( S \) is the symmetric solution of Riccati equation as follows:

\[
\bar{A}_k^T S + S \bar{A}_k - S \bar{B}_k R^{-1} \bar{B}_k^T S + \bar{C}^T Q \bar{C} = 0.
\]

(41)

When the state \( x(t) \) in (38) is not available for measurement, we need to construct an observer to estimate the state \( x(t) \), denoted by \( \hat{x}(t) \) as

\[
\dot{\hat{x}}(t) = \bar{A}_k \hat{x}(t) + \bar{B}_k u(t) + L_{c,k}(y - \bar{C} \hat{x}(t)),
\]

(42)

where \( L_{c,k} \in \mathbb{R}^{p \times p} \) is the observer gain. We define the state estimation error as

\[
x(t) = x(t) - \hat{x}(t).
\]

(43)

By differentiating (43) and using (36) and (42), we have

\[
\dot{\hat{x}}(t) = (\bar{A}_k - L_{c,k} \bar{C}) \hat{x}(t).
\]

(44)

Comparing (44) and (39) with \( r(t) = 0 \) and utilizing the fact,

\[
(\bar{A}_k - L_{c,k} \bar{C})^T = \bar{A}_k^T - \bar{C}^T L_{c,k}^T,
\]

(45)

we conclude that the observer design is a dual process of the state-feedback design. It means that the same theory developed for designing the analog-feedback gain \( K_{c,k} \) can be used to design the observer gain \( L_{c,k} \) as

\[
L_{c,k} = P_{ok} \bar{C}^T R_o^{-1},
\]

(46)

where \( P_{ok} \) is the symmetric and positive-definite solution of the following Riccati equation:

\[
\bar{A}_k P_{ok} + P_{ok} \bar{A}_k^T - P_{ok} \bar{C}^T R_o^{-1} \bar{C} P_{ok} + Q_o = 0.
\]

(47)
5. Derivation of the observer-based digital tracker for the linear sampled-data system

We consider a controllable and observable optimally linearized model of a nonlinear system, which is described by

\[
\begin{aligned}
\dot{x}(t) &= \begin{bmatrix} A & B \end{bmatrix} x(t) + \begin{bmatrix} 0 & C \end{bmatrix} r(t), \\
y_c(t) &= Cx_c(t),
\end{aligned}
\]  

(48)

where \( x_c(t) \in \mathbb{R}^n, u_{c,k}(t) \in \mathbb{R}^m \) and \( y_c(t) \in \mathbb{R}^p \), and \( \begin{bmatrix} A & B \end{bmatrix}, \begin{bmatrix} 0 & C \end{bmatrix} \) are constant matrices of suitable dimensions. Let the continuous-time state-feedback control law be

\[
u_{c,k}(t) = -K_{c,k} x_c(t) + E_{c,k} r(t),
\]  

(49)

where the feedback gain \( K_{c,k} \in \mathbb{R}^{m \times n} \), the forward gain \( E_{c,k} \in \mathbb{R}^{m \times m} \) have been given or obtained for some tracking objective, and \( r(t) \) is an \( m \times 1 \) reference input vector. The controlled system is

\[
\dot{x}_c(t) = \begin{bmatrix} A & B \end{bmatrix} x_c(t) + \begin{bmatrix} 0 & C \end{bmatrix} r(t), \quad x_c(0) = x_0,
\]  

(50)

where \( \begin{bmatrix} A & B \end{bmatrix} = \begin{bmatrix} A_{c,k} & B_{c,k} \end{bmatrix} \). The configuration of the analogously controlled system is shown in Fig. 5.

Let the corresponding state equation of the sampled-data system be described by

\[
\dot{x}_{d,k}(t) = \begin{bmatrix} A_{c,k} & B_{c,k} \end{bmatrix} x_{d,k}(t) + \begin{bmatrix} 0 & C \end{bmatrix} r(t), \quad x_{d,k}(0) = x_0,
\]  

(51)

where \( u_{d,k}(t) \in \mathbb{R}^m \) is a piecewise-constant input vector, satisfying \( u_{d,k}(t) = u_{d,k}(kT) \), for \( kT \leq t < (k+1)T \) and \( T > 0 \) is the sampling period. Let the discrete-time state-feedback control law be

\[
u_{d,k}(kT) = -K_{d,k} x_{d,k}(kT) + E_{d,k} r^*(kT),
\]  

(52)

where \( E_{d,k} \in \mathbb{R}^{m \times m} \) is digital forward gain, \( K_{d,k} \in \mathbb{R}^{m \times n} \) is a digital state-feedback gain, and \( r^*(kT) \in \mathbb{R}^m \) is a piecewise-constant reference input vector to be determined in terms of \( r(kT) \) for tracking purpose. The digitally controlled closed-loop system thus becomes

\[
\dot{x}_{d,k}(t) = \begin{bmatrix} A_{c,k} & B_{c,k} \end{bmatrix} x_{d,k}(t) + \begin{bmatrix} 0 & C \end{bmatrix} r^*(kT), \quad x_{d,k}(0) = x_0,
\]  

(53)

for \( kT \leq t < (k+1)T \), where a zero-order-hold device is used. The digital redesign problem is reduced to finding digital controller gains \( K_{d,k}, E_{d,k} \) in (52) from analog gains \( K_{c,k}, E_{c,k} \) in (49), so that the closed-loop state \( x_{d,k}(t) \) in (53) can closely match the closed-loop state \( x_c(t) \) in (50) at all the sampling instants for a given \( r(t) = r(kT) \), \( k = 0, 1, 2, \ldots \).

The solution \( x_c(t) \) of (50) at \( t = t_v = kT + vT \) for \( 0 \leq v \leq 1 \) is found to be

\[
x_c(t_v) = \exp(\begin{bmatrix} A_{c,k} & B_{c,k} \end{bmatrix} (kT + vT)) x_c(kT) + \int_{kT}^{kT + vT} \exp(\begin{bmatrix} A_{c,k} & B_{c,k} \end{bmatrix} (\tau)) \begin{bmatrix} 0 & C \end{bmatrix} r(\tau) d\tau.
\]  

Let \( u_{c,k}(t_v) \) be a piecewise-constant input. Then, Eq. (50) reduces to

\[
x_c(t_v) \approx \exp(\begin{bmatrix} A_{c,k} vT \end{bmatrix} x_c(kT)) + \int_{kT}^{kT + vT} \exp(\begin{bmatrix} A_{c,k} (kT + vT - \tau) \end{bmatrix} \begin{bmatrix} 0 & C \end{bmatrix} u_{c,k}(\tau)) d\tau u_{c,k}(t_v)
\]  

\[
= G^{(v)}_k x_c(kT) + H_k^{(v)} u_{c,k}(t_v),
\]  

(55)
where
\[ H_k^{(v)} = \int_{t_v}^{t_v + vT} \exp(\bar{A}_k(t_v - \tau)) \bar{B}_k d\tau = \int_0^T \exp(\bar{A}_k \tau) \bar{B}_k d\tau = [G_k^{(v)} - I_n] \bar{A}_k^{-1} \bar{B}_k, \]
\[ G_k^{(v)} = \exp(\bar{A}_k(t_v - KT)) = \exp(\bar{A}_kvT) = (\exp(\bar{A}_kT))^v = (G_k)^v. \]

It is noted that \([G_k^{(v)} - I_n] \bar{A}_k^{-1}\) is a shorthand notation, which is well-defined and can be proved by a cancellation of \(\bar{A}_k^{-1}\) in the series expansion of the term \([G_k^{(v)} - I_n]\). This convenient notation will be exercised throughout the paper. Also, the state \(x_d(t)\) of (53) at \(t = t_v = kT + vT\) for \(0 \leq v \leq 1\) is obtained as
\[ x_d(t_v) = \exp(\bar{A}_k(t_v - kT))x_d(kT) + \int_{kT}^{t_v} \exp(\bar{A}_k(t_v - \tau)) \bar{B}_k d\tau u_{d,k}(kT) \]
\[ = G_k^{(v)} x_d(kT) + H_k^{(v)} u_{d,k}(kT), \] (56)
thus, from (51) and (52) it follows that to obtain the state \(x_d(t_v) = x_d(t_0)\) under the assumption of \(x_c(kT) = x_d(kT)\), it is essential to have \(u_{d,k}(kT) = u_{c,k}(t_0)\). This brings about the prediction-based digital controller:
\[ u_{d,k}(kT) = u_{c,k}(t_v) = -K_{c,k}x_c(t_v) + E_{c,k}r(t_v) = -K_{c,k}x_d(t_v) + E_{c,k}r(t_v). \] (57)
where the future state \(x_d(t_v)\) needs to be predicted based on the available causal signals \(x_d(kT)\) and \(u_{d,k}(kT)\).

Substituting the predicted state \(x_d(t_v)\) in (56) into (57), then we have
\[ u_{d,k}(kT) = (I_m + K_{c,k}H_k^{(v)})^{-1} [-K_{c,k}G_k^{(v)} x_d(kT) + E_{c,k}r(t_v)]. \] (58)
For this reason, the desired predicted digital controller (57) is found from (58) to be
\[ u_{d,k}(kT) = -K_{d,k}^{(v)} x_d(kT) + E_{d,k}^{(v)} r^{*}(kT), \] (59)
where, for tracking purpose, \(r^{*}(kT) = r(kT + vT)\) and
\[ K_{d,k}^{(v)} = (I_m + K_{c,k}H_k^{(v)})^{-1} K_{c,k}G_k^{(v)}, \] (60)
\[ E_{d,k}^{(v)} = (I_m + K_{c,k}H_k^{(v)})^{-1} E_{c,k}. \] (61)
For simplicity, we choose \(v = 1\). Thus, we have the discrete-time system and the one-step ahead discrete-time controller as
\[ x_d(kT + T) = G_k x_d(kT) + H_k u_{d,k}(kT), \] (62a)
and
\[ u_{d,k}(kT) = -K_{d,k} x_d(kT) + E_{d,k} r^{*}(kT), \] (62b)
where
\[ G_k = e^{\bar{A}_k T}, \] (63a)
\[ H_k = [G_k - I_n] \bar{A}_k^{-1} \bar{B}_k, \] (63b)
\[ K_{d,k} = (I_m + K_{c,k}H_k)^{-1} K_{c,k}G_k, \] (63c)
\[ E_{d,k} = (I_m + K_{c,k}H_k)^{-1} E_{c,k}, \] (63d)
\[ r^{*}(kT) = r(kT + T). \] (63e)

In selecting an appropriate sampling period for the digital redesign method, a bisection searching method is proposed to find an suitable long sampling period, so that the reasonable tradeoff between the closed-loop response (i.e., the matching of the states \(x_c(kT)\) in (55) and \(x_d(kT)\) in (56)) and the stability of the closed-loop system can be achieved.
Fig. 6. The digitally controlled system.

Fig. 7. The observer-based analog tracker for the continuous-time linear system.

Here, \( r^*(kT) \) is an alternative form of the original reference input \( r(t) \) at time step \( t = kT \) with one-step ahead amplitude \( r(kT + T) \), where \( T \) is the sampling time. The configuration of the digitally controlled system is shown in Fig. 6, where Z.O.H. denotes the zero-order hold. When the state variables \( x_c(t) \) are not available, we have to construct an observer to estimate the unavailable state variables, denoted by \( \hat{x}_c(t) \), i.e. \( x_c(t) \approx \hat{x}_c(t) \). The observer is represented as

\[
\dot{\hat{x}}_c(t) = A_k \hat{x}_c(t) + B_k u_{c,k}(t) + L_{c,k}(y_c(t) - \tilde{C}\hat{x}_c(t)).
\]  

Thus, the original state-feedback controlled system shown in Fig. 5 is now modified to be the observer-based state-feedback controlled system shown in Fig. 7 as

\[
\dot{x}_c(t) = A_k x_c(t) + B_k u_{c,k}(t), \quad x_c(0) = x_0,
\]

\[
y_c(t) = \tilde{C} x_c(t),
\]

\[
u_{c,k}(t) = -K_{c,k} \hat{x}_c(t) + E_{c,k} r(t).
\]

Based on the digitally redesigned controller obtained in (62), we only need the discrete-time state, instead of continuous-time state, for digital implementation of the obtained digital controller. Hence, we need to carry out digital redesign of the analog observer in (64). First, we define the continuous-time and discrete-time state estimate errors, respectively, as

\[
\tilde{x}_c(t) = x_c(t) - \hat{x}_c(t),
\]

\[
\tilde{x}_d(kT) = x_d(kT) - \hat{x}_d(kT).
\]

By using the observer design method discussed in Section 4, we are able to obtain \( \tilde{x}_c(t)|_{t=kT} \approx x_c(t)|_{t=kT} \). Then, by using the digital redesign concept discussed in Section 5, we can design a digital observer from the analog observer in (64) such that \( \tilde{x}_d(kT) \approx \tilde{x}_c(t)|_{t=kT} \). As a result, we have \( \tilde{x}_d(kT) \approx \tilde{x}_c(t)|_{t=kT} \approx x_c(t)|_{t=kT} \). The digitally redesigned observer can be determined using the dual concept of the digitally redesigned controller as follows.

Using the duality once again, one can find the discrete-time state estimation error dynamics of (44) from (62) as follows

\[
\tilde{x}_d(kT + T) = (G_k - M_k N_k)\tilde{x}_d(kT),
\]
Fig. 8. The practically implementable observer-based tracker for the sampled-data linear system.

where
\[ G_k = e^{A_k T}, \]  
\[ M_k = (G_k - I)A_k^{-1} L_{c,k}, \]  
\[ N_k = (I + C M_k)^{-1} C G_k. \]  

Further defining \( L_{d,k} = M_k (I + C M_k)^{-1} \), one can write \( M_k N_k = L_{d,k} C G_k \) and with the substitution of (66) into (67), it becomes

\[ x_d(kT + T) - \hat{x}_d(kT + T) = (G_k - L_{d,k} C G_k)[x_d(kT) - \hat{x}_d(kT)]. \]  

By substituting the following identities into (69)

\[ x_d(kT + T) = G_k x_d(kT) + H_k u_{d,k}(kT), \]
\[ y_d(kT) = C x_d(kT), \]
\[ C G_k x_d(kT) = C x_d(kT + T) - C H_k u_{d,k}(kT) = y_d(kT + T) - C H_k u_{d,k}(kT), \]

and solving the result for \( \hat{x}_d(kT) \), one obtains the digitally redesigned observer for system (42)

\[ \hat{x}_d(kT + T) = G_{d,k} \hat{x}_d(kT) + H_{d,k} u_{d,k}(kT) + L_{d,k} y_d(kT + T), \]

or

\[ \hat{x}_d(kT) = G_{d,k} \hat{x}_d(kT - T) + H_{d,k} u_{d,k}(kT - T) + L_{d,k} y_d(kT), \]

where

\[ L_{d,k} = (G_k - I)A_k^{-1} L_{c,k}[I + C (G_k - I)A_k^{-1} L_{c,k}]^{-1}, \]
\[ G_{d,k} = G_k - L_{d,k} C G_k, \]
\[ H_{d,k} = H_k - L_{d,k} C H_k, \]

with \( G_k = e^{A_k T} \) and \( H_k = (G_k - I)A_k^{-1} B_k \). Then, the practically implementable observer-based tracker for the sampled-data linear system is shown in Fig. 8.

6. A linear conditioning technique for hybrid chaotic systems

The windup phenomenon is usually caused by the mismatch between process input and controller output. During the time when the actuator is saturated, anti-windup method is often utilized to reduce the output of controller, or to modify the controller states, so that the saturating inputs can quickly return to the unsaturating inputs. However, majority of the developed anti-windup controllers (AWCs) are in analog settings, which are not suitable for implementation. For practical applications, it is often required to convert the analog AWC into a digital AWC, which preserves the functions of the analog AWC. The typical waveforms of digitally redesigned controller \( u_{d,k}(kT) \) and
original analog controller $u_{c,k}(t)$ are shown in Fig. 9. It is observed that the amplitude of the piecewise-constant digital controller $u_{d,k}(kT)$ is often smaller than the maximal amplitude of the analog controller $u_{c,k}(t)$ in each sampling period $T$. Nevertheless, in some sampled periods, the digitally redesigned controller $u_{d,k}(kT)$ may still exceed the bound of the actuator. Therefore, we still need an AWC to get over the windup phenomenon.

The designed controller with unconstrained states may cause actuator saturation and performance deterioration [24]. This can be overcome by modifying the dynamics of the internal states of the controller when the actuator is saturated. The structure of the proposed observer-based digitally redesigned dual-rate control scheme for hybrid linear system is shown in Fig. 10. In the proposed scheme, we develop a theoretical analog observer with the high-gain property to accurately and quickly estimate the states of the linear system of interest. Then, a low-gain digital observer is developed based on the dual concept of the proposed prediction-based digital redesign method for the tracker design. When the digitally redesigned low-gain tracker together with the digitally redesigned low-gain observer is still over the constrained bound of the actuator, we add a fast-rate inner-state compensator at the inputs of the linear system to deal with the windup phenomenon. The linear conditioning scheme is described as follows.

The fast-rate sampling period is defined as

$$ T_f = T/N, $$

where $T_f$ is the fast sampling period and $N$ is an integer. Also, we define $1 \leq k_f \leq N$. From Fig. 10, the model of the fast-rate inner-state compensator is represented as

$$ u_{df}(k_f T_f) = u_{dlin}(kT) - \tilde{u}_{df}(k_f T_f), $$

$$ u_{dlin}(kT) = u_{d,k}(kT) = -K_{d,k} \hat{x}_d(kT) + E_{d,k} r^*(kT), $$

$$ \tilde{x}_{df}(k_f T_f + T_f) = G_{f,k_f} \tilde{x}_{df}(k_f T_f) + H_{f,k_f} \tilde{u}_d(k_f T_f), $$

where $G_{f,k_f} = e^{A_k T_f}$ and $H_{f,k_f} = [G_{f,k_f} - I_n] \tilde{A}_k^{-1} \tilde{B}_k$.

The mechanism of the linear conditioning scheme can be described in the following.

From Fig. 10, we observe that when the amplitude of the piecewise-constant signal $u_{dlin}(kT)$ of the state-feedback control signal $u_{d,k}(kT)$ is within the linear range of the limiter, or the actuator signal $\tilde{u}_d(t)$ is not saturated, the error signal, $\tilde{u}_{df}(k_f T_f) = u_{df}(k_f T_f) - \tilde{u}_d(k_f T_f)$, which is produced by the limiter, becomes the input of the fast-rate inner-state compensator in (74) and its value is zero. As a result, the fast-rate inner-state compensator in (74) with a zero-initial state and a zero-input signal will not be activated. Hence, $u_{dlin}(k_f T_f) = u_{df}(k_f T_f) = \tilde{u}_d(k_f T_f)$.

When the amplitude of $u_{dlin}(kT)$ is out of the linear range of the limiter, the error signal $\tilde{u}_d(k_f T_f)$ becomes non-zero and activates the fast-rate inner-state compensator to produce a fast-rate output signal $\tilde{u}_{df}(k_f T_f)$, which acts as a disturbance rejection signal to reduce the amplitude of $u_{dlin}(kT)$ until $\tilde{u}_d(k_f T_f) = 0$. As soon as $\tilde{u}_d(k_f T_f) = 0$, the actuator is no more saturated and the bump-transfer effects have been reduced. It is noticed that the initial state of the fast-rate inner-state compensator should be set to zero when $\tilde{u}_d(k_f T_f) = 0$ to avoid the production of the unnecessary disturbance rejection signal $\tilde{u}_{df}(k_f T_f)$. The process of reducing the bump-transfer effects should be completed within a slow sampling period $T$. The development of the conditioning bumpless-transfer scheme can be derived as follows.
Following the digital redesign technique shown in Section 5, we get the fast-rate digital controller as

\[
\tilde{u}_d(k_f T_f) = u_{df}(k_f T_f) - \bar{u}_d(k_f T_f),
\]

and

\[
\tilde{u}_{df}(k_f T_f) = K_{df,f} \tilde{x}_{df}(k_f T_f),
\]

where \(K_{df,f} \in \mathbb{R}^{m \times n}\) is obtained based on the fast-rate digital redesign scheme. The fast-rate digital redesign gain has the same formula as shown in (48) except that the fast-rate sampling period \(T_f\) should be utilized instead of the slow-rate sampling period \(T\). The fast-rate control gain in (76) becomes

\[
K_{df,f} = (I_m + K_{c,k} H_{f,k})^{-1} K_{c,k} G_{f,k}.
\]

From (72)–(76), we have

\[
\tilde{x}_{df}(k_f T_f + T_f) = G_{f,k_f} \tilde{x}_{df}(k_f T_f) + H_{f,k_f} [u_{df}(k_f T_f) - \bar{u}_d(k_f T_f)]
\]

\[
= G_{f,k_f} \tilde{x}_{df}(k_f T_f) + H_{f,k_f} [u_{dlin}(k_f T_f) - \bar{u}_d(k_f T_f) - \bar{u}_d(k_f T_f)]
\]

\[
= [G_{f,f} - H_{f,k_f} K_{df,f,k_f}] \tilde{x}_{df}(k_f T_f) + H_{f,k_f} \tilde{r}(k_f T_f),
\]

where \(\tilde{r}(k_f T_f)\) is the reference input of the inner-state compensator and can be written as

\[
\tilde{r}(k_f T_f) = u_{dlin}(k_f T_f) - \bar{u}_d(k_f T_f).
\]

It is noted that \(u_{dlin}(kT) = u_{dlin}(k_f T_f)\).

The inner-state compensator can be considered as a fictional sub-system shown in Fig. 6. This sub-system can be described as follows:

\[
\tilde{x}_{df}(k_f T_f + T_f) = G_{f,k_f} \tilde{x}_{df}(k_f T_f) + H_{f,k_f} \tilde{u}_d(k_f T_f),
\]

\[
\tilde{u}_d(k_f T_f) = -K_{df,f} \tilde{x}_{df}(k_f T_f) + E_{df,f,k} \tilde{r}(k_f T_f),
\]
where the control law $\tilde{u}_d(k_f T_f)$ with $E_{df,k_f} = I$ can be determined by

$$
\tilde{u}_d(k_f T_f) = u_{df}(k_f T_f) - \bar{u}_d(k_f T_f)
= u_{dlin}(k_f T_f) - K_{df,k_f} \tilde{x}_{df}(k_f T_f) - \bar{u}_d(k_f T_f)
= -K_{df,k_f} \tilde{x}_{df}(k_f T_f) + \tilde{r}(k_f T_f).
$$

The desired control law $\tilde{u}_d(k_f T_f)$ in (80) can be obtained from this sub-system (79), which can be considered as a disturbance rejection filter. So $\tilde{u}_d(k_f T_f)$ can be considered as an amplitude reduction signal when the saturation occurs. It systematically reduces the amplitude of $u_{d,k}(kT)$ to the linear region of the actuator to avoid saturation.

The decision-making logic (DML) shown in Fig. 10 resets the initial state of the inner-state compensator to null whenever $u_{dlin}(kT)$ is functioning under the linear situation so that the inner-state compensator will not be activated.

The feedback gain $K_{df,k_f} \in R^{m\times n}$ in (80) of the sub-system in (79) can be designed such that $\tilde{u}_d(k_f T_f)$ will track the reference input, i.e.,

$$D_{ic}(u_{dlin}(k_f T_f) - \bar{u}_d(k_f T_f)) = D_{ic}\tilde{r}(k_f T_f),$$

in a few steps (dead-beat type). The constant gain $D_{ic}$ can be determined at the steady state as follows:

From (78a), we have

$$\tilde{x}_{df}(k_f T_f + T_f)|_{\text{steady state}} = \tilde{x}_{df}(k_f T_f)$$

$$= [G_{f,k_f} - H_{f,k_f} K_{df,k_f}] \tilde{x}_{df}(k_f T_f) + H_{f,k_f} \tilde{r}(k_f T_f),$$

hence,

$$\tilde{x}_{df}(k_f T_f) = [I - (G_{f,k_f} - H_{f,k_f} K_{df,k_f})]^{-1} H_{f,k_f} \tilde{r}(k_f T_f).$$

Also, from (76), we get

$$\tilde{u}_d(k_f T_f) = K_{df,k_f} \tilde{x}_{df}(k_f T_f)$$

$$= K_{df,k_f}[I - G_{f,k_f} + H_{f,k_f} K_{df,k_f}]^{-1} H_{f,k_f} \tilde{r}(k_f T_f)$$

$$= D_{ic}\tilde{r}(k_f T_f).$$

Hence, from the above equations the desired constant gain $D_{ic}$ becomes

$$D_{ic} = K_{df,k_f}[I - G_{f,k_f} + H_{f,k_f} K_{df,k_f}]^{-1} H_{f,k_f}. \quad (85)$$

For the constant gain $D_{ic}$ in (85), we observe that if a dead-beat type controller is designed, then $[I - (G_{f,k_f} - H_{f,k_f} K_{df,k_f})]^{-1} \cong I$ and $D_{ic} \cong K_{df,k_f} H_{f,k_f}$. Hence, $D_{ic}$ is minimal. Also, it is observed that when the controller $u_{dlin}(kT)$ is back to the linear region of actuator saturation, the stability of the designed system is insured because the state $\tilde{x}_d(kT)$ for the controller $u_{d,k}(kT)$ in (52) can be evaluated from the discretized model, $\tilde{x}_d(kT + T) = G_k \tilde{x}_d(kT) + H_k \bar{u}_{d,k}(kT)$.

The overall configuration of the digitally redesigned sampled-data system is shown in Fig. 11.

7. The CEPA tracker scheme for uncertain nonlinear time-invariant systems

The proposed minimal-maximal principle of CEPA can be utilized for finding the practically implementable “best” nominal tracker and the corresponding worst case of the system. In the proposed processes, we design the best tracker based on the minimal principle. This process is called “design level”. Meanwhile, we would search the “worst” system in the pool based on the maximal principle. This process is called “test level”. Both processes are summarized and described as follows.

Level I: Design level — design the tracker.

1. Generate a $p$-dimensional initial population $P$ of size $N$, denoted by $IP = \{P_{d,0,i} : i = 1, 2, \ldots, N\}$, and a spare population of size $N$, denoted by $SP = \{P_{d,0,i} : i = N + 1, N + 2, \ldots, 2N\}$. Here, the index 0 is the initial generation index $g = 0$ and $d$ indicates that the quantity is at the design level. This task is done by using QRS to initialize each individual $P_{d,0,i} \in IP \cup SP$, for $i = 1, \ldots, N, N + 1, \ldots, 2N$. 

Fig. 11. The observer-based digitally redesigned tracker of linear conditioning with inner-state compensator for the hybrid chaotic system.

(2) Search for the worst case system parameters for each selected nominal tracker based on the maximal principle at the test level (see Level II below). Use the proposed optimal linearization formulas (35), in which the realized tracker of each individual \( P_{d,0,i} \) is constructed based on (63c) and (63d).

(3) Assign to each \( P_{d,0,i} \) an objective function (OF) score:

\[
\max J(k) = \text{OF}(P_{t,g,i';}^{*}; P_{d,g,i'}, K_{d,g,i'}^*)
\]

where the index \( t \) indicates that the quantity is at the test level. This OF can be the one defined in (20). By going through the test level, we can find the above maximal objective function value.

(4) Receive the message from the test level about the nominal tracker

\[
(K_{c,k}(\hat{x}_d(kT)), E_{c,k}(\hat{x}_d(kT)), K_{d,k}(\hat{x}_d(kT)), E_{d,k}(\hat{x}_d(kT)))
\]

to see if it satisfies the stability requirement. If not, this matrix has to be replaced by one from the spare population \( SP \), until stability is achieved.

(5) Calculate Fitness and Probability functions and apply the minimal principle operator of the EPA to mutate a new population of higher quality.

(6) Carry out the chaotic search.

(7) Go to Step (2) at this level and repeat the steps, until the maximal value of \( \max J(k) \) is reached. This resulting stage will provide the associated “best” innovation tracker

\[
(K_{c,k}(\hat{x}_d(kT)), E_{c,k}(\hat{x}_d(kT)), K_{d,k}(\hat{x}_d(kT)), E_{d,k}(\hat{x}_d(kT))).
\]

Terminate the process at this stage, if the corresponding min–max \( \max J(k) \) cannot be further improved or the allowable tolerance is met.

Level II: Test level — test the designed tracker.

(1) Generate a \( \rho \)-dimensional initial population of size \( N \), by using QRS to initialize each individual \( P_{t,g',i'} \in S \), for \( i' = 1, 2, \ldots, N \).

(2) Assign to each \( P_{t,g',i'} \in S \), an objective function (OF) score. This OF can be the one defined in (20). If some OF score is much higher than others, it means the tracker being tested is infeasible. In this case, the stability is consequently not guaranteed, so send a message to Level I about this situation and then terminate the process at this level; otherwise, continue the process.

(3) Calculate Fitness and Probability functions and apply the maximal principle operator of the EP to mutate a new population of higher quality.

(4) Carry out the chaotic search.

(5) Go to Step (2) at this level and repeat the steps, until the maximal value of \( J(k) \) is reached. This resulting stage will provide the max \( J(k) \) under the realization of the interval system in terms of \( P_{t,g',i'}^{*} \), which cannot be further improved.
Fig. 12. The attractor of the Chen’s chaotic system, plotted in the $x_3$–$x_1$–$x_2$ space.

(6) Inform Level I about the finding of an individual with the highest quality at this level, $P^*_t g' i'$ (which is actually the worst case of estimation error and will be minimized at Level I, as discussed above).

8. An illustrative example

We utilize the recently discovered chaotic attractor \[11,12\] as an illustrative example:

$$\begin{align*}
\dot{x}_1(t) &= a(x_2(t) - x_1(t)), \\
\dot{x}_2(t) &= (c - a)x_1(t) - x_1(t)x_3(t) + cx_2(t), \\
\dot{x}_3(t) &= x_1(t)x_2(t) - bx_3(t),
\end{align*}$$

(87)

where the nominal parameters are given as $a = 35$, $b = 3$ and $c = 28$. It has been verified that this chaotic system is not topologically equivalent to the familiar Lorenz system \[12\] and is known to be more complex dynamically. Represent system (87) by a simple state equation,

$$\dot{x}_{\text{chaos}}(t) = f(x_{\text{chaos}}(t)), \quad x_{\text{chaos}}(t) \in \mathbb{R}^3.$$  

In our illustrative example, we set the fast sampling period to be $T_f = 0.005$ s, the slow sampling period to be $T = 0.01$ s, the final simulation time to be $T_{\text{final}} = 60$ s, and the initial conditions to be $x_1(0) = -10$, $x_2(0) = 0$, $x_3(0) = 37$. In general, a suitable compromise between the pre-specified performance and the selections of the sampling time $T$ and $T_f$ should be considered. The bisection search technique or other sophisticated search techniques such as the genetic algorithm and evolutionary programming can be utilized for this goal. Besides, some results for selecting the appropriate sampling period can also be referred to in \[25\]. The output simulation is shown in Fig. 12.

Step 1: Specify the target reference $r(t)$.

Now, we select the target reference signal $r(t)$ as a periodic orbit embedded within the attractor of the Chen’s chaotic system. Then, we collect those data $y(hT_f)$ for fast sampled periods $h = 848, \ldots, 991$. To obtain a closed orbit, we smooth the connection between the starting point and the ending point via a simple linear interpolation:

$$y((991 + i)T_f) = y(991T_f) + \frac{i}{N}[y(848T_f) - y(991T_f)],$$

where $i = 1, 2, \ldots, N - 1$ ($N = 4$ in our simulation). Thus, for the system output to travel through this closed orbit $r(hT_f)$, it takes $(991 - 848 + N) \times T_f = 0.735$ s. Then, repeat the desired $r(hT_f)$ periodic curve
cycle-by-cycle to fit the final simulation time $T_{\text{final}}$. The two-dimensional phase portrait of the reference $r(t)$ is given in Fig. 13.

Step 2. Construct the optimally linearized model for the chaotic system by using the estimated states $\hat{x}_d(kT)$.

Now based on the optimal linearization approach shown in Section 3, we have the linear model of observer-based sampled-data chaotic system at the sampling time $t = kT$, for $k = 0, 1, 2 \ldots$, as

$$\dot{x}_e(t) = \overline{A}_k x_e(t) + \overline{B}_k u_{e,k}(t), \quad x_e(0) = x_0,$$

$$y_e(t) = \overline{C} x_e(t),$$

Here, we set $\overline{B}_k = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $\overline{C} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

$$\overline{A}_k = \begin{bmatrix}
-a & \frac{\hat{x}_{d,1}(k)\hat{x}_{d,3}(k)}{\|\hat{x}_d(k)\|^2_2} & \frac{a}{\|\hat{x}_d(k)\|^2_2} & 0 \\
(c - a) - \hat{x}_{d,3}(k) & \frac{\hat{x}_{d,2}(k)\hat{x}_{d,3}(k)}{\|\hat{x}_d(k)\|^2_2} & \frac{\hat{x}_{d,1}(k)\hat{x}_{d,2}(k)}{\|\hat{x}_d(k)\|^2_2} & \frac{c + \hat{x}_{d,1}(k)\hat{x}_{d,3}(k)}{\|\hat{x}_d(k)\|^2_2} \\
\hat{x}_{d,2}(k) - \frac{\hat{x}_{d,1}(k)\hat{x}_{d,2}(k)}{\|\hat{x}_d(k)\|^2_2} & \frac{\hat{x}_{d,1}(k)\hat{x}_{d,2}(k)}{\|\hat{x}_d(k)\|^2_2} & \hat{x}_{d,1}(k) - \frac{\hat{x}_{d,1}(k)\hat{x}_{d,3}(k)}{\|\hat{x}_d(k)\|^2_2} & \frac{-\hat{x}_{d,1}(k)\hat{x}_{d,3}(k)}{\|\hat{x}_d(k)\|^2_2} \\
\hat{x}_{d,1}(k) & \frac{\hat{x}_{d,1}(k)\hat{x}_{d,2}(k)}{\|\hat{x}_d(k)\|^2_2} & \frac{\hat{x}_{d,1}(k)\hat{x}_{d,3}(k)}{\|\hat{x}_d(k)\|^2_2} & \frac{-\hat{x}_{d,1}(k)\hat{x}_{d,3}(k)}{\|\hat{x}_d(k)\|^2_2}
\end{bmatrix}.$$
where $\hat{x}_d(k) = [\hat{x}_{d,1}(k), \hat{x}_{d,2}(k), \hat{x}_{d,3}(k)]^T$ is the digitally redesigned observer state of the analogously designed observer state $\hat{x}_c(t)$. Note that the notation “$\hat{x}_d(k)$” represents “$\hat{x}_d(kT)$” for simplicity, which should not cause any confusion.

**Step 3.** Determine the analog optimal tracker based on the above linearization model

Let the analog optimal tracker for the linear model at the sampling time $t = kT$ be given by

$$u_{c,k}(t) = -K_{c,k}x_c(t) + E_{c,k}r(t),$$

(90)

where

$$K_{c,k} = R^{-1}B_k^TP_k,$$  
(91a)

$$E_{c,k} = -R^{-1}B_k^T((A_k - B_kK_{c,k})^{-1})^T C^T Q,$$  
(91b)

and $P_k$ is the solution of the following matrix Riccati equation:

$$A_k^TP_k + P_kA_k - P_kB_kR^{-1}B_k^TP_k + C^TQ\overline{C} = 0.$$  
(92)

Here, we choose $Q = 10^4 I_2$ and $R = I_2$ in the simulation. Notice that the subscript $k$ represents the corresponding values of the observer-based tracker for the sampled-data chaotic system at the sampling time $t = kT$ for simplicity. When the state variables $x_c(t)$ in (90) are not available for measurement, we need to estimate the unavailable state variables, so that $x_c(t) \approx \hat{x}_c(t)$, where $\hat{x}_c(t)$ is the estimate state of $x_c(t)$ from the following analog observer:

$$\dot{\hat{x}}_c(t) = \overline{A}_k\hat{x}_c(t) + \overline{B}_ku_{c,k}(t) + L_{c,k}(y_c(t) - \overline{C}\hat{x}_c(t)).$$  
(93)
Fig. 15. The time series of input control signals for constrained systems with the inner-state compensator. (a) The first input control signal. (b) The second input control signal.

Utilizing the dual concept of the controller design method yields the observer gain $L_{ck}$ as

$$L_{ck} = P_{ok} C^T R^{-1},$$

where $P_{ok}$ is the solution of the following matrix Riccati equation:

$$\overline{A}_k P_{ok} + P_{ok} \overline{A}_k^T - P_{ok} C^T R^{-1} C P_{ok} + Q_o = 0,$$

in which $Q_o = 10^4 I_3$ and $R_o = I_2$.

**Step 4.** Perform the digital redesign for the observer-based optimal tracker.

According to the estimated states and gain matrices (48) and (56), we can find $E_{d,k}, L_{d,k}, G_{d,k}, K_{d,k}$ and $H_{d,k}$. Notice that the exact linear model $\{\overline{A}_k, \overline{B}_k, K_{c,k}, E_{c,k}, L_{c,k}, G_k, H_k\}$ at the $k$th operating point should be constructed based upon the on-line observer-based sampled-data chaotic system state $\hat{x}_d(kT)$ to have the on-line parameterization of the digitally redesigned feedback and feed-forward gains $\{K_{d,k}, E_{d,k}\}$. The three-dimensional phase portraits of $\hat{x}_c(t)$ and $\hat{x}_d(t)$ are shown in Fig. 14.

**Step 5.** Check whether if there is an input saturation during each sampling time.

Let all inputs of plant be constrained between 60% of the maximum inputs and the minimum inputs. Fig. 15 shows the process input signals for the constrained systems with the inner-state compensator. It can be clearly seen that the
inner-state compensator indeed changes the internal state of the controller. It makes the controller output quickly leave the saturated region and thus the controller works in a linear region and the constrained system with the inner-state compensators can follow the desired trajectory.

Step 6. Extend the afore-mentioned steps developed for a nominal chaotic system to an interval chaotic system.

The Chen’s chaotic system with interval parameters is given as:

\[
\begin{align*}
\dot{x}_1(t) &= a^I (x_2(t) - x_1(t)), \quad (96a) \\
\dot{x}_2(t) &= (c^I - a^I)x_1(t) - x_1(t)x_3(t) + c^I x_2(t), \quad (96b) \\
\dot{x}_3(t) &= x_1(t)x_2(t) - b^I x_3(t), \quad (96c)
\end{align*}
\]

where \( a^I = [31.5 \quad 38.5] \), \( b^I = [2.7 \quad 3.3] \) and \( c^I = [25.2 \quad 30.8] \). It is observed that the variations of the interval parameters are chosen as 10% of the nominal parameters in the Chen’s chaotic system shown in (87). Employing the afore-mentioned steps and the CEPA shown in Section 7, we can obtain the “best” tracker via the set of chaotic system parameters as \( [a, b, c] = [36.1063, 3.3000, 29.1976] \) and the “worst” system parameters as \( [a, b, c] = [31.8281, 2.8185, 29.5008] \). Both of them were found by the CEPA optimization process. Besides, it is well-known that the high-gain controller and observer can suppress system uncertainties such as nonlinear
perturbations, parameter variations, modeling errors and external disturbances. For this reason, the sub-optimal controller and observer with a high-gain property is adopted in our approach. The high-gain property can be obtained by choosing a sufficiently high ratio of $Q$ to $R$ in (37) so that the system output can closely track the reference input robust. The two-dimensional phase portraits of $r(t)$ and $y(t)$ are shown in Fig. 16. It can be clearly seen that the observed-based tracker can accurately follow the trajectories of reference signal $r(t)$.

9. Conclusion

A dual-rate observer-based digital tracker for a hybrid interval chaotic system with saturating inputs, by using the newly proposed CEPA is presented. First, we transform a chaotic system into an optimal linearization model, which has the exact dynamics of the original system at the operating points of interest and with minimal modeling errors in the vicinity of those operating points. Then, we develop a digitally redesigned low-gain tracker/observer with the high-gain property to accurately track/estimate the trajectory/states of the given system. The design of the digital tracker/observer is based on the prediction-based digital redesign technique that provides a predication property. Additionally, we deal with the windup phenomenon by adding a fast-rate inner-state compensator. Finally, we use the minimal–maximal principle of CEPA to find the “best” nominal digital tracker/observer for the hybrid interval chaotic systems with saturating inputs.

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