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Hawking radiation of black holes in the $z = 4$ Horava–Lifshitz gravity

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ABSTRACT

We investigate the Hawking radiation of $(3+1)$- and $(4+1)$-dimensional black holes in the $z = 4$ Horava–Lifshitz gravity with fermion tunnelling. It turns out that the Hawking temperatures are recovered and are in consistence with those obtained by calculating surface gravity of the black holes. For the $(3+1)$-dimensional black holes, the Hawking temperatures are related to the fundamental parameters of Horava–Lifshitz gravity.

1. Introduction

Motivated by the Lifshitz theory, a different field theory model for a UV complete theory of gravity was recently proposed by Horava [1–4]. This model is based on a scaling symmetry which treats space and time differently. It has the same dynamical degrees of freedom as in general relativity, but without the full diffeomorphism invariance. The power counting renormalizability of this theory indicates that fundamental theory could be specified by a finite number of UV parameters which are in principle measurable in the IR. In this theory, the divergence of the speed of light in the UV may resolve the horizon and flatness problems. Moreover, this model is helpful to solve some singularity problems, for instance, black hole singularity and cosmological singularity. Since it was proposed, this Horava–Lifshitz model has attracted considerable attention. In the literature, the properties of Horava–Lifshitz gravity have been extensively studied. The black hole solutions in the $z = 3$ and $z = 4$ cases were obtained in [5–10] and [11], respectively. The cosmology solutions of the theory were addressed in [12–17] and cosmological perturbation in the theory was discussed in [18–21]. The properties of black hole solutions were researched in [22–27] and other properties of Horava–Lifshitz gravity were investigated in [28–36].

The research on the $z = 3$ Horava–Lifshitz gravity has been carried out thoroughly. In [11] and the references therein, it is believed that the new massive gravity can be used to construct a $z = 4$ Horava–Lifshitz gravity and the spectral dimension calculated using the numerical Causal Dynamical Triangulations approach for quantum gravity in $3 + 1$ dimensions prefers the $z = 4$ Horava–Lifshitz gravity in the UV. With this reason, we investigate the thermodynamic properties of $(3 + 1)$-dimensional and $(4 + 1)$-dimensional black holes in the $z = 4$ Horava–Lifshitz gravity by fermion tunnelling in this Letter. Recently the Hawking radiation of general black holes has been studied [37–59]. In Refs. [37,38], the Hawking radiation of has been researched by Parikh–Wilczek's tunnelling method and anomaly cancellation method, respectively. In Ref. [43], the Hawking radiation has been investigated by Hamilton–Jacobi method. The black hole radiates not only scalar particles but also Dirac particles. Subsequently the tunnelling effect of spin 1/2 particles in the static spacetime was first studied in Ref. [47] by Hamilton–Jacobi method, and this work was extended to four-dimensional spacetime and higher-dimensional spacetime [48–59].

This Letter is organized as follows. In Section 2, the solution of $(3+1)$-dimensional black holes in the $z = 4$ Horava–Lifshitz gravity is reviewed and the Hawking radiation of the black holes is discussed with fermion tunnelling. In Section 3, the Hawking radiation of fermions in $(4 + 1)$-dimensional black holes in the $z = 4$

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Horava–Lifshitz gravity is studied and the Hawking temperatures are reproduced. Section 4 is devoted to discussions and conclusion.

2. Hawking radiation of $(3+1)$-dimensional black holes in the $z=4$ Horava–Lifshitz gravity

In this section, we investigate the Hawking radiation of $(3+1)$-dimensional black holes in the $z=4$ Horava–Lifshitz gravity theory. We first review the solution of black holes that was obtained by Cai et al. [11]. According to the ADM decomposition of the metric, the $(D+1)$-dimensional metric can be written as

$$ds^2 = -N^2c^2\,dt^2 + g_{ij}\big(dx^i - N^i\,dt\big)\big(dx^j - N^j\,dt\big), \tag{1}$$

where $i=1,\dots,D$ and c is the speed of light. The kinetic term of the action is given by

$$S_K = \frac{2}{\kappa^2}\int dt\,d^Dx\,\sqrt{g}\,N\big(K_{ij}K^{ij} - \lambda K^2\big), \tag{2}$$

where $K_{ij} = \frac{1}{2N}(\partial_t g_{ij} - \nabla_i N_j - \nabla_j N_i)$, κ is the coupling constant and λ is a dimensionless parameter and should be equal to 1 in the IR to restore general relativity here. From the detailed balance principle, the potential term is obtained as

$$S_V = \frac{\kappa^2}{8}\int dt\,d^Dx\,\sqrt{g}\,N E^{ij}\varsigma_{ijkl}E^{kl}, \tag{3}$$

with E^{ij} coming from a D-dimensional relativistic action in the form $E^{ij} = \frac{1}{\sqrt{g}}\frac{\delta W_D[g_{ij}]}{\delta g_{ij}}$, and $\varsigma_{ijkl} = \frac{1}{2}(g_{ik}g_{jl} + g_{il}g_{jk}) - \tilde\lambda g_{ik}g_{jl}$, $\tilde\lambda = \frac{\lambda}{D\lambda - 1}$. Using the kinetic term and potential term, the action of the $(3+1)$-dimensional Horava–Lifshitz theory is expressed as

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_0 + \mathcal{L}_1,\\
\mathcal{L}_0 &= \sqrt{g}\,N\left\{\frac{2}{\kappa^2}\big(K^{ij}K_{ij} - \lambda K^2\big) + \frac{\kappa^2\mu^2(\Lambda_W R - 3\Lambda_W^2)}{8(1-3\lambda)}\right\},\\
\mathcal{L}_1 &= -\sqrt{g}N\frac{\kappa^2}{8}\left\{\frac{4}{\omega^4}C^{ij}C_{ij} - \frac{4\mu}{\omega^2}C^{ij}R_{ij} - \frac{4}{\omega^2 M}C^{ij}L_{ij}\right.\\
&\quad + \mu^2 G_{ij}G^{ij} + \frac{2\mu}{M}G^{ij}L_{ij} + \frac{2\mu}{M}\Lambda_W L + \frac{1}{M^2}L_{ij}L^{ij}\\
&\quad \left. - \tilde\lambda\left(\frac{L^2}{M^2} - \frac{\mu L}{M}(R - 6\Lambda_W) + \frac{\mu^2R^2}{4}\right)\right\},
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
C^{ij} &= \varepsilon^{ikl}\nabla_k\left(R^j_l - \frac{1}{4}R\delta^j_l\right), \qquad G^{ij} = R^{ij} - \frac{1}{2}g^{ij}R,\\
L^{ij} &= (1+2\beta)\big(g^{ij}\nabla^2 - \nabla^i\nabla^j\big)R + \nabla^2 G^{ij}\\
&\quad + 2\beta R\left(R^{ij} - \frac{1}{4}g^{ij}R\right) + 2\left(R^{imjn} - \frac{1}{4}g^{ij}R^{mn}\right)R_{mn},\\
L &\equiv g^{ij}L_{ij} = \left(\frac{3}{2} + 4\beta\right)\nabla^2 R + \frac{1}{2}\beta R^2 + \frac{1}{2}R^{ij}R_{ij},
\end{aligned}$$

μ, ω^2, M and β are coupling constants. To get the explicit black hole solution, the value of β is chosen as $-3/8$ in this section [11]. In order to obtain the general relativity in the IR region, the effective coupling is related to the speed of light c, the Newton coupling G and the effectively cosmological constant Λ as

$$c = \frac{\mu\kappa^2}{4}\sqrt{\frac{\Lambda_W}{1-3\lambda}}, \qquad G_N = \frac{\kappa^2 c}{32\pi}, \qquad \Lambda = \frac{3}{2}\Lambda_W. \tag{5}$$

From the action, the spherically symmetric black hole solution can be obtained as

$$ds^2 = -f(r)c^2\,dt^2 + \frac{1}{f(r)}dr^2 + r^2\,d\Omega_k^2, \tag{6}$$

with $f(r) = k + \frac{\tilde\mu x^2}{2\tilde\beta}\big(1 - \sqrt{1 - \frac{4\tilde\beta}{x^2\tilde\mu^2}(\tilde\mu x^2 - \sqrt{c_0 x})}\big)$, $\tilde\mu = -\mu\Lambda_W$, $\tilde\beta = \frac{\Lambda_W^2}{4M}$, $x = \sqrt{-\Lambda_W}\,r$, $k = 0, \pm 1$, Λ_W is the cosmological constant. The mass of the black hole is expressed as $m = \frac{1}{16}k^2\Omega_k c_0(-\Lambda_W)^{-\frac{3}{2}}$, $c_0 \geqslant 0$ is an integration constant and can be expressed in terms of horizon radius as $c_0 = \frac{1}{x_+}(\frac{\tilde\beta k^2}{x_+^2} + \tilde\mu(k + x_+^2))^2$ and $d\Omega_k^2$ is the line element for a two-dimensional Einstein space with constant scalar curvature $2k$ and volume Ω_k. To have a well-defined vacuum solution, it should let $\frac{4\tilde\beta}{\tilde\mu} \leqslant 1$. The solution is asymptotically AdS_4. When $\tilde\beta$ goes to zero, we can get $f = k + x^2 - \frac{1}{\tilde\mu}\sqrt{c_0 x}$ from the equations of motion given in Ref. [11]. Meanwhile when an appropriate integration constant is chosen and $k=1$, the metric (6) reduces to the solutions given in Refs. [6] and [5], respectively. In the following, we investigate the Hawking radiation of spin 1/2 particles in Horava–Lifshitz gravitation theory. To explore the fermion tunnelling, we introduce Dirac equation

$$i\gamma^\mu(\partial_\mu + \Omega_\mu)\Psi + \frac{m}{\hbar}\Psi = 0, \tag{7}$$

where $\Omega_\mu = \frac{i}{2}\Gamma^{\alpha\beta}_\mu\sum_{\alpha\beta}$, $\sum_{\alpha\beta} = \frac{i}{4}[\gamma^\alpha, \gamma^\beta]$, γ^μ matrices satisfy $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}I$, and m is the mass of the emission fermion. For getting the solution of the Dirac equation, we first choose γ^μ matrices. There are many ways to choose them, and our choice is given as

$$\begin{aligned}
\gamma^t &= \frac{1}{\sqrt{f(r)}}\begin{pmatrix}0 & I\\ -I & 0\end{pmatrix}, \qquad \gamma^\theta = \sqrt{g^{\theta\theta}}\begin{pmatrix}0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix},\\
\gamma^r &= \sqrt{f(r)}\begin{pmatrix}0 & \sigma^3\\ \sigma^3 & 0\end{pmatrix}, \qquad \gamma^\phi = \sqrt{g^{\phi\phi}}\begin{pmatrix}0 & \sigma^2\\ \sigma^2 & 0\end{pmatrix},
\end{aligned} \tag{8}$$

σ^μ is the Pauli sigma matrix. For a spin 1/2 particle, there are two states, which correspond to spin up state and spin down state, respectively. When we measure the spin states along r direction, the spin up case is along r direction and the spin down case has opposite direction. In this Letter, we only investigate the spin up case. The spin down case is similar as that of spin up state and the same result can be gotten [47]. The wave function of the spin up case is given by

$$\psi_{(\uparrow)} = \begin{pmatrix}A(t,r,\theta,\phi)\\ 0\\ B(t,r,\theta,\phi)\\ 0\end{pmatrix}\exp\left(\frac{i}{\hbar}I_\uparrow(t,r,\theta,\phi)\right). \tag{9}$$

Inserting the wave function and γ^μ matrices into the Dirac equation, dividing the exponential term and multiplying by $\hbar$, we can get the resulting equations to leading order in $\hbar$ as

$$\frac{B}{\sqrt{f(r)}}\partial_t I_\uparrow + B\sqrt{f(r)}\partial_r I_\uparrow - mA = 0, \tag{10}$$

$$\frac{-A}{\sqrt{f(r)}}\partial_t I_\uparrow + A\sqrt{f(r)}\partial_r I_\uparrow - mB = 0, \tag{11}$$

$$B\sqrt{g^{\theta\theta}}\,\partial_\theta I_\uparrow + iB\sqrt{g^{\phi\phi}}\,\partial_\phi I_\uparrow = 0, \tag{12}$$

$$A\sqrt{g^{\theta\theta}}\,\partial_\theta I_\uparrow + iA\sqrt{g^{\phi\phi}}\,\partial_\phi I_\uparrow = 0. \tag{13}$$

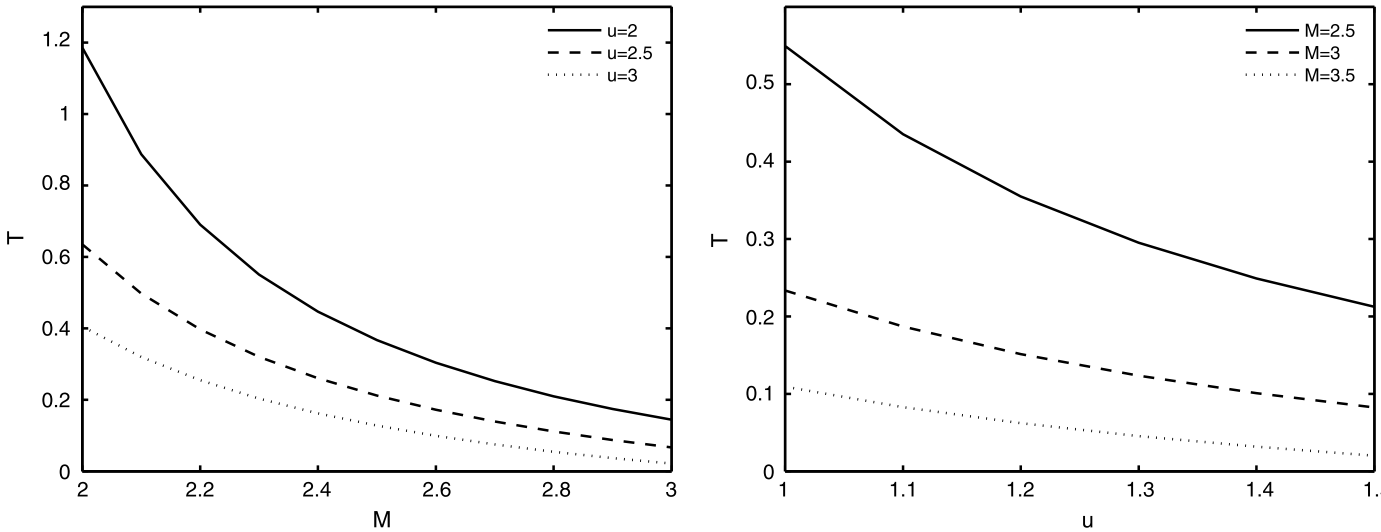

Fig. 1. The Hawking temperature for the case of $k=1$.

It is difficult to solve the action of the emission fermion. From above equations, we know the action can be separated into radial term and transverse term. Considering the symmetry of the spacetime, we carry out the separation of variables as

$$I_{\uparrow} = -\omega t + W(r) + \Theta(\theta, \phi), \tag{14}$$

where ω is the energy of the particle. The contribution of imaginary part of the action is mainly produced by two parts, namely $W(r)$ and $\Theta(\theta,\phi)$, but the contribution of $\Theta(\theta,\phi)$ could be canceled out in the calculation of the tunnelling probability. So we only take care of the action of the radial direction here. Inserting Eq. (14) into (10) and (11) yields

$$-\frac{B\omega}{\sqrt{f(r)}} + B\sqrt{f(r)}\,\partial_r W(r) - mA = 0, \tag{15}$$

$$\frac{A\omega}{\sqrt{f(r)}} + A\sqrt{f(r)}\,\partial_r W(r) - mB = 0. \tag{16}$$

When $m=0$, it denotes the Hawking radiation of the massless particle. While $m \neq 0$, it is the Hawking radiation of the massive fermion. The final results (tunnelling probability and Hawking temperature) are not related to the mass of the particle. Without loss the generality in the discussion, we choose $m \neq 0$. Solving the action of radial direction and canceling out A and B, we get

$$W_{\pm}(r) = \pm\int \frac{\sqrt{\omega^2 + m^2 f(r)}}{f(r)} dr = \pm\frac{i\pi\omega}{f'(r_+)},$$

$$f'(r_+) = \frac{\sqrt{-\Lambda_W}}{2x_+}\frac{3x_+^2 - k - \frac{5\tilde{\beta}k^2}{\tilde{\mu}x_+^2}}{1 + \frac{2\tilde{\beta}k}{\tilde{\mu}x_+^2}}, \tag{17}$$

where $+/-$ correspond to the outgoing/ingoing solutions. From the WKB approximation, we know the tunnelling probability is related to the imaginary part of the action. Thus the tunnelling probability of the emission fermion is obtained as

$$\Gamma = \frac{P(emission)}{P(absorption)} = \frac{\exp(-2\,\mathrm{Im}\,I_+)}{\exp(-2\,\mathrm{Im}\,I_-)} = \frac{\exp(-2\,\mathrm{Im}\,W_+)}{\exp(-2\,\mathrm{Im}\,W_-)}$$
$$= \exp\left[-\frac{4\pi\omega}{f'(x_+)}\right], \tag{18}$$

which means the Hawking temperature is

$$T = \frac{f'(x_+)}{4\pi} = \frac{\sqrt{-\Lambda_W}}{8\pi x_+}\frac{3x_+^2 - k - \frac{5\tilde{\beta}k^2}{\tilde{\mu}x_+^2}}{1 + \frac{2\tilde{\beta}k}{\tilde{\mu}x_+^2}}, \tag{19}$$

where x_+ is the black hole horizon obtained from $f(r)=0$. When $\tilde{\beta} \to 0$, it is just the Hawking temperature of the black hole in Horava–Lifshitz gravity with $z=3$ [6] and recovers the Hawking temperature of the black hole in Horava–Lifshitz gravity where $k=1$ that was obtained by Lü [5]. When $k=0$, the Hawking temperature is $T = \frac{3x_+\sqrt{-\Lambda_W}}{8\pi}$. In the extreme case, when $k \neq 1$ and $\frac{\tilde{\beta}}{\tilde{\mu}} = \frac{x_+^2(3x_+^2 - k)}{5}$, the Hawking temperature vanishes. The Hawking temperature vanishing means the surface gravity of the black hole vanishes, but the entropy of the black hole doesn't vanish. When $k=1$ and $\frac{\tilde{\beta}}{\tilde{\mu}} > \frac{x_+^2(3x_+^2-1)}{5}$, the temperature is negative. When $k=-1$ and $\frac{x_+^2}{2} > \frac{\tilde{\beta}}{\tilde{\mu}} > \frac{x_+^2(3x_+^2+1)}{5}$, the temperature is also negative. (Note the condition $4\frac{\tilde{\beta}}{\tilde{\mu}} \leqslant 1$ of the vacuum solution is considered here [11]). The negative temperature is not allowed in black hole physics. Therefore we have to avoid the negative temperature. When we consider $\tilde{\mu} = -\mu\Lambda_W$, $\tilde{\beta} = \frac{\Lambda_W^2}{4M}$ and the expression of the black hole horizon x_+, we find the Hawking temperatures are related to the fundamental parameters (μ, M, Λ_W). Therefore we must restrict the fundamental parameters of Horava–Lifshitz gravity to avoid negative temperatures. This is an important constraint on the theory.

The Hawking temperatures change with the parameters is expressed in the following figures. There both c_0 and Λ_W are choose to equal 1 for visibility. From the Figs. 1 and 2, we find the temperatures become lower with the increase of M (or μ). When the lower temperatures.

In the calculation, we only investigated the Hawking radiation of the spin up state. For the spin down case, we can adopt analogous step to investigate it and get the same result. In the investigation, the background spacetime of black holes was seen as fixed with the particle emission and the back reaction is neglected. The true tunnelling picture implies the background should fluctuate with particles emission. Therefore considering the true tunnelling picture, the Hawking temperature should be corrected. Meanwhile we neglected the higher order terms of the action in the calculation. When it was considered, the Hawking temperature would be

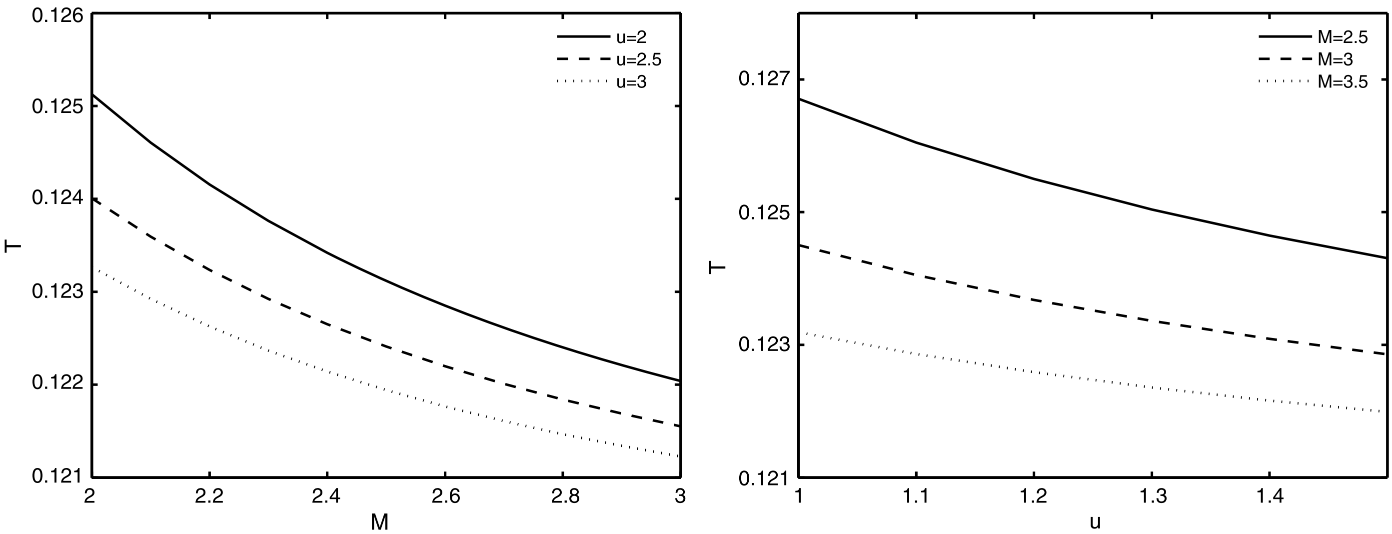

Fig. 2. The Hawking temperature for the case of $k=-1$.

also corrected. This view has been mentioned in Refs. [50,52], and we don't correct the temperature in the current Letter.

3. Hawking radiation of $(4+1)$-dimensional black holes in the $z=4$ Horava–Lifshitz gravity

In this section, we investigate the fermions tunnelling effect in $(4+1)$-dimensional spacetime in the $z=4$ Horava–Lifshitz gravity. This may be interesting to study the AdS_5/CFT_4 correspondence in the framework of this theory.

The Lagrangian of the $z=4$ Horava–Lifshitz gravity in $(4+1)$-dimensional spacetime is given by

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_0+\mathcal{L}_1,\\ \mathcal{L}_0 &= \sqrt{g}N\left\{\frac{2}{\kappa^2}\left(K^{ij}K_{ij}-\lambda K^2\right)+\frac{\kappa^2\mu^2\Lambda_W(R-2\Lambda_W)}{4(1-4\lambda)}\right\},\\ \mathcal{L}_1 &= -\sqrt{g}N\frac{\kappa^2}{8}\left\{\mu^2G_{ij}G^{ij}+\frac{2\mu}{M}G^{ij}L_{ij}+\frac{2\mu}{M}\Lambda_W L+\frac{1}{M^2}L^{ij}L_{ij}\right.\\ &\quad\left.-\tilde{\lambda}\left(\frac{L^2}{M^2}-\frac{2\mu L}{M}(R-4\Lambda_W)+\mu^2R^2\right)\right\},\end{aligned} \tag{20}$$

where $L=2(1+3\beta)\nabla^2R$, the expressions of L^{ij} and G^{ij} are given in Section 2. In order to get back to general relativity in the IR region, the effective couplings should be related to the light speed, the Newton coupling and the effectively cosmological constant as

$$c=\frac{\mu\kappa^2}{\sqrt{8}}\sqrt{\frac{\Lambda_W}{1-3\lambda}},\qquad G_N=\frac{\kappa^2c}{32\pi},\qquad \Lambda=\Lambda_W. \tag{21}$$

In the IR region, to get back to general reactivity, we let λ be equal to 1, and then Λ_W should be negative for the speed light having the physical meaning. It is difficult to get the exact solutions for the general β. In this section, the value of β is chosen as $-1/3$, and it comes from an unsuccessful attempt to generalize the New Massive Gravity to the case in four dimensions. From the action, the solution of $(4+1)$-dimensional black holes solution in Horava–Lifshitz gravity was given by [11]

$$ds^2=-F(r)c^2\,dt^2+\frac{1}{F(r)}dr^2+r^2\,d\Omega_k^2, \tag{22}$$

where $F(r)=k+\frac{x^2}{3}\pm\sqrt{c_0}$, $c_0\geqslant 0$ is an integration constant, $d\Omega_k^2$ is the three-dimensional Einstein manifold with constant scalar curvature $6k$, which one can choose $k=0,\pm1$, without loss of generality. The metric (22) denotes a five-dimensional Anti de Sitter spacetime obtained from the Horava–Lifshitz gravity. It cannot reduce to 5D-Schwarzschild metric. The quantum inheritance principle is an important principle in the construction of the Horava–Lifshitz gravity. If the Horava–Lifshitz gravity is quantum gravity, it is very important to understand the AdS/CFT correspondence in the frame work of Horava–Lifshitz theory and it is interested to further study the asymptotically AdS solution in five-dimensional spacetime. Therefore we further study the Hawking radiation of fermions from these black holes in this section.

To investigate the Hawking radiation of emission fermions, we still use the Dirac equation expressed in Eq. (7) and spin up wave function (9). For the five-dimensional spacetime, we choose the following gamma matrices

$$\begin{aligned}&\gamma^t=\frac{1}{\sqrt{F(r)}}\begin{pmatrix} i & 0\\ 0 & -i\end{pmatrix},\qquad \gamma^r=\sqrt{F(r)}\begin{pmatrix} 0 & \sigma^3\\ \sigma^3 & 0\end{pmatrix},\\ &\gamma^\theta=\sqrt{g^{\theta\theta}}\begin{pmatrix} 0 & \sigma^2\\ \sigma^2 & 0\end{pmatrix},\qquad \gamma^\varphi=\sqrt{g^{\varphi\varphi}}\begin{pmatrix} 0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix},\\ &\gamma^\phi=\sqrt{g^{\phi\phi}}\begin{pmatrix} -I & 0\\ 0 & I\end{pmatrix}.\end{aligned} \tag{23}$$

Inserting the spin up wave function and gamma matrices into the Dirac equation, dividing the exponential term and multiplying by $\hbar$, we can get the resulting equations to leading order in $\hbar$ as

$$\frac{iA}{\sqrt{F(r)}}\,\partial_t I_\uparrow+B\sqrt{F(r)}\,\partial_r I_\uparrow-A\sqrt{g^{\phi\phi}}\partial_\phi I_\uparrow-mA=0, \tag{24}$$

$$\frac{-iB}{\sqrt{F(r)}}\partial_t I_\uparrow+A\sqrt{F(r)}\,\partial_r I_\uparrow+B\sqrt{g^{\phi\phi}}\,\partial_\phi I_\uparrow-mB=0, \tag{25}$$

$$iB\sqrt{g^{\theta\theta}}\,\partial_\theta I_\uparrow+B\sqrt{g^{\varphi\varphi}}\,\partial_\varphi I_\uparrow=0, \tag{26}$$

$$iA\sqrt{g^{\theta\theta}}\,\partial_\theta I_\uparrow+A\sqrt{g^{\varphi\varphi}}\,\partial_\varphi I_\uparrow=0. \tag{27}$$

It is difficult to solve the above equations. Similar to that in Section 2, considering the symmetry of the spacetime, we carry out the separation of variable as

$$I_\uparrow=-\omega t+W(r)+\Xi(\theta,\varphi,\phi). \tag{28}$$

Insert Eq. (28) into (24)–(27). Considering the contribution of the imaginary part of $\Xi(\theta,\varphi,\phi)$ would be canceled out in the calculation of the tunnelling probability, we still only take care of the radial direction part. Solving $W(r)$ yields

$$W_{\pm}(r) = \pm \int \frac{\sqrt{\omega^2 + (\sqrt{g^{\phi\phi}}\partial_\phi \Xi(\theta,\varphi,\phi) - m^2 - \frac{2m\omega}{\sqrt{F(r)}})F(r)}}{F(r)} dr = \pm \frac{i\pi\omega}{F'(r)} = \pm \frac{i3\pi\omega}{2x_+\sqrt{-\Lambda_W}}, \tag{29}$$

where +/− correspond to the outgoing/ingoing solutions. Applying the WKB approximation, we obtain the tunnelling probability of the emission fermion in $(4+1)$-dimensional spacetime as

$$\Gamma = \frac{P(emission)}{P(absorption)} = \frac{\exp(-2\,\mathrm{Im}\, I_+)}{\exp(-2\,\mathrm{Im}\, I_-)} = \frac{\exp(-2\,\mathrm{Im}\, W_+)}{\exp(-2\,\mathrm{Im}\, W_-)} = \exp\left(-\frac{6\pi\omega}{\sqrt{-\Lambda_W}x_+}\right), \tag{30}$$

which results the Hawking temperature as

$$T = \frac{\sqrt{-\Lambda_W}x_+}{6\pi}. \tag{31}$$

This result is in consistence with that obtained by directly calculating surface gravity of the black holes. For the spin down case, we can use analogous process to explore the Hawking radiation of the black holes and get the same temperature. The unfixed background spacetime and back reaction were also neglected in this section. When these are considered, the Hawking temperature should be corrected.

4. Discussion and conclusion

In this Letter, we have investigated the Hawking radiation of $(3+1)$- and $(4+1)$-dimensional black holes in the $z=4$ Horava–Lifshitz gravity by fermion tunnelling. As a result, the Hawking temperatures were recovered and are in consistence with that obtained by directly calculating surface gravity of the black holes. The result shows the Hawking temperatures of $(3+1)$-dimensional black holes in the $z=4$ Horava–Lifshitz gravity are related to the fundamental parameters (μ, M, Λ_W) of Horava–Lifshitz gravity. Considering the negative temperature is not allowed in black hole physics, therefore we have to restrict the fundamental parameters to avoid the negative temperatures. For the $(4+1)$-dimensional black hole, the Hawking temperatures are related to the cosmological constant. This is our expectation. In the investigation, the unfixed background spacetime and back reaction were neglected, and then the obtained Hawking temperatures are only leading term. When they are taken into account, the Hawking temperatures should be corrected. Recently Banerjee and Majhi et al. made a correction to the Hawking temperature by Hamilton–Jacobi method. They extended the action of the emission particle in a power of $\hbar$ and made a correction to the action, and then obtained the correction temperatures. When the action of emission particle is corrected in this Letter, the corrected temperatures can also be obtained.

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