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Mixed Integer Linear Programming (MILP) approach to deal with spatio-temporal water allocation

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Abstract

Water of sufficient quantity and quality is indispensable for multiple purposes like domestic use, irrigated agriculture, hydropower generation and ecosystem functioning. However, in many regions of the world water availability is limited and even declining. Moreover, water availability is variable in space and time so that it does not match with the spatio-temporal use pattern of the water consumers. To overcome the temporal discrepancy between availability and consumption, reservoirs are constructed. Monitoring and predicting the water available in the reservoirs, the needs of the consumers and the losses throughout the water distribution system are necessary requirements to fairly allocate the available water for the different consumers. In this article, the water allocation problem is considered as a Network Flow Optimization Problem (NFOP) to be solved by a spatio-temporal optimization approach using Mixed Integer Linear Programming (MILP) techniques.

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Keywords: Optimization; allocation; water; chain; Gurobi

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1. Introduction

Water is required for multiple purposes: domestic use, irrigated agriculture, industry, hydropower generation and ecosystem functioning. The availability of surface water is variable in space and time and this variability often does not match with the spatially distributed and temporal use pattern of the water consumers. For example, agriculture requires irrigation when precipitation is low and power needs to be generated throughout the year, also when river discharge is low. To overcome the temporal discrepancy between water availability and consumption, reservoirs are constructed. Monitoring water present in the reservoirs and the needs of the consumers is indispensable to decide a fair allocation. Since water management systems typically consist of more than one reservoir, each with its own characteristics and temporal dynamics, decisions must be based on spatio-temporal data of water availability and needs. The collection and processing of these data is a tedious issue not only because of the spatially and temporally distributed nature but also due to the fact that various measurements and communication devices are needed. This kind of issues is even more noticeable in areas were the dry and wet seasons are very different.

Water allocation problems have challenged water managers for decades [1]. Allocation can become controversial when competition for water increases between multiple water users such as municipalities, industry and irrigated agriculture. Increased population shifts and shrinking water supplies magnify this type of user competition in many regions across the globe. This competition will be aggravated if natural conditions become more unpredictable and as concern for water quantity and quality grows. A poorly-planned system for allocating water can be the origin of serious problems under disadvantageous climate and river-flow conditions.

All this has led decision-makers to the point that tools are needed for optimizing the water resource allocation. It is now recognized that the efficiency, equity and environmental soundness of water allocation and management must be improved by developing innovative techniques for environmental policy implementation including water allocation for various levels of complexity [2].

De Meyer et al. [3], give an overview of optimization methods and models focusing on decisions regarding the design and management of the upstream segment of the biomass-for-bioenergy supply chain. The aim of this paper is to identify existing optimization methods or models that satisfy specific requirements. The authors also give a further description of the biomass supply chain identified in [4] and [5]. Also, this paper presents an optimization model classification according to (1) the mathematical optimization methodology used, (2) the decision level and decision variables addressed and (3) the objective to be optimized.

Labadie [6] compared the most used methods to optimize water distribution: 1) Implicit stochastic optimization, 2) Linear programming models, 3) Network flow optimization models, 4) Nonlinear programming models, 5) Discrete dynamic programming models, 6) Explicit stochastic optimization, 7) Real-time control with forecasting and 8) Heuristic programming models. Morsi et al. [7], introduced a mixed integer linear modeling approach for the optimization of dynamic water supply networks based on the piecewise linearization of nonlinear constraints. One advantage of applying mixed integer linear techniques is that these methods are nowadays very mature, that is, they are fast, robust, and are able to solve problems with up to a huge number of variables. Other major point is that these methods have the potential of finding globally optimal solutions or at least to provide guarantees for the solution's quality. In this paper a demonstration of the applicability of their approach on example networks has been made. Tinoco et al. [8] [9], studied the Macul basin in the north of Ecuador. The objective of his research is to optimize the operation of a reservoir system. This system includes three reservoirs. Water will be transferred from one reservoir to others in order to allocate it in an optimal way to the different irrigation projects. This modelling approach consists of a trial-and-error process to identify the optimal amount of water and consists of the following steps: 1) River/reservoir system modeling in order to simulate and to optimize water availability for a period of historical data, 2) Post-statistical analyses of each of the resultant reservoir outflows and reservoir water levels, and 3) Extreme value analysis of the minimum reservoir water levels. Authors state that the approach consists in an easy and practical way to optimize water allocation from reservoirs. Mula et al. [10], present a review of mathematical programming models for supply chain production and transport planning with possible application to water. They made a classification based on eight aspects: 1) supply chain structure, 2) decision level, 3) supply chain modeling approach, 4) purpose, 5) shared information, 6) model limitations, 7) novelty contributed and 8) practical application. They concluded that: 1) most of the models are related with transportation planning, 2) the most widely modeling approach is mixed integer linear programming (MILP) where the use of heuristics and meta-heuristics stands out. 3) The main objective of the model is to minimize total supply chain costs as well as maximize the total income.

This paper presents a mixed integer linear programming (MILP) model for optimizing the allocation of the water supplied by multiple rivers to a reservoir to multiple downstream uses such as irrigation, hydropower generation, human consumption, ecosystem, industrial use, etc. The aim is to apply the model to the Los Rios province in Ecuador characterized by two distinct seasons along the year. Also, due to climate change there is a increasing water consumption competition between the different end users.

2. Materials and Methods

2.1. Generic representation of water supply chain

The water supply chain (WSC) as conceived in this paper is an abstraction of the activities related to the water management. This WSC is taking in account the hydrology of the river basin since it is assumed that sensor measurements of the discharges of water in the rivers and the amounts of water present in the reservoirs are available at a sufficiently high temporal resolution. Additionally, this WSC includes an 'Ecosystem' which stands for the water left in the river after all demands have been satisfied, part of it is ultimately flowing into the ocean.

2.2. Optimization Model

The optimization problem is about managing the reservoir levels and allocating the available water resources in such a way that the spatially and temporally distributed demands for water are optimally met. To this end, this research consider the problem to be of the type "Minimum cost flow problem (MCFP)" (Winston and Goldberg [11]) and to be solved with mathematical programming techniques (Linear, Integer and Mixed Integer linear programming). In line with the considered configuration of the WSC (Fig. 1), the objective function of the optimization model is the following (equation 1):

$$\min \sum_{k} \sum_{t} \left(\mathrm{P_{Dk}} * \mathrm{S_{k,t}}\right) \tag{1}$$

Constraints are introduced to regulate the water flow in the studied WSC. These constraints can be grouped into different types: (1) the constraints regulating the mass balance in the water flow between locations, (2) the constraints considering the physical and regulatory limitations such as capacity restrictions.

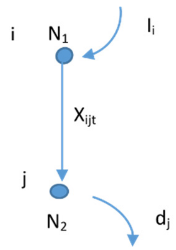

Fig. 1. Graphical representation of a segment and its input and output nodes in the network configuration

Subject to:

$$\sum_{j} X_{ijt} = I_{it}\ , \forall_{i,t} \tag{2}$$

$$\sum_{i} a_{ijt} X_{ijt} + S_{jt} = d_{jt},\ \forall_{i,t} \tag{3}$$

$$V_{i,t} = V_{i,t-1} - \sum_j X_{ijt} + \sum_j a_{jit} X_{jit}, \forall_{i,t} \quad (4)$$

$$\sum_j X_{ijt} = \sum_j X_{jit} \cdot a_{jit}, \forall_{i,t} \quad (5)$$

$$V_{i,t} \geq 500, \forall_{i,t} \quad (6)$$

$$V_{i,t} \leq 10000, \forall_{i,t} \quad (7)$$

$$\sum_i a_{ijt} X_{ijt} + S_{jt} = d_{jt}, \forall_{i,t} \quad (8)$$

$$\beta_{it} \sum_j X_{jit} = \sum_j X_{ijt}, \forall_{i,t} \quad (9)$$

Where:
Parameters
P_{Dk} [monetary units]: the penalty of not meeting one unit of demand (k).
a_{ijt}[cubic meters]: is what remains coming from node (i) moving to node j at time (t).
β_{jt}: A percent value, so $(1 - \beta_{jt})$ will be the loss of the corresponding entering the special sink node.
d_{jt}[cubic meters]: Is the amount of water going out from the node (j) at time (t).
Variables
I_{it}[cubic meters]: Is the amount of water arriving at the node (i) at time (t).
S_{jt}[cubic meters]: Is the amount of water that cannot be allocated to demand (j) at time (t)
$V_{i,t}$[cubic meters per second]: Volume in source (j) at time (t).
$V_{i,t-1}$ [cubic meters per second]: Previous volume in source (i) at time (t-1).
X_{ijt}[cubic meters]: Decision variables, and is the flow between two nodes, (i) and (j) are the nodes defining the segment at time (t).

Equations 2, 3, 4 and 5 represent the amount of water coming in and out of a node. Equation 2, takes in account water coming directly from a river (I_{it}) to a specific node. Equation 3, is more related to the amount of water allocated to a specific demand (d_{jt}). This equation takes into account the water that cannot be allocated (S_{jt}) and the remaining water from the previous node (a_{ijt}). Equation 4, establish the amount of water present at a certain time (t) in a reservoir node(i). This equation considers the previous volume ($V_{i,t-1}$) and the current one ($V_{i,t}$). Finally, equation 5 models the amount of water passing by each node.

Equations 6, 7, 8 and 9 are related to the constraints. Equations 6 and 7 establish the minimum and maximum amount of water that has to be present in a reservoir at a specific time. Equations 8 and 9 model the node when a portion of water is used to meet a demand and the remaining water returns to the following node.

2.3. Alternative definition

The network configuration studied in this paper is shown in Fig. 2. It consists of one reservoir that stores the water coming from several rivers (inputs I1 to I4). The water to be allocated is the amount of water that is present through time in the reservoir (volume R1), and that is added downstream of the reservoir (I5 to I7). The reservoir has both maximum and minimum capacity constraints to avoid upstream floods and to guarantee a correct aquatic ecosystem functioning of the reservoir.

Several users (D1 to D7) demand water either directly from the reservoir or from the river downstream of the reservoir. The main objective of the model is to optimally meet those demands given the available water and a set of penalties related to not meeting the demands. These penalty values are established in an arbitrary way but ensuring the assignment a big value to the demand that has the highest priority. If it is not possible to meet all demands the model minimizes the sum of the penalties.

Additionally, each segment (node) of the river (N1 to N6), reservoirs and rivers have evaporation problems. This means that some water is lost during the transportation process; to deal with this a loss function has been included based on the length of the segment and on the temperature of the area. The configuration of the alternative is shown in Fig. 2 and its characteristics are detailed in Table 1 in the numeral 8.

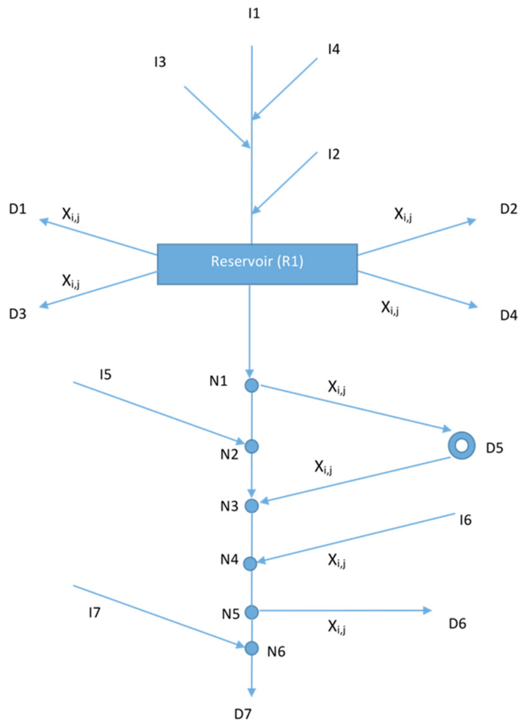

Fig. 2. Network configuration of the alternatives

2.4. Alternatives

In order to test and improve the optimization model, eight alternatives (AT) of increasing complexity have been addressed (Table 1).

Table 1. Description of the alternatives addressed

Alternative	Description
1	This alternative includes four inputs (I1, I2, I3 and I4), 1 reservoir and four demands (D1, D2, D3, D4) connected directly to the reservoir (Fig. 2). The objective is to minimize the sum of the penalties of not meeting a specific demand. Penalties values are equal for all demands. These values are: 3, 5, 4, 2 USD per m3.
2	Almost the same as alternative 1, but including a maximum and minimum capacity constraint for the reservoir.
3	This alternative includes four inputs (I1, I2, I3 and I4) and seven demands (D1, D2, D3, D4, D5, D6 and D7). The main difference with alternative 2 is that the three extra demands are located downstream of the reservoir. In the following alternatives CPLEX[12] and GUROBI[13] were used as solvers. Penalty values are: 3, 5, 4, 2, 3, 3, 10 USD per m3.
4	This alternative is the same as alternative 3, but setting the same penalty value (1 USD per m3) to each demand.

5	Alike AT 3 and 4, this alternative has four rivers as inputs and also includes seven demands. The main objective is to minimize the penalties of not meeting the water demands. The reservoir has a minimum capacity of 500 m3. Additionally, there is a demand called "D7" that is related with the minimum amount of water that has to be present in the river after allocation, to serve the ecosystem. Penalty values are: 3, 5, 4, 2, 3, 3, 10 USD per m3.
6	This alternative is identical to AT 5, but penalty values are equal.
7	This alternative is similar to AT 5. The only difference is the presence of extra inputs (I5, I6 and I7) downstream of the reservoir.
8	The alternative 8 is detailed in the following sections and the configuration of it is shown in Fig. 2. This model has the following configuration: 4 rivers coming to a reservoir, these rivers are considered as inputs (I1 to I4) and 3 extra-inputs downstream of the reservoir (I5, I6, I7). 7 uses or water demands (D1 to D7). The reservoir has to have at least 500 m3 of water at the end of the time step. A penalty value has to be paid if a demand is not met. Penalty values are: 3, 5, 4, 2, 3, 3, 10 USD per m3. Demand 7 is considered as a river and has to have at least 600 m3 of water. Each segment has a loss function associated, this function depends mainly in the length of the segment, temperature in the zone, etc. The maximum capacity of the reservoir is set to 10000 m3.

Those alternatives include several nodes; each node will receive a certain amount of water in a specific time step, this means that the amount of water present in a specific node at time t will be different on the same node at time t + 1. For simplicity, in the present alternatives this time value is set to zero assuming that the same amount of water will be present at the following nodes without a delay. The main objective of creating these alternatives is to determine if mixed integer linear programming methods are commonly applied to the water management networks and also if those result are good enough.

3. Results and Discussion

From the application of the optimization model and the execution of the solver (Gurobi). This last alternative (8) includes four inputs (I1, I2, I3 and I4) directly to the reservoir and three extra-inputs downstream of the reservoir (I5, I6 and I7) and four demands (D1, D2, D3, D4) directly from the reservoir, two more along of the river (D5 and D6) and D7 related directly to the ecosystem. Also, a value related to the amount of water that is lost during the transportation from the reservoir to the demand through the river has been added to each demand. In this sense, the model has to allocate the demand value plus this small loss value.

In Table 2, each demand value has increased a small value between 0.1 and 0.9 m3(columns allocate D1 to allocate D7) of water, this, in order to meet the demand and to avoid losses. In order to run the model a year (2014) dataset has been used. The results about the optimized water allocation are summarized in Table 2 and Fig. 3 (one month only). From results in Table 2, it is clear that the first day no water has been allocated, this is due to the constraint related to the minimum required amount of water in the reservoir at the end of the day (Equation 6 and 7). Additionally, it is clear that the model is mainly satisfying D7 (the river), which has the highest penalty value if the demand is not met, since the ecosystem has been given a high priority. Almost every day, this demand is receiving the demand of 600 m3 per day at the expense of the other demand nodes. Only one day (28/01/2014), the amount of water available in the reservoir was not sufficient to meet this demand and only 504 m3 could be allocated. This means that at the end of the day a specific amount has to be paid as a penalty (Fig. 3).

In Fig. 3, there are four periods in which a penalty has to be paid. In the beginning, the model has to be adjusted to store enough water for the next days. The second one is present exactly on 11/01/2014, the third one between 17/01/2014 and 22/01/2014 and finally at the end of the month on with an extreme 28/01/2014 where the highest penalty has to be paid.

Table 2. Optimization model results

Date	Total Supply	Allocate D1	Allocate D2	Allocate D3	Allocate D4	Allocate D5	Allocate D6	Allocate D7 (River)	Total Allocated
01/01/2014	500	0	0	0	0	0	0	0	0
02/01/2014	2300	190.1	50.1	100.1	70.1	90.1	140.1	600.8	1241.4
03/01/2014	2533.6	180.1	80.1	160.1	40.1	30.1	90.1	600.8	1181.4
04/01/2014	2881.2	160.1	70.1	90.1	50.1	180.1	70.1	600.8	1221.4
05/01/2014	3152.8	180.1	180.1	20.1	170.1	180.1	160.1	600.8	1491.4
06/01/2014	1790.4	170.1	140.1	50.1	90.1	10.1	40.1	600.8	1101.4
07/01/2014	1911	80.1	10.1	20.1	10.1	160.1	180.1	600.8	1061.4
08/01/2014	2382.6	70.1	40.1	130.1	50.1	80.1	40.1	600.8	1011.4
09/01/2014	2291.2	70.1	20.1	180.1	50.1	100.1	200.1	600.8	1221.4
10/01/2014	2287.8	160.1	200.1	110.1	50.1	60.1	90.1	600.8	1271.4
11/01/2014	1260.4	0	0	0	130.1	0	47.5	600.8	778.4
12/01/2014	2354	120.1	80.1	160.1	30.1	30.1	180.1	600.8	1201.4
13/01/2014	2172.6	140.1	130.1	60.1	70.1	140.1	40.1	600.8	1181.4
14/01/2014	2119.2	130.1	80.1	140.1	120.1	180.1	180.1	600.8	1431.4
15/01/2014	1924.8	190.1	130.1	10.1	20.1	100.1	190.1	600.8	1241.4
16/01/2014	2024.4	160.1	50.1	50.1	100.1	190.1	160.1	600.8	1311.4
17/01/2014	1583	40.1	95.7	110.1	160.1	60.1	50.1	600.8	1117
18/01/2014	1504	20.1	49.7	30.1	180.1	120.1	20.1	600.8	1021
19/01/2014	1404	37.9	0	190.1	50.1	0	70.1	600.8	949
20/01/2014	1904	20.1	60.1	70.1	190.1	10.1	70.1	600.8	1021.4
21/01/2014	1166.6	0	0	0	73.8	0	0	600.8	674.6
22/01/2014	1604	100.1	130.1	50.1	150.1	40.1	50.1	600.8	1121.4
23/01/2014	1506.6	20.1	100.1	10.1	100.1	90.1	100.1	600.8	1021.4
24/01/2014	1667.2	80.1	80.1	80.1	30.1	50.1	30.1	600.8	951.4
25/01/2014	1594.8	120.1	0	190.1	50.1	61.6	110.1	600.8	1132.8
26/01/2014	1654	126.9	0	80.1	200.1	0	180.1	600.8	1188
27/01/2014	1304	0	0	22	50.1	0	160.1	600.8	833
28/01/2014	954	0	0	0	0	0	0	504	504
29/01/2014	1704	80.1	20.1	170.1	40.1	60.1	160.1	600.8	1131.4
30/01/2014	1643.6	20.1	111.3	150.1	90.1	50.1	140.1	600.8	1162.6
31/01/2014	1304	0	0	0	170.1	0	55.1	600.8	826

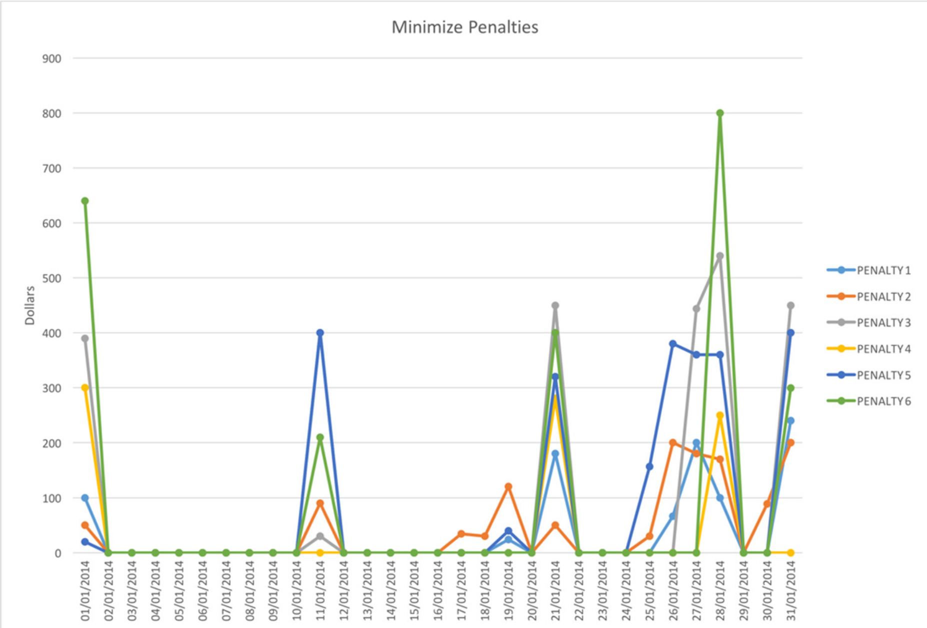

Fig. 3. Graphical representation of the penalties to be paid

4. Conclusions and future work

The use of mathematical models in the optimization of natural resources, water, in this case, can be a good option for this kind of techniques have been already tested in many studies and alternatives. Additionally, mathematical optimization models can include more restrictions over time including constraints without modifying the basic model. The use of mixed integer linear programming (MILP) methods can be considered as a good option when there is enough information and the problem to be solved is well known to model each related activity. To work with this kind of models, it is highly recommendable to work with at least two datasets that will be used for calibration and validation.

The generic representation of the water supply chain allows classifying each activity occurring in the water supply chain to a specific type of activity group. This allows defining a generic optimization model that will be able to optimize water supply chains in all kinds of regions independent of spatial and temporal scale. This also implies that case specific constraints can be included easily to be able to define a specific water supply chain correctly and include new activities that were not present before.

This research work is the basis to create an advanced and complete optimization model that will represents realistic situations. In this sense, the following steps will be related with the use of data from a specific area of Ecuador. At this moment, this model can be used in a strategic way in order to determine the best locations for future reservoirs as well as their capacity (maximum and minimum levels).

As a part of this research, real time data from sensors will be included to move towards an operational planning system. As a drawback, this kind of operational planning models require very accurate and detailed information. This requires sufficient and valid information in order to obtain a valid result. Additionally, the next step to follow in this research is to adapt/create a model to work with different time steps (variable (t) from all equations with a non-zero value). Additionally, a sensitivity analysis and uncertainty analysis will be included in order to determine the most suitable parameters and variables.

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