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AN ASSEMBLY CELL WITH AN AUTOMATED QUALITY CONTROL STATION

EXPONENTIALLY DISTRIBUTED PROCESSING TIMES

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Abstract—The performance of an assembly cell consisting of a set of machines, a sufficiently large local storage, an adjacent automated quality control station, a loading and an unloading station is modeled by a $G/M/1$ queuing system with generally distributed interarrival time, a single Markovian server, the first come first served queuing discipline, and with a fixed delay Bernoulli feedback mechanism, in steady state. In a queuing system with a fixed delay Bernoulli feedback mechanism, a fraction of the departing units will merge with the incoming arrival units to be reprocessed, after being delayed for a fixed length of time. The performance of this system is approximated by a recursive algorithm. Furthermore, approximation outcomes are compared against those from a simulation study.

1. INTRODUCTION

The primary objectives of this paper are to develop a model and test an approximation algorithm for analyzing the performance of an assembly cell. This model could then be used to quantify the effect of the quality control mechanism on the performance of the assembly cell. In this system, the assembly cell consists of a set of machines, capable of processing workpieces belonging to the same family of parts, contains a local storage with a sufficiently large capacity, and is placed adjacent to an inspection station. In this paper, it is assumed that no matter how many stages of operation each workpiece requires at different machines of the assembly cell, the aggregated duration of the processing time at the work station can be approximated by an exponentially distributed random variable. This aggregation assumption is based on the group technology concept. That is, the assembly cell is intelligently designed such that a compatible group of machines are placed in it for processing various stages of operation of a compatible group of workpieces. For a review of the literature of the group technology, see Waghodekar and Sahu (1983). Additionally, the inspection station is used to identify the defective parts. In general, the workpieces which are transported to the assembly cell, after being processed, leave the assembly cell and arrive at the inspection station; there after being inspected, the non-defective workpieces leave the system and the defective workpieces after a fixed inspection time at the quality control station, are rerouted to the assembly cell to be reprocessed.

To develop the model, the performances of the assembly cell and the inspection station are jointly modeled by an infinite capacity $G/M/1$ queuing system with generally distributed interarrival time, a single Markovian server, the first come first served queuing discipline (e.g. dispatching rule), and a Bernoulli feedback mechanism. That is, the performance of the inspection station is modeled by a Bernoulli filter which operates as a binary mechanism for identifying defective or non-defective workpieces. It is noted that this problem has never been considered in the literature. However, when the length of the delay time is equal to zero, the same problem has been considered in the literature by Kuehn (1979) and Whitt (1982), and algorithms different than the one presented in this paper are proposed by the latter authors for approximating the performance of a $G/M/1$ queuing system with an instantaneous Bernoulli feedback mechanism.

The motivation behind developing the proposed algorithm is that the performance of the above system is too complex, and its performance cannot be analytically quantified. Hence, to be able to understand how the above system behaves, a heuristic algorithm is proposed.

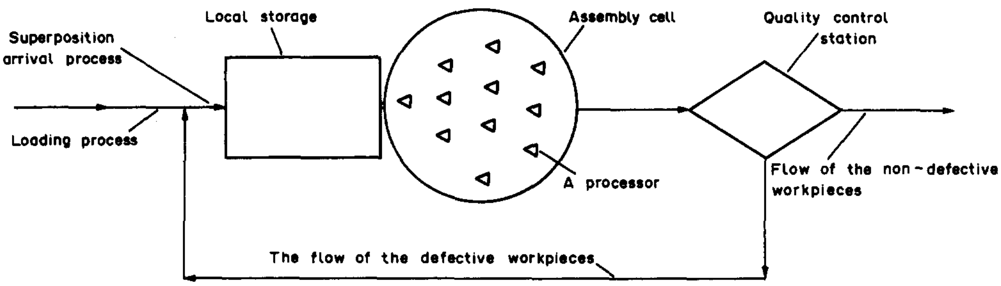

Fig. 1. An assembly system consisting of an assembly cell and a quality control station.

Queuing systems with an instantaneous Bernoulli feedback mechanism have previously been considered in the literature, see Kuehn (1979) and Whitt (1983). For a review of the performance modeling literature of the manufacturing systems, see Buzacott and Yao (1986).

Notice that the approximation algorithm which has been provided by Kuehn (1979) and Whitt (1983) does not perform very well under light traffic conditions. Notice that Kuehn's (1979) algorithm and Whitt's (1983) algorithm are almost identical, the only difference between these algorithms in approximating a $G/M/1$ queuing system is expression (36) of Kuehn (1979) which has been modified in expression (17) of Whitt (1983). In this paper, it will be demonstrated that under low traffic intensities, for approximating the performance of a $G/M/1$ queuing system with a Bernoulli feedback mechanism, the proposed algorithm performs better than Kuehn's (1979) and Whitt's (1983) algorithms.

The organization of this paper is as follows. In Section 2, the components of the model are delineated. In Section 3, numerical results are presented. Finally, in Section 4, the concluding remarks are discussed.

2. THE MODEL

2(a) Assumptions

In this paper, the following assumptions are made. First, no matter how many stages of operation each workpiece requires at different machines of the assembly cell, the aggregated duration of the processing time at the assembly cell can be approximated by an exponentially distributed random variable. This aggregation assumption is based on the group technology concept. Second, the loading process is a stationary process with identically distributed interarrival times. Third, the non-renewal superposition arrival process can be approximated with à renewal process. Fourth, the flow of the defective parts can be approximated with a thinned renewal process. Fifth, the fixed inspection time does not influence the performance of the system in steady state.

2(b) Notation

Throughout the paper, the following notation will be used.

j—Iteration index. It is noted that this index counts the number of times a fraction of the departing units will merge with the arrival units.

e—The minimum required number of iterations for reaching the steady state.

$A(t)$, λ^a and c^a—The distribution of the interarrival time of the incoming arrival process, the arrival rate, and coefficient of variation (c.v.) of the distribution of the interarrival time, respectively.

$F_j(t)$, λ_j^s and c_j^s—The distribution of the interarrival time of the superposition arrival process at iteration j, the superposition arrival rate at iteration j, the c.v. of the corresponding process at iteration j, respectively.

$D_j(t)$, λ_j^d and c_j^d—The distribution of the interdeparture time at iteration j, the departure rate at iteration j, the c.v. of the distribution of the interdeparture time at iteration j, respectively.

$\hat{\lambda}_j^d$ and $\hat{c}_j^d$—The departure rate and the c.v. of the distribution of the interdeparture time of the thinned departure process which superimposes with the incoming arrival process, at iteration j.

μ, ρ_j, $\bar{N}_j$ and $\bar{W}_j$—The service rate, the service utilization factor at iteration j, the average queue length at iteration j, and the average waiting time in the queue at iteration j, respectively.

p—The fraction of the departure units which will not merge with the incoming arrival units.

z—The fixed inspection time at the quality control station.

Also, let $D_j^*(s)$ and $(-1)^m D_j^{*m}(0)$ be the Laplace–Stieltjes transform of $D_j(t)$ and its mth moment, respectively.

Before describing the approximation steps, it is noted that at every iteration, a fraction of departing units will merge with the incoming arrival units to form a new arrival process. We call the latter process, the superposition arrival process.

2(c) *Algorithm*

To approximate the performance of a $G/M/1$ queuing system with a Bernoulli feedback mechanism in steady state, the following steps will be implemented.

Step 1. At iteration j, ignore the dependencies among the interarrival times of the superposition arrival process. That is, approximate the superposition arrival process by a renewal process. Notice that at the first iteration, the superposition arrival process is identical to the original arrival process.

Step 2. Given that at least the first two moments of the distribution of the interarrival time of the superposition arrival process are known, fit a compatible (e.g. a phase type) distribution function to those moments, see the Appendix A. That is, at iteration j, if c_j^s is less than one, approximate the performance of the $G/M/1$ queuing system with a Bernoulli feedback mechanism by a compatible $E_k/M/1$ queuing system with a Bernoulli feedback mechanism, a renewal arrival process and hypoexponentially distributed interarrival time with k parameters; and for c_j^s greater than or equal to one, the performance of the queuing system is approximated by a compatible $H_2/M/1$ queuing system with a Bernoulli feedback mechanism, a renewal arrival process, and hyperexponentially distributed interarrival time with two parameters and balanced means. For further details, see the Appendices A and B.

Step 3. Obtain at least the first two moments of the distribution of the interdeparture time from the queuing system, see Appendix B.

Step 4. Ignore the dependencies among the interdeparture times. That is, treat the departure process as a renewal process.

Step 5. Approximate both the departure processes corresponding to the departing units which will permanently leave the system and the departing units which will merge with the incoming arrival units. For this purpose, the approximated renewal departure process can be thinned based on Gnedenko and Kovalenko's (1968) results on thinning of a renewal process. The parameters of ($\hat{\lambda}_j^d$ and $\hat{c}_j^d$) of the thinned departure process which superimposes with the incoming arrival process can be obtained as follows:

$$\hat{\lambda}_j^d = (1-P)\lambda_j^d, \quad j \geqslant 1, \tag{1}$$

$$\hat{c}_j^d = [p + (1-p)(c_j^d)^2]^{1/2}, \quad j \geqslant 1. \tag{2}$$

Step 6. Superimpose the thinned departure process with the incoming arrival process to form a new superposition arrival process at the next iteration. The resulted non-renewal superposition arrival process can be approximated based on the asymptotic method of Whitt (1982) for approximating a finite number of stationary processes, as follows:

$$\lambda_{j+1}^s = \hat{\lambda}_j^d + \lambda^a, \quad j \geqslant 1 \tag{3}$$

$$c_{j+1}^s = \left[\frac{\hat{\lambda}_j^d(\hat{c}_j^d)^2 + \lambda^a(c^a)^2}{\lambda_{j+1}^s}\right]^{1/2}, \quad j \geqslant 1 \tag{4}$$

It is noted that at iteration $j = 1$, $\lambda_1^s = \lambda^a$ and $c_1^s = c^a$.

Notice that to approximate the performance of a $G/M/1$ queuing system with a fixed delay Bernoulli feedback mechanism: the previous steps can be applied and the algorithm for approximating a $G/M/1$ queuing system with an instantaneous Bernoulli feedback can be used to approximate a $G/M/1$ queuing system with a delayed Bernoulli feedback mechanism. For this purpose, the length of time which takes for a departing unit which is going to be superimposed with the arrival units will be set equal to zero (e.g. $z = 0$), and then the thinned departure process will be superimposed with the incoming arrival process to form a new superposition arrival process at iteration $j + 1$, same as before. Based on a simulation analysis, as will be shown in the next section, it has been demonstrated that the value of "z" does not significantly influence the approximation results in steady state.

Step 7. Repeat the previous steps until a steady state is reached at iteration "e". The steady state is identified as follows. Let δ be a sufficiently small value (e.g. 1×10^{-3}) and

$$m = \min(j; \lambda^s_{j+1} - \lambda^s_j \leqslant \delta), \tag{5}$$

$$n = \min(j; c^s_{j+1} - c^s_j \leqslant \delta), \tag{6}$$

then

$$e = \max(m, n). \tag{7}$$

Step 8. After reaching the steady state, approximate the performance of the system, based on the average queue length, the average waiting time in the queue, and the service utilization factor, see Appendix B. It is noted that for $\delta = 0.001$ (0.000001), it only takes $e = 3$ ($e = 7$) iterations to approximate the performance of the system in steady state.

For a summary of the approximation steps, see Fig. 2.

3. NUMERICAL RESULTS

In this section, several examples are presented for $\mu = 1.0$, $p = 0.9$, and the approximation outcomes are compared against those from both a simulation study and Whitt's (1983) results.

In the simulation study, the performance of the system with both an instantaneous Bernoulli feedback mechanism and the fixed delay Bernoulli feedback mechanism are quantified. In the latter case, in the simulation study, we set $z = 100$. The numerical results are presented in Figs 3–14. The approximation outcomes are obtained based on the proposed algorithm and the results provided in the Appendix B. The simulation results are obtained based on using the SLAM simulation package of Pritsker and Pegden (1979). Each simulation outcome is obtained based on 50,000 departing units and two independent runs.

To generate the approximation outcomes based on our algorithm, the interarrival times were generated based on one of the following three distributions: a hyperexponential distribution with two parameters and balanced means [i.e. expressions (A.1)–(A.4)], or an exponential distribution, or a hypoexponential distribution [i.e. expressions (A.5)–(A.7)]. In Figs 3–14, the service rate is set equal to one. Hence, because $\rho_e = \lambda^d_e/\mu$, the values of the departure rate from the system and the service utilization factor are identical, also see expression (B.9).

To generate the simulation outcomes, for the cases with the c.v. of the distribution of the interarrival time greater than or equal to (less than) one, a hyperexponential (shifted exponential) distribution with two parameters and balanced means was used, see expressions (A.1)–(A.4) [(A.8)–(A.10)] in the Appendix A. The reason for not using a hypoexponential distribution in the simulation study was that a shifted exponential distribution can easily be simulated and as can be observed in Figs 3–14, it can accurately approximate the behavior of a compatible hypoexponential distribution.

We conclude this section by making the following observations based on Figs 3–14: firstly, the fixed inspection time does not significantly influence the values of any of the performance measures; secondly, under low to medium traffic intensities, the proposed algorithm performs better than Kuehn's (1979) and Whitt's (1983) algorithms. The reason for the superior performance of our algorithm is that the approximate outcomes in Figs 3–14 of Kuehn's (1979) and Whitt's (1983) are

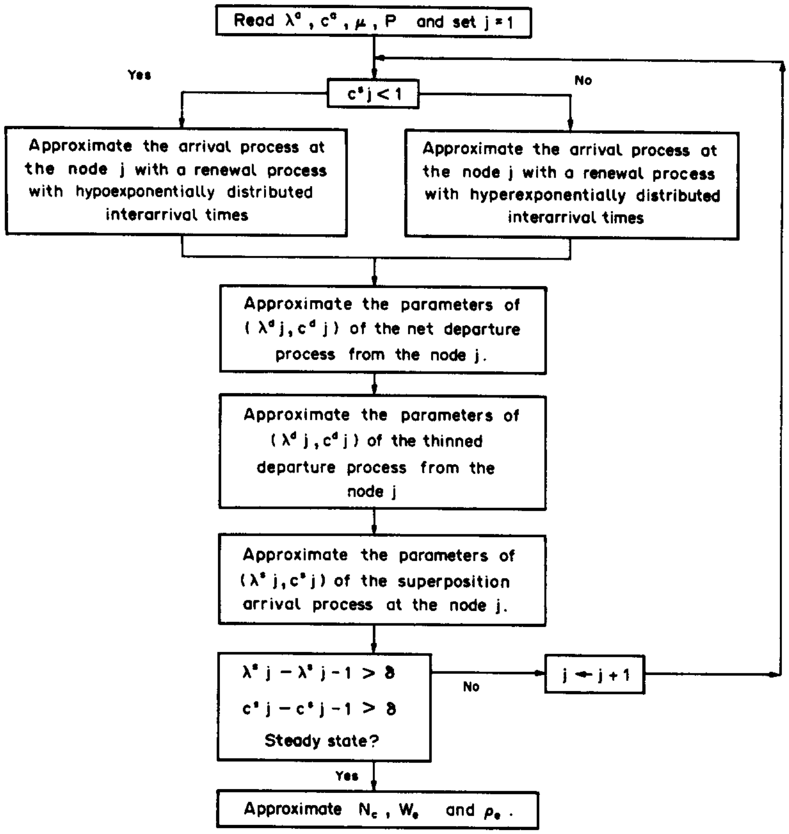

Fig. 2. The flow chart of the heuristic recursive algorithm.

based on Marshall's (1968) approximate expression for the variance of the interdeparture times of a $GI/M/1$ queuing system, but our algorithm is based on the exact expression for the variance of the interdeparture times of a $GI/M/1$ queuing system. Notice that even in our algorithm, the exact expression for the variance of the interdeparture times of a $GI/M/1$ queuing system serves only as an

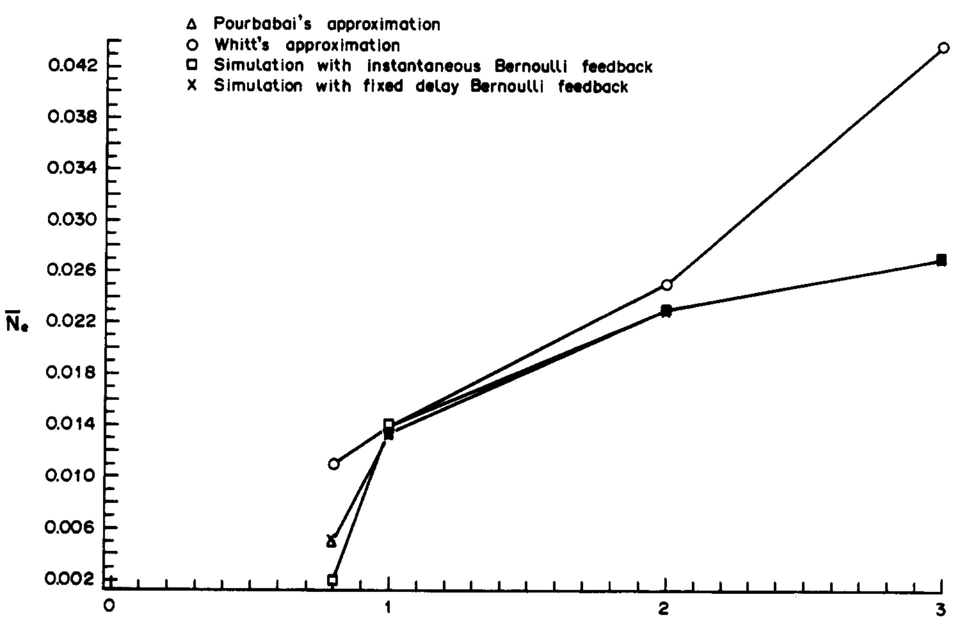

Fig. 3. Average queue length vs the c.v. of the distribution of the interarrival times for $\mu = 1$ and $\lambda^a = 0.1$.

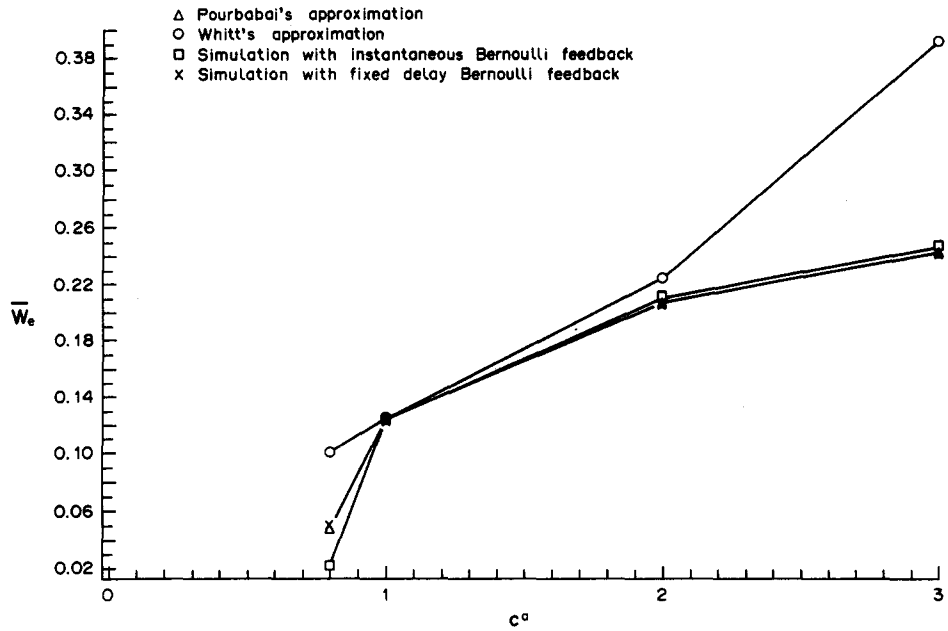

Fig. 4. Average waiting time in the queue vs the c.v. of the distribution of the interarrival time for $\mu = 1$ and $\lambda^a = 0.1$.

approximate expression for the variance of the interdeparture times of a $G/M/1$ queuing system with a non-renewal input. However, it is interesting to note that even after ignoring the dependencies among the interarrival times, as can be seen in Figs 3–14, the proposed algorithm performs well; thirdly, for each value of the c.v. of the distribution of the interarrival time, as the arrival rate increases, the values of $\bar{N}_e$, $\bar{W}_e$ and ρ_e also increase; fourthly, for each value of the arrival rate, as the c.v. of the distribution of the interarrival time increases, the values of $\bar{N}_e$ and $\bar{W}_e$ also increase, but the value of ρ_e does not change. It is noted that the c.v. of the distribution of the interarrival time does not influence the value of the utilization factor. Because, $\rho_e = \lambda_e^d/\mu$, $\lambda_e^d = \lambda_e^s$ and $\lambda_e^s = \lambda^a + P\lambda_e^d$. That is, the variability of the interarrival times does not affect the departure rate.

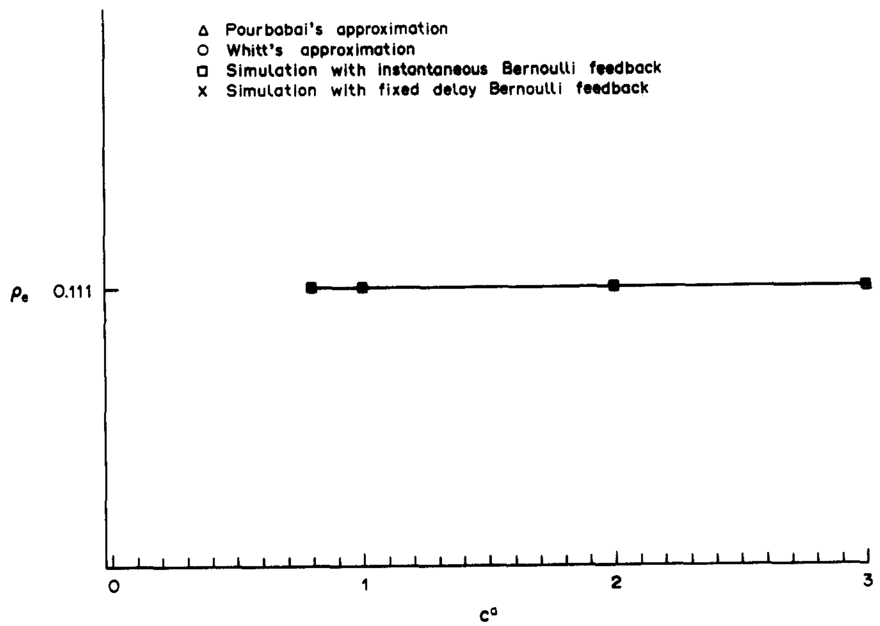

Fig. 5. Utilization factor vs the c.v. of the distribution of the interarrival time for $\mu = 1$ and $\lambda^a = 0.1$.

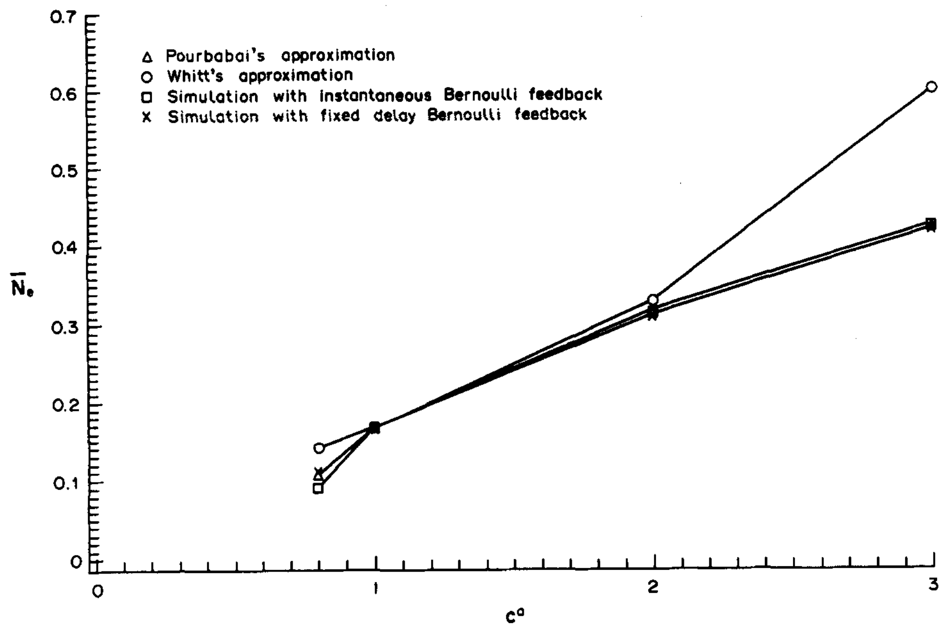

Fig. 6. Average queue length vs the c.v. of the distribution of the interarrival time for $\mu = 1$ and $\lambda^a = 0.3$.

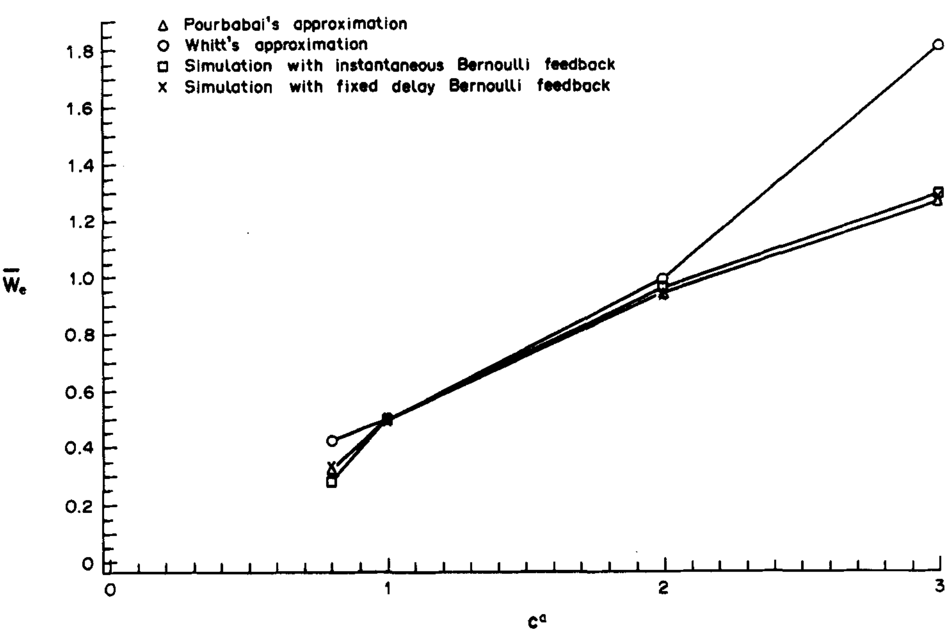

Fig. 7. Average waiting time vs the c.v. of the distribution of the interarrival time for $\mu = 1$ and $\lambda^a = 0.3$.

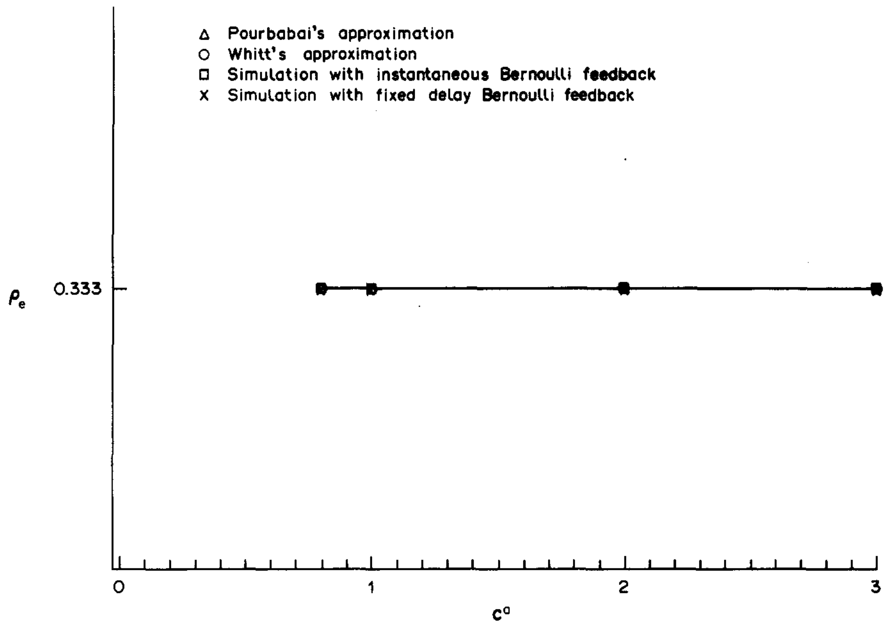

Fig. 8. Utilization factor vs the c.v. of the distribution of the interarrival time for $\mu = 1$ and $\lambda^a = 0.3$.

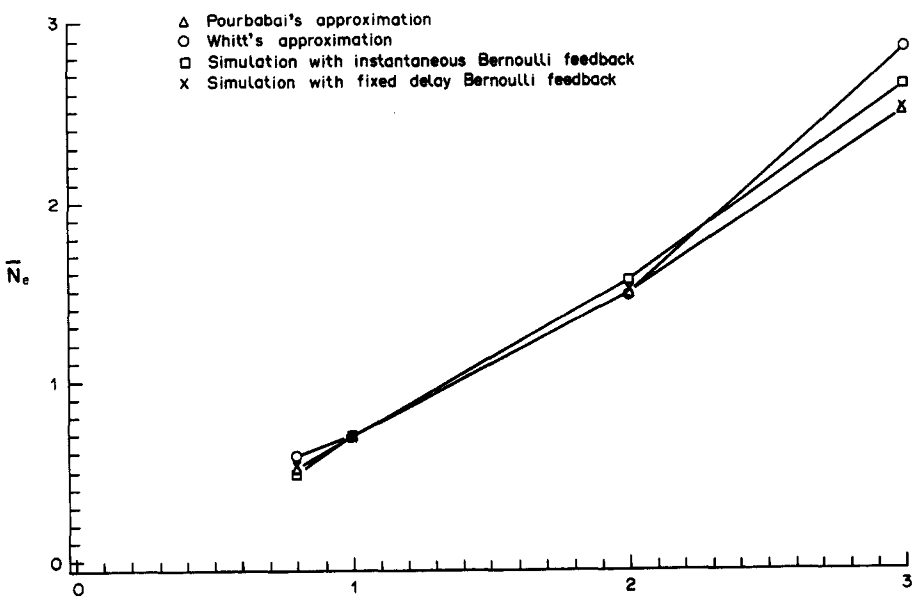

Fig. 9. Average queue length vs the c.v. of the distribution of the interarrival time for $\mu = 1$ and $\lambda^a = 0.5$.

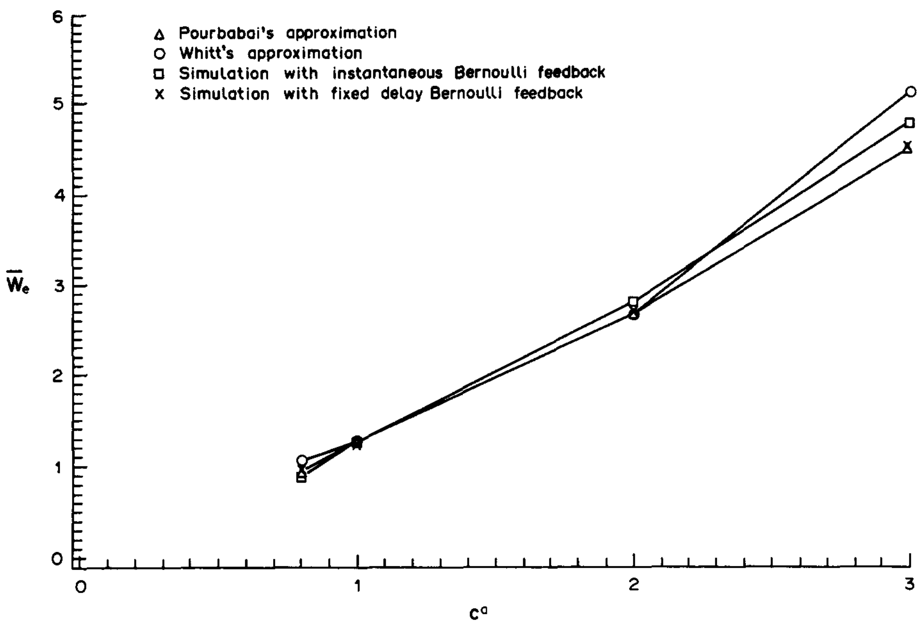

Fig. 10. Average waiting times vs the c.v. of the distribution of the interarrival time for $\mu = 1$ and $\lambda^a = 0.5$.

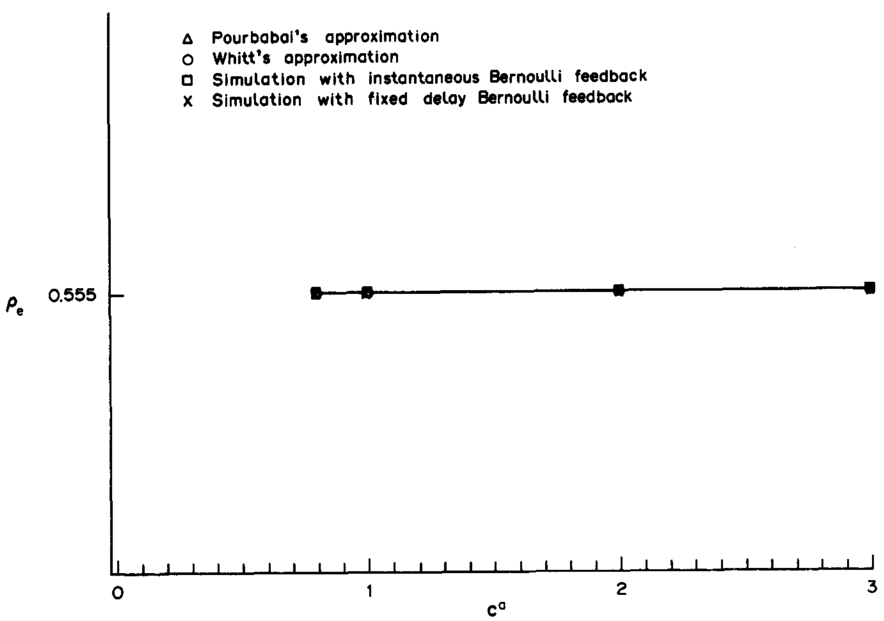

Fig. 11. Utilization factors vs the c.v. of the distribution of the interarrival time for $\mu = 1$ and $\lambda^a = 0.5$.

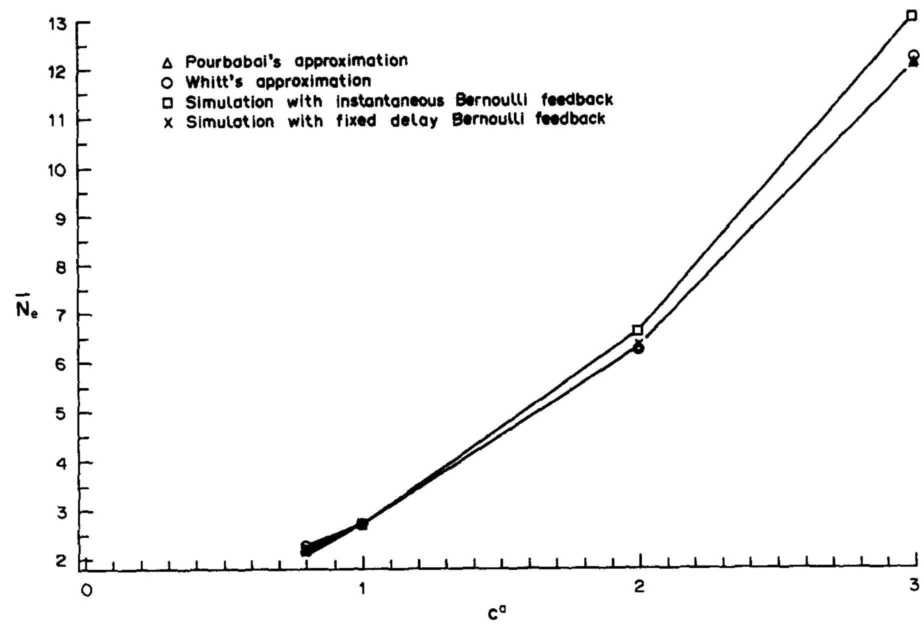

Fig. 12. Average queue length vs the c.v. of the distribution of the interarrival time for $\mu = 1$ and $\lambda^a = 0.7$.

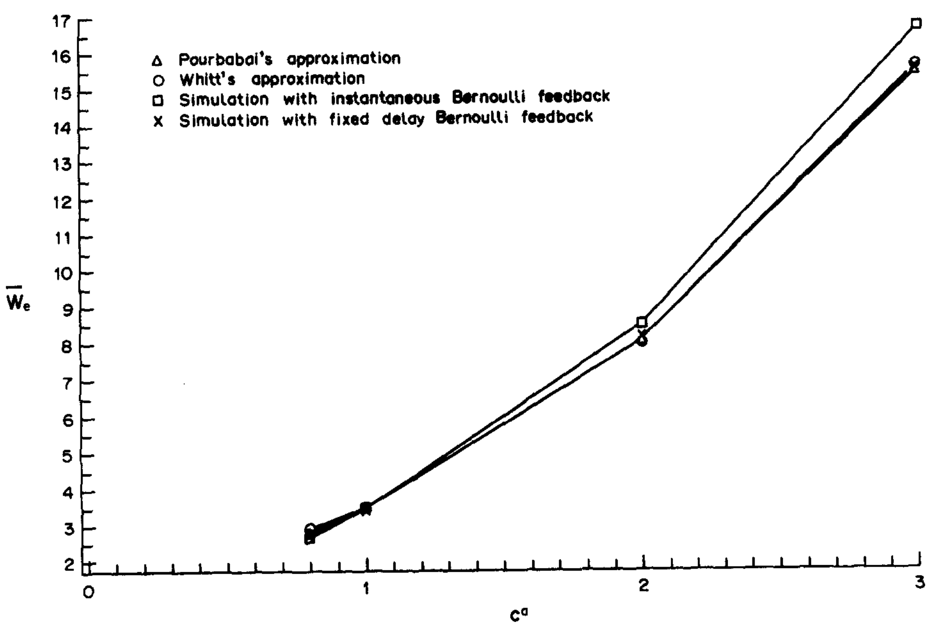

Fig. 13. Average waiting time vs the c.v. of the distribution of the interarrival time for $\mu = 1$ and $\lambda^a = 0.7$.

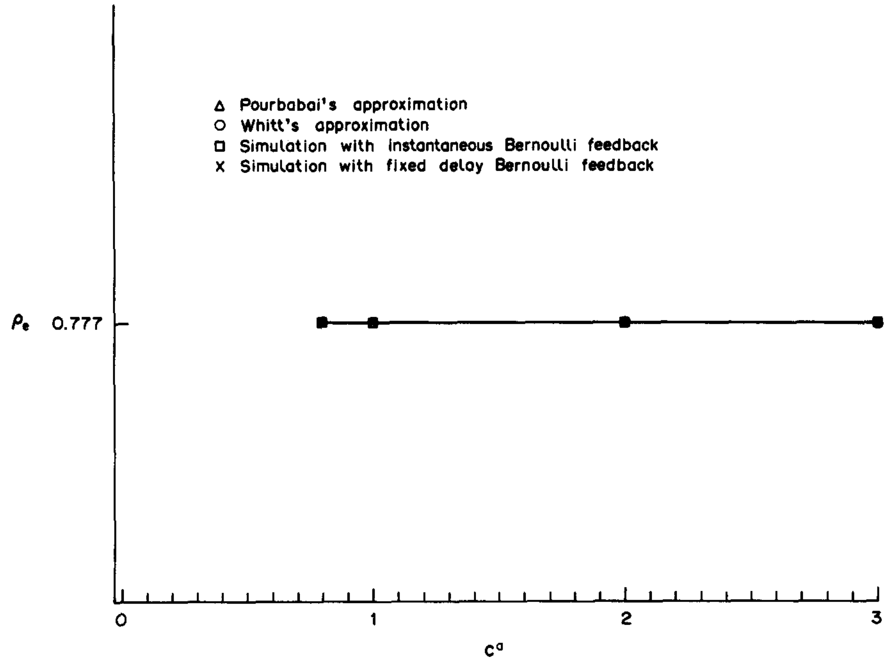

Fig. 14. Utilization factor vs the c.v. of the distribution of the interarrival time for $\mu = 1$ and $\lambda^a = 0.7$.

Notice that the other advantage of using the proposed algorithm is that it is flexible and if necessary, it can be improved to use more than the first two moments of the respective distributions of the interarrival and the interdeparture times. This capability does not exit in Kuehn's (1979) and Whitt's (1983) algorithms. Hence, if necessary, the accuracy of the proposed algorithm can be improved.

4. CONCLUDING REMARKS

In this paper, the performance of an assembly cell with an automated quality control station is modeled. For this purpose, a recursive algorithm is developed for approximating the performance of a $G/M/1$ queuing system with either an instantaneous Bernoulli feedback mechanism or a fixed delay Bernoulli feedback mechanism, in steady state. As is demonstrated in the previous section this algorithm is computationally efficient and simple to work with, and more importantly, it is relatively accurate (i.e. the approximation errors are less than 5%). However, the system performance can be improved, if more than the first two moments of the distribution of the interarrival time of the superposition arrival process is used.

Finally, we conclude this paper by pointing out that the most significant aspect of our algorithm was to ignore the dependencies among the interarrival times and the interdeparture times, to be able to approximate the distribution of the interarrival time and the distribution of the interdeparture time, in steady state. Based on our numerical results in Section 3, it is obvious that ignoring the dependencies among the interarrival times and the interdeparture times do not significantly influence the accuracy of the proposed algorithm.

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APPENDIX A

As discussed in Kuehn (1979) and Whitt (1982), to approximate the distribution of the interarrival time of a stationary arrival process with the arrival rate λ and the c.v. of the distribution of the interarrival time $c \geqslant 1$, the following hyperexponential distribution function with two parameters and balanced means can be used:

$$H_2(\theta, \bar{\gamma};\quad t) = 1 - \theta \exp(-\gamma_1 t) - (1-\theta)\exp(-\gamma_2 t), \quad t \geqslant 0, \tag{A.1}$$

where, the shape parameter is

$$\theta = \left[\left(\frac{c^2-1}{c^2+1}\right)^{1/2} + 1\right]\Big/2 \tag{A.2}$$

and the intensity parameters are,

$$\gamma_1 = 2\theta\lambda \tag{A.3}$$

$$\gamma_2 = 2(1-\theta)\lambda. \tag{A.4}$$

When

$$\frac{1}{\sqrt{2}} < c < 1,$$

the following hypoexponential distribution function with two parameters can be used,

$$E_2(\bar{\alpha};\quad t) = 1 - \frac{\alpha_2}{\alpha_2 - \alpha_1}\exp(-\alpha_1 t) - \frac{\alpha_1}{\alpha_1 - \alpha_2}\exp(-\alpha_2 t), \quad t \geqslant 0, \tag{A.5}$$

where the intensity parameters are,

$$\alpha_1 = 2\lambda/[1 + (2c^2 - 1)^{1/2}], \tag{A.6}$$

$$\alpha_2 = 2\lambda/[1 - (2c^2 - 1)^{1/2}]. \tag{A.7}$$

On the other hand, when $0 \leqslant c \leqslant 1$, the following shifted exponential distribution function can be used:

$$M'(b, \beta; t) = 1 - \exp[-\beta(t-b)], \quad t \geqslant b, \tag{A.8}$$

where the intensity parameter is,

$$\beta = \frac{\lambda}{c} \tag{A.9}$$

and the shift parameter is

$$b = \frac{1}{\lambda} - \frac{1}{\beta}. \tag{A.10}$$

APPENDIX B

Proposition 1

Consider a $GI/M/1$ queuing system. Then, the distribution of the interdeparture time for $0 \leqslant \sigma < 1$ is,

$$D(x) = \int_0^x \left[A(u) + \int_u^\infty \exp[(-\mu(1-\sigma)(t-u)]\,\mathrm{d}A(t)\right]\mu\exp[(-\mu(x-u)]\,\mathrm{d}u, \tag{B.1}$$

where

$$\sigma = A^*(\mu - \mu\sigma). \tag{B.2}$$

Proof. See Daley (1968).

Proposition 2

Consider a $GI/M/1$ queuing system.

(i) Let the r.v. n denote the number of units found in this system. Then

$$P(n = k) = (1-\sigma)\sigma^k, \quad k \geqslant 0 \tag{B.3}$$

(ii) Let $\bar{N}$ and $\bar{N}'$ be the average queue length and the average number of units in the system, respectively. Then,

$$\bar{N}' = \frac{\lambda\sigma}{\mu(1-\sigma)}, \tag{B.4}$$

$$\bar{N} = \bar{N}' - \frac{\lambda}{\mu}. \tag{B.5}$$

(iii) Let the r.v. w denote the waiting time in the system. Then

$$P(w \leqslant t) = 1 - \sigma\exp[-\mu(1-\sigma)t], \quad t \geqslant 0 \tag{B.6}$$

(iv) Let $\bar{W}$ and $\bar{W}'$ be the average waiting time at the queue and at the system, respectively. Then,

$$\bar{W}' = \frac{\sigma}{\mu(1-\sigma)}, \tag{B.7}$$

$$\bar{W} = \bar{W}' - \frac{1}{\mu}. \tag{B.8}$$

(v) The utilization factor is

$$\rho = \frac{\lambda}{\mu}. \tag{B.9}$$

Proof. See Kleinrock (1975, Chap. 6).

Proposition 3

The Laplace–Stieltjes transform of the distribution of the interdeparture time of an $H_2/M/1$ queuing system is,

$$D^*(s) = \frac{\mu}{\mu+s} + K_1\left(\frac{\lambda_1}{\lambda_1+s} - \frac{\mu}{\mu+s}\right) + K_2\left(\frac{\lambda_2}{\lambda_2+s} - \frac{\mu}{\mu+s}\right), \tag{B.10}$$

where

$$K_1 = \frac{(A_1-\tau)\mu}{\lambda_1-\mu}, \tag{B.11}$$

$$K_2 = \frac{(A_2-1+\tau)\mu}{\lambda_2-\mu}, \tag{B.12}$$

$$A_1 = \frac{\tau\lambda_1}{\lambda_1+(1-\sigma)\mu}, \tag{B.13}$$

$$A_2 = \frac{(1-\tau)\lambda_2}{\lambda_2+(1-\sigma)\mu}, \tag{B.14}$$

where

$$\sigma = \{(\lambda_1+\lambda_2+\mu) \pm \sqrt{(\lambda_1+\lambda_2+\mu)^2 - 4\{\lambda_1\lambda_2 + \mu[\tau\lambda_1 + (1-\tau)\lambda_2]\}}\}/2\mu. \tag{B.15}$$

Proof. Directly resulted from Proposition 1.

Proposition 4

The first two moments of the distribution of the interdeparture time of an $H_2/M/1$ queuing system are,

$$-D^{*\prime}(0) = -\frac{1}{\mu} - K_1\left(\frac{1}{\lambda_1} - \frac{1}{\mu}\right) - K_2\left(\frac{1}{\lambda_2} - \frac{1}{\mu}\right), \tag{B.16}$$

$$D^{*\prime\prime}(0) = \frac{2}{\mu^2} + 2K_1\left(\frac{1}{\lambda_1^2} - \frac{1}{\mu^2}\right) + 2K_2\left(\frac{1}{\lambda_2^2} - \frac{1}{\mu^2}\right), \tag{B.17}$$

where K_1 and K_2 are similar as those in Proposition 3.

Proof. Directly resulted from proposition 1.

Proposition 5

The Laplace–Stieltjes transform of the distribution of the interdeparture time of an $E_2/M/1$ queuing system is,

$$D^*(s) = \frac{\mu}{\mu+s} + L_1\left(\frac{\lambda_1}{\lambda_1+s} - \frac{\mu}{\mu+s}\right) + L_2\left(\frac{\lambda_2}{\lambda_2+s} - \frac{\mu}{\mu+s}\right), \tag{B.18}$$

where

$$L_1 = \frac{(A_1-B_1)\mu}{\lambda_1-\mu}, \tag{B.19}$$

$$L_2 = \frac{(A_2-B_2)\mu}{\lambda_2-\mu}, \tag{B.20}$$

$$B_1 = \frac{\lambda_2}{\lambda_2-\lambda_1}, \tag{B.21}$$

$$B_2 = \frac{\lambda_1}{\lambda_1-\lambda_2}, \tag{B.22}$$

$$\mathrm{A}_1 = \frac{B_2\lambda_1}{\lambda_1+(1-\sigma)\mu} \tag{B.23}$$

and

$$A_2 = \frac{B_2\lambda_2}{\lambda_2+(1-\sigma)\mu}, \tag{B.24}$$

where

$$\sigma = \left[(\lambda_1 + \lambda_2 + \mu) \pm \sqrt{(\lambda_1 + \lambda_2 + \mu)^2 - 4\lambda_1\lambda_2}\right] \Big/ 2\mu. \tag{B.25}$$

Proof. Directly derived from Proposition 1.

Proposition 6

The first two moments of the distribution of the interdeparture time of an $E_2/M/1$ queuing system are,

$$-D^{*\prime}(0) = -\frac{1}{\mu} - L_1\left(\frac{1}{\lambda_1} - \frac{1}{\mu}\right) - L_2\left(\frac{1}{\lambda_2} - \frac{1}{\mu}\right), \tag{B.26}$$

$$D^{*\prime\prime}(0) = \frac{2}{\mu^2} + 2L_1\left(\frac{1}{\lambda_1^2} - \frac{1}{\mu^2}\right) + 2L_2\left(\frac{1}{\lambda_2^2} - \frac{1}{\mu^2}\right), \tag{B.27}$$

where L_1 and L_2 are similar to those in Proposition 5.

Proof. Directly derived from Proposition 1.