Lepton flavor violating muon decays in a model of electroweak-scale right-handed neutrinos

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The small neutrino mass observed in neutrino oscillations is nicely explained by the seesaw mechanism. Rich phenomenology is generally expected if the heavy neutrinos are not much heavier than the electroweak scale. A model with this feature built in has been suggested recently by Hung. The model keeps the standard gauge group but introduces chirality-flipped partners for the fermions. In particular, a right-handed neutrino forms a weak doublet with a charged heavy lepton, and is thus active. We analyze the lepton flavor structure in gauge interactions. The mixing matrices in charged currents (CC) are generally non-unitary, and their deviation from unitarity induces flavor changing neutral currents (FCNC). We calculate the branching ratios for the rare decays $\mu \to e\gamma$ and $\mu \to ee\bar{e}$ due to the gauge interactions. Although the former is generally smaller than the latter by three orders of magnitude, parameter regions exist in which $\mu \to e\gamma$ is reachable in the next generation of experiments even if the current stringent bound on $\mu \to ee\bar{e}$ is taken into account. If light neutrinos dominate for $\mu \to e\gamma$, the latter cannot set a meaningful bound on unitarity violation in the mixing matrix of light leptons due to significant cancelation between CC and FCNC contributions. Instead, the role is taken over by the decay $\mu \to ee\bar{e}$.

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Neutrino oscillation has provided the first evidence of physics beyond the Standard Model (SM) that the neutrinos are massive, non-degenerate and mix. The large or even maximal mixing angles measured in solar and atmospheric neutrinos should in principle allow the detection of lepton flavor violating (LFV) effects in the charged lepton sector, e.g., the observation of the muon decays, $\mu \to e\gamma$ and $\mu \to ee\bar{e}$. In simple extensions of SM that incorporate only right-handed neutrino singlets, this is not the case. As far as those loop-induced processes of charged leptons are concerned, all neutrinos can be considered as degenerate and their leading-order contribution is thus removed by the unitarity of the mixing matrix. The tiny mass of neutrinos diminishes the contribution further via the leptonic GIM mechanism: all such effects are suppressed by the tiny ratio of neutrino masses squared over those of weak gauge bosons and are therefore not observable in the foreseeable future [1].

It would be physically more interesting if LFV effects could also be observed beyond neutrino oscillations. The current limits on LFV muon decays are already stringent, with the branching ratios Br$(\mu \to e\gamma) < 1.2 \times 10^{-11}$ [2] and Br$(\mu \to ee\bar{e}) < 1.0 \times 10^{-12}$ [3]. The former one will likely be pushed to 10^{-13}–10^{-14} in the com-

ing years [4]. Significant progress has also been made in LFV τ decays, although the constraints are not comparable to the muon's in the near future. An observation of such processes will unambiguously point to non-trivial new physics. There are indeed many alternatives for new physics that contain new sources of lepton flavor violation. For instance, the LFV decays could be large enough to be observable in supersymmetric models [5], in the extension of SM by a Higgs triplet [6], and in the littlest Higgs model with T-parity [7], to just mention a few among many [8]. For a model-independent, leading logarithmic QED correction to the decay $\mu \to e\gamma$, see Ref. [9].

The extreme smallness of neutrino mass can be understood in the elegant seesaw mechanism [10]. In its standard implementation, this is done by assuming a Dirac mass of order charged leptons' and a huge mass of heavy neutrinos typically of order grand unification scale. But then the heavy neutrinos that are at the heart of new physics are beyond direct experimental accessibility. Richer phenomenology would be possible if heavy neutrinos had a mass not much greater than the electroweak scale so that they could be detected at high energy colliders.

A model with the above desired feature built in has been suggested recently by Hung [11]. (See also Ref. [12] for an alternative model building with neutrinos at the electroweak scale.) The model retains the SM gauge group albeit in a 'vector-like' manner: the SM (ordinary) fermions are augmented with mirror fermions

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that carry the same charges as their SM partners but with chirality flipped. In particular, a right-handed neutrino that is sterile in many models now becomes a member of a weak doublet of mirror leptons. A tiny Dirac mass for neutrinos is provided by a scalar singlet whose vacuum expectation value is not necessarily associated with the electroweak scale, while a Majorana mass of order the electroweak scale is introduced by a scalar triplet. As we shall describe in detail, this model has a rich flavor structure in weak gauge couplings as well as in Yukawa couplings. The weak charged couplings are generally non-unitary with or without restricting to the subspace of light leptons, and flavor changing neutral currents (FCNC) occur in a way that is controlled by the weak charged couplings. It is the purpose of this work to explore their implications for the LFV muon decays. We find that there exist parameter regions where the decay $\mu \to e\gamma$ is accessible in the planned experiments when the current upper bound on $\mu \to ee\bar{e}$ is almost saturated.

We start with a brief description of the model relevant to our later analysis; for a full account of it, see Ref. [11]. We consider three generations and use slightly different notations from the reference. The SM and mirror leptons with quantum numbers under the gauge group $SU(2) \times U(1)_Y$ are:

$$F_L = \begin{pmatrix} n_L \\ f_L \end{pmatrix} (\mathbf{2}, Y = -1), \qquad f_R(\mathbf{1}, Y = -2);$$

$$F_R^M = \begin{pmatrix} n_R^M \\ f_R^M \end{pmatrix} (\mathbf{2}, Y = -1), \qquad f_L^M(\mathbf{1}, Y = -2); \tag{1}$$

where the subscripts L, R refer to chirality and the superscript M to mirror. For anomaly cancelation, the quark sector also has mirror partners that are of no interest here. Besides the SM scalar doublet Φ, the model contains the new scalars

$$\phi(\mathbf{1}, 0), \qquad \chi(\mathbf{3}, 2), \tag{2}$$

plus an additional triplet $\xi(\mathbf{3}, 0)$ that together with χ preserves the custodial symmetry [13] but is irrelevant here.

The Yukawa couplings of leptons are, with the generation indices suppressed,

$$-\mathscr{L}_\Phi = y\overline{F_L}\Phi f_R + y_M \overline{F_R^M}\Phi f_L^M + \text{h.c.},$$

$$-\mathscr{L}_\phi = x_F \overline{F_L}F_R^M \phi + x_f \overline{f_R}f_L^M \phi + \text{h.c.},$$

$$-\mathscr{L}_\chi = \frac{1}{2}z_M \overline{(F_R^M)}^C (i\tau^2)\chi F_R^M + \text{h.c.}, \tag{3}$$

where $\psi^C = \mathscr{C}\gamma^0\psi^*$, $\mathscr{C} = i\gamma^0\gamma^2$, and

$$\chi = \frac{1}{\sqrt{2}}\vec{\tau}\cdot\vec{\chi} = \frac{1}{\sqrt{2}}\begin{pmatrix} \chi^+ & \sqrt{2}\chi^{++} \\ \sqrt{2}\chi^0 & -\chi^+ \end{pmatrix}. \tag{4}$$

A potential Majorana coupling of χ to F_L is forbidden by imposing an appropriate $U(1)$ symmetry [11]. Suppose the vacuum expectation values have the structure:

$$\langle\Phi\rangle = \frac{v_2}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \langle\phi\rangle = v_1, \qquad \langle\chi\rangle = v_3\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \tag{5}$$

where $v_{2,3}$ contribute to the masses of weak gauge bosons and are naturally of order the electroweak scale while v_1 is not necessarily related to it. In the basis of f, f^M, the charged lepton mass terms are

$$-\mathscr{L}_m^f = (\overline{f_L}, \overline{f_L^M})m_f\begin{pmatrix} f_R \\ f_R^M \end{pmatrix} + \text{h.c.},$$

$$m_f = \begin{pmatrix} \frac{v_2}{\sqrt{2}}y & v_1 x_F \\ v_1 x_f^\dagger & \frac{v_2}{\sqrt{2}}y_M^\dagger \end{pmatrix}, \tag{6}$$

while the neutrino mass terms are

$$-\mathscr{L}_m^n = \frac{1}{2}(\overline{n_L}, \overline{(n_R^M)}^C)m_n\begin{pmatrix} n_L^C \\ n_R^M \end{pmatrix} + \text{h.c.},$$

$$m_n = \begin{pmatrix} 0 & v_1 x_F \\ v_1 x_F^T & v_3 z_M \end{pmatrix}. \tag{7}$$

The seesaw mechanism operates for a Majorana mass of order the electroweak scale and a Dirac mass proportional to v_1 that can be chosen small. This relaxes in some sense the tension in ordinary seesaw models between the generation of a light neutrino mass and the observability of heavy neutrinos at colliders [11].

The lepton mass matrices are diagonalized by unitary transformations ($a = L, R$):

$$\begin{pmatrix} f \\ f^M \end{pmatrix}_a = X_a \ell_a, \qquad X_L^\dagger m_f X_R = m_\ell = \text{diag}(m_\alpha),$$

$$\begin{pmatrix} n_L^C \\ n_R^M \end{pmatrix} = Y\nu_R, \qquad Y^T m_n Y = m_\nu = \text{diag}(m_i), \tag{8}$$

where $\alpha = e, \mu, \tau, \ldots$ denotes the mass eigenstates of the charged leptons and $i = 1, 2, 3, \ldots$ those of the neutrinos with the first (last) three being light (heavy). The neutrinos are of Majorana-type, $\nu = \nu_R + \nu_L$ with $\nu_L = \nu_R^C$. There is a constraint on their masses from the zero texture, $\sum_{k=1}^6 m_k Y_{ik} Y_{jk} = 0$, for $i, j = 1, 2, 3$.

The above diagonalizing matrices will enter the gauge (and Yukawa) interactions of leptons. Some algebra yields,

$$\mathscr{L}_g = g_2(j_W^{+\mu}W_\mu^+ + j_W^{-\mu}W_\mu^- + J_Z^\mu Z_\mu) + e J_{\text{em}}^\mu A_\mu, \tag{9}$$

where the currents are ($P_{L,R} = (1 \mp \gamma_5)/2$)

$$\sqrt{2}j_W^{+\mu} = \bar{\nu}\gamma^\mu(V_L P_L + V_R P_R)\ell,$$

$$c_W J_Z^\mu = \frac{1}{2}\bar{\nu}\gamma^\mu(V_L V_L^\dagger P_L + V_R V_R^\dagger P_R)\nu$$

$$\qquad - \frac{1}{2}\bar{\ell}\gamma^\mu(V_L^\dagger V_L P_L + V_R^\dagger V_R P_R)\ell + s_W^2 \bar{\ell}\gamma^\mu\ell,$$

$$J_{\text{em}}^\mu = -\bar{\ell}\gamma^\mu\ell, \tag{10}$$

and $c_W = \cos\theta_W$, $s_W = \sin\theta_W$ with θ_W being the Weinberg angle. To relate the matrices V_L, V_R to X_a, Y, it is convenient to decompose the latter into the up and down 3×6 blocks,

$$X_a = \begin{pmatrix} X_a^u \\ X_a^d \end{pmatrix}, \qquad Y = \begin{pmatrix} Y^u \\ Y^d \end{pmatrix}, \tag{11}$$

then

$$V_L = Y^{uT}X_L^u, \qquad V_R = Y^{d\dagger}X_R^d, \tag{12}$$

with $V_L^T V_R = 0$. These matrices are generally non-unitary and the deviation from unitarity induces FCNC in both sectors of neutrinos and charged leptons:

$$V_L V_L^\dagger = Y^{uT}Y^{u*}, \qquad V_R V_R^\dagger = Y^{d\dagger}Y^d;$$

$$V_L^\dagger V_L = X_L^{u\dagger}X_L^u, \qquad V_R^\dagger V_R = X_R^{d\dagger}X_R^d. \tag{13}$$

X_L^d, X_R^u do not enter the charged currents (CC) since f_L^M, f_R are $SU(2)$ singlets. Although f_L^M, f_R carry $U(1)_Y$ charges, their electromagnetic currents are vector-like and their neutral currents (NC) are also vector-like when combined with those of f_R^M, f_L so that X_L^d, X_R^u do not enter these currents either.

The gauge interactions displayed above will induce LFV processes at both tree and loop levels. The leading contribution to the decay $\mu \to e\gamma$ occurs at one loop as shown in Fig. 1. The two diagrams corresponding to CC and FCNC gauge interactions give the following on-shell amplitudes:
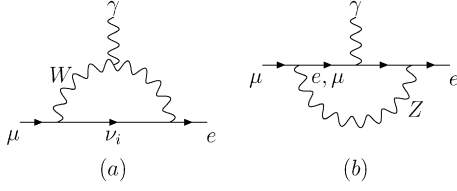
Fig. 1. Diagrams contributing to $\mu \to e\gamma$.

$$\mathscr{A}_W = \frac{e}{(4\pi)^2}\sqrt{2}G_F q^\beta \epsilon^{\alpha*}\bar{u}_e i\sigma_{\alpha\beta}\big[m_\mu(V_1 P_R + V_2 P_L)\mathscr{F}(r_i)$$
$$+ m_i(V_3 P_L + V_4 P_R)\mathscr{G}(r_i)\big]u_\mu,$$
$$\mathscr{A}_Z = \frac{e}{(4\pi)^2}\sqrt{2}G_F q^\beta \epsilon^{\alpha*}\bar{u}_e i\sigma_{\alpha\beta}m_\mu\frac{2}{3}\big[-2(1+s_W^2)V_1 P_R$$
$$+ (3-2s_W^2)V_2 P_L\big]u_\mu, \tag{14}$$

where ϵ and q are respectively the polarization and momentum of the photon, and $u_{e,\mu}$ the lepton spinors. The ratio $r_i = m_i^2/m_W^2$, and the mixing matrix elements are

$$V_1 = (V_L^\dagger)_{ei}(V_L)_{i\mu}, \qquad V_2 = (V_R^\dagger)_{ei}(V_R)_{i\mu},$$
$$V_3 = (V_R^\dagger)_{ei}(V_L)_{i\mu}, \qquad V_4 = (V_L^\dagger)_{ei}(V_R)_{i\mu}. \tag{15}$$

The summation over the neutrino index i is understood in both amplitudes. The loop functions are found to be

$$\mathscr{F}(r) = \frac{1}{6(1-r)^4}\big[10 - 43r + 78r^2 - 49r^3 + 4r^4 + 18r^3\ln r\big],$$
$$\mathscr{G}(r) = \frac{1}{(1-r)^3}\big[-4 + 15r - 12r^2 + r^3 + 6r^2\ln r\big]. \tag{16}$$

We have taken $m_e = 0$ and kept m_μ only until its linear term that is required for chirality flip. This is a good approximation even for the τ decays, $\tau \to e\gamma, \mu\gamma$. In the Z diagram we have ignored smaller contributions from other charged leptons and small corrections to the diagonal Z vertex so that we stay at the same precision level as the W diagram.

An interesting technical point is in order. It is simplest to work in unitarity gauge. For the Z diagram, this is all right both because the would-be Goldstone boson contributes at a higher order in the lepton masses than kept in the above and because the diagram is convergent enough for the relevant Lorentz structure. But this is not automatically true with the W diagram which is more ultraviolet divergent due to the triple gauge coupling. There is no guarantee in this case that the order of removing the ultraviolet regulator commutes with that of taking the unitarity gauge limit. As a matter of fact, although the diagram is convergent in both unitarity and R_ξ gauges, there is a finite difference in the terms linear in the lepton masses between the results obtained in the two gauges. This caveat is restricted to the mentioned terms because terms of a higher order are convergent enough to allow the free interchange of taking the limits. In the conventional case of unitary, pure left-handed couplings, the linear terms are killed by the unitarity of V_L so that an identical result can be reached in either gauge [15]. This is no more the case here. Considering this, we have replaced the terms linear in either m_μ or m_i obtained in unitarity gauge by those obtained in R_ξ gauge whose ξ dependence is canceled as expected.

The above amplitude involves several neutrino masses and many mixing matrix elements. In our later numerical analysis, we shall make some approximations. First, the light neutrinos can be safely treated as massless. Then, $\mathscr{F} \to \frac{5}{3}$, and the $\mathscr{G}$ term multiplied by m_i can be ignored. In simple extensions of SM, the leading term of $\mathscr{F}$ is removed by the unitarity of the CC mixing matrix of light leptons while the $\mathscr{G}$ term does not appear, leaving

behind a significantly GIM suppressed term that is not observable [1]. This is not the case in the type of models considered here. From the phenomenological point of view, neutrino oscillation experiments that are the main source of the lepton mixing matrix so far, are not yet precise enough to test its unitarity. Instead, it is exactly the lepton flavor changing transitions studied here that provide the most stringent constraint on the unitarity. Second, we assume that the heavy neutrinos are almost degenerate. We checked that the leading terms of $\mathscr{F}(r)$ and $\mathscr{G}(r)$ in the limit $r \to \infty$ deviate significantly from the exact values for m_i of order the electroweak scale or slightly higher. We shall thus retain their exact forms for numerical analysis. As a bonus of the approximations, the amplitude depends on the products of matrix elements summed over light and heavy neutrinos respectively:

$$V_1^l = \sum_{i=1}^{3}(V_L^\dagger)_{ei}(V_L)_{i\mu}, \qquad V_2^l = \sum_{i=1}^{3}(V_R^\dagger)_{ei}(V_R)_{i\mu},$$
$$V_3^l = \sum_{i=1}^{3}(V_R^\dagger)_{ei}(V_L)_{i\mu}, \qquad V_4^l = \sum_{i=1}^{3}(V_L^\dagger)_{ei}(V_R)_{i\mu}, \tag{17}$$

and similarly for $V_{1,2,3,4}^h$ with i summed over 4, 5, 6.

The branching ratio is then

$$\text{Br}(\mu \to e\gamma) = \frac{3\alpha}{8\pi}\big(|h_L|^2 + |h_R|^2\big), \tag{18}$$

where, denoting the common heavy neutrino mass as m_h and $r_h = m_h^2/m_W^2$,

$$h_L = \frac{5}{3}V_2^l + V_2^h\mathscr{F}(r_h) + \frac{m_W}{m_\mu}V_3^h\sqrt{r_h}\mathscr{G}(r_h)$$
$$+ \frac{2}{3}(3-2s_W^2)(V_2^l + V_2^h),$$
$$h_R = \frac{5}{3}V_1^l + V_1^h\mathscr{F}(r_h) + \frac{m_W}{m_\mu}V_4^h\sqrt{r_h}\mathscr{G}(r_h)$$
$$- \frac{4}{3}(1+s_W^2)(V_1^l + V_1^h). \tag{19}$$

Note in passing that the heavy neutrinos do not necessarily decouple in the heavy mass limit. For $r \to \infty$, $\mathscr{F}(r) \to \frac{2}{3}$ and $\mathscr{G}(r) \to -1$. The explicit factor m_h appearing in front of $\mathscr{G}(r_h)$ is actually canceled by m_h^{-1} coming from $V_{3,4}^h$, since the latter are proportional to $v_1 x_F/m_h$ with $v_1 x_F$ being independent of m_h to good precision. The contribution to the same process from the heavy charged leptons-ϕ loop has recently been considered in Ref. [14] in the heavy lepton limit. The singlet scalar ϕ has been assumed not to mix with other scalars. Note that even with this simplifying assumption the coupling matrices involved in the two types of contributions cannot be mutually obtained. In particular, the neutrino diagonalizing matrix Y does not enter into the ϕ diagram.

Now we turn to the decay $\mu \to ee\bar{e}$ whose leading term occurs at the tree level via FCNC. There are two diagrams due to identical fermions appearing in the final state. Once again, we ignore the small correction to the diagonal $Ze\bar{e}$ vertex in SM. Taking into account a factor of $\frac{1}{2}$ in the phase space, the branching ratio is

$$\text{Br}(\mu \to ee\bar{e}) = \frac{1}{2}|V_1^l + V_1^h|^2\big[(1-2s_W^2)^2 + 2s_W^4\big]$$
$$+ \frac{1}{4}|V_2^l + V_2^h|^2\big[(1-2s_W^2)^2 + 8s_W^4\big]. \tag{20}$$

The two branching ratios involve the following unknown parameters: the six complex matrix elements in the form of $V_{1,2}^l$, $V_{1,2,3,4}^h$ plus one heavy neutrino mass m_h. Roughly speaking, for
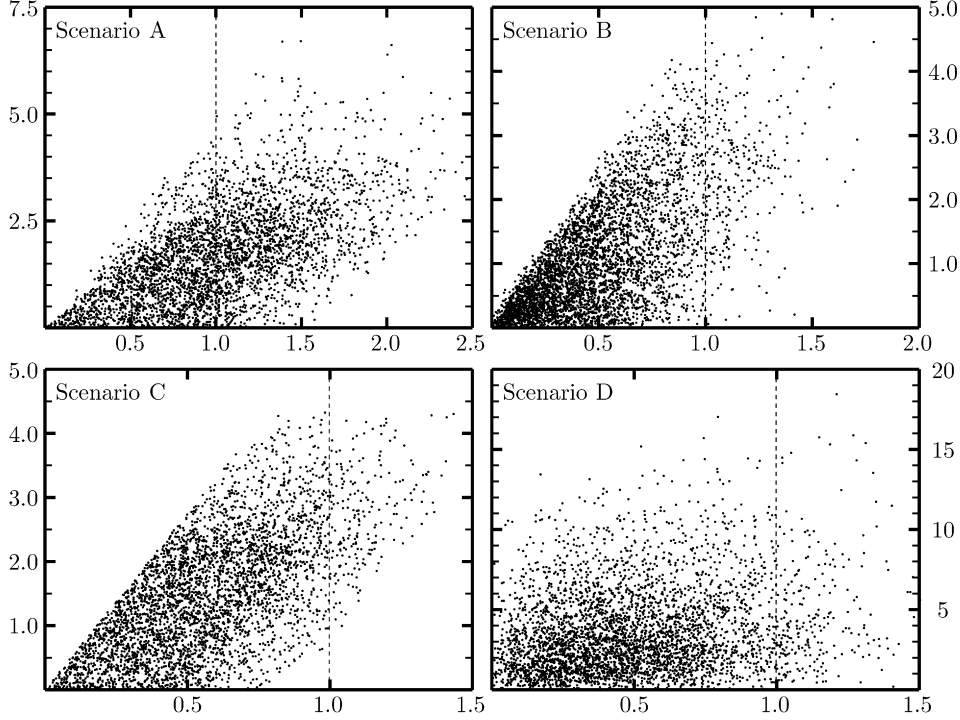
Fig. 2. Sampled points for Br($\mu \to ee\bar{e}$) (horizontal, in units of 10^{-12}) and Br($\mu \to e\gamma$) (vertical, in units of 10^{-14}) for the four scenarios described in the text. The dashed vertical line shows the current upper bound on Br($\mu \to ee\bar{e}$).

all matrix elements of similar order and m_h deviating not much from m_W, we have Br($\mu \to e\gamma$)/Br($\mu \to ee\bar{e}$) $\sim \frac{\alpha}{\pi} \sim 2 \times 10^{-3}$. One cannot get better quantitative feel of the effects without making some further simplifications. To demonstrate the physical relevance of our results, we choose to present our numerical results by sampling m_h and the matrix elements in certain ranges. We consider the following scenarios for the purpose of illustration. For the standard input parameters, we use $\alpha = 1/137.04$, $m_W = 80.2$ GeV, $m_\mu = 0.1056$ GeV, $s_W^2 = 0.23$.

We find an algebraically simple case after some inspection. Suppose the upper-right 3×3 block of Y is real. In this scenario A, our special neutrino spectrum (three almost massless plus three almost degenerate and heavy) implies that the two off-diagonal 3×3 blocks of Y vanish, the lower-right block is trivially identity and the upper-left one is unitary. Then, $V_1^l = (x_L^\dagger x_L)_{e\mu}$, $V_2^h = -(x_R^\dagger x_R)_{e\mu}$ while all others vanish, where $x_{L,R}$ are the upper-left 3×3 blocks of $X_{L,R}$ respectively. Since we have no idea of their magnitudes, we sample randomly the real and imaginary parts of V_1^l, V_2^h between -2×10^{-6} and $+2 \times 10^{-6}$, keeping an eye on the current upper bound on Br($\mu \to ee\bar{e}$). For the heavy neutrino mass we choose $m_h = 50, 100$ up to 1000 GeV. The combined result is shown in Fig. 2. For Br($\mu \to ee\bar{e}$) $< 10^{-12}$, most points drop in the region where Br($\mu \to e\gamma$) is at the edge of precision available in the next generation of experiments, $\sim 10^{-14}$.

In scenario B, we sample the real and imaginary parts of $V_{1,2}^l$, $V_{1,2}^h$ in the range $[-10^{-6}, 10^{-6}]$ while keeping $V_3^h = V_4^h = 0$ and assuming the value of m_h as in scenario A. The matrix elements are chosen smaller than in scenario A in order that most points would not break the current bound on Br($\mu \to ee\bar{e}$). The terms from the four elements tend to interfere constructively so that Br($\mu \to e\gamma$) is slightly larger than in scenario A.

We assume in scenario C that only the contribution of light neutrinos is important while that of heavy ones is suppressed for some reason. The real and imaginary parts of V_1^l, V_2^l run randomly in the range from -1.5×10^{-6} to $+1.5 \times 10^{-6}$, and the result is independent of m_h. We find that Br($\mu \to e\gamma$) $\lesssim$ a few $\times 10^{-14}$ for

Br($\mu \to 3e$) $< 10^{-12}$ in most regions of the parameter space. Actually, this scenario can be better treated analytically. The branching ratios are

$$\text{Br}(\mu \to e\gamma) \approx 10^{-4}\left[0.0064|V_1^l|^2 + 102|V_2^l|^2\right],$$
$$\text{Br}(\mu \to ee\bar{e}) \approx 0.20|V_1^l|^2 + 0.18|V_2^l|^2. \tag{21}$$

The very small coefficient, $(1 - 4s_W^2)^2 = 0.0064$, of $|V_1^l|^2$ in Br($\mu \to e\gamma$) arises from the destructive interference between the W and Z graphs. If the W graph were only present, the coefficient would be 25. This is indeed the case in the models where FCNC does not appear in the charged lepton sector; and the light neutrino contribution to $\mu \to e\gamma$ via pure left-handed CC gauge interactions (i.e., $V_2^l = 0$) has been employed in Ref. [16] to set a stringent upper bound on unitarity violation in the mixing matrix of light leptons (i.e., $|V_1^l|^2$). However, for the type of new physics as discussed here in which FCNC occurs in both sectors of leptons, we can no longer utilize the decay to set a useful bound on $|V_1^l|^2$ as its effect has been diminished by a factor of $25/0.0064 \sim 3900$. In this case, the decay $\mu \to ee\bar{e}$ studied here sets a much more stringent bound, $|V_1^l|^2 < 5 \times 10^{-12}$. This means that we can ignore V_1^l for $\mu \to e\gamma$. Using again the bound from $\mu \to ee\bar{e}$, this implies in turn an upper bound on $\mu \to e\gamma$ in this scenario:

$$\text{Br}(\mu \to e\gamma) \approx 10^{-2}|V_2^l|^2 < 5.7 \times 10^{-13}. \tag{22}$$

The best one can have is to saturate the above bound on $\mu \to e\gamma$ while sitting at the current experimental bound on $\mu \to ee\bar{e}$. It is impossible in particular to approach a branching ratio of 10^{-12} for both decays simultaneously.

To get some feel on the mixed effect between left- and right-handed CC currents involving light charged leptons and heavy neutrinos, we consider scenario D. The real and imaginary parts of $V_1^{l,h}$, $V_2^{l,h}$ are allowed to run randomly in the range from -10^{-6} to $+10^{-6}$ while the range of $V_{3,4}^h$ is smaller by a factor of 10^{-3}. The latter two are likely smaller than others since they involve

the Dirac neutrino mass term proportional to $v_1 x_F$ where v_1 is small [11]. It seems difficult to get an exact handle of the orders of magnitude on the involved matrix elements since the heavy charged lepton masses also set in through the diagonalizing matrices $X_{L,R}$. We thus choose to illustrate our results by assuming a value for m_h from 50 GeV to 500 GeV at a step of 50 GeV when sampling V's. We do not assume a larger value for it to avoid amplifying artificially the heavy neutrino term because as we mentioned earlier $V_{3,4}^h$ is proportional to m_h^{-1} to good precision. We find that Br$(\mu \to e\gamma)$ can reach the level of 10^{-13} for Br$(\mu \to 3e) < 10^{-12}$.

The small neutrino mass is naturally explained by the seesaw mechanism. Physics would be phenomenologically more interesting if heavy neutrinos have a mass close to the electroweak scale. In that case, they would be directly accessible at high energy colliders. On the other hand, the large leptonic mixing observed in neutrino oscillations does not imply large lepton flavor violation in the charged lepton sector if the neutrino mass is incorporated in a trivial manner. An observation of LFV charged lepton decays would thus point to non-trivial new physics related to the origin of neutrino mass. This is encouraged especially by experimental advances expected in the near future. Motivated by this observation, we have studied the rare decays $\mu \to e\gamma, ee\bar{e}$ in a model suggested recently in which non-trivial new physics does appear with heavy neutrinos at the electroweak scale. Although Br$(\mu \to e\gamma)$ is generally smaller than Br$(\mu \to ee\bar{e})$ by three orders of magnitude, there exists a significant portion of the parameter space in which Br$(\mu \to e\gamma)$ reaches or is within the sensitivity available in the new generation of experiments without breaking the current bound on $\mu \to ee\bar{e}$. But it is generally impossible to reach the level of 10^{-12} for both decays simultaneously. When the direct contribution from heavy neutrinos enters, it is difficult to make a definite quantitative prediction due to too many free parameters. But if for some reason the effect of heavy neutrinos is strongly suppressed compared to light neutrinos, the situation becomes transparent. Due to the destructive interference between the CC and FCNC interactions, the decay $\mu \to e\gamma$ is insensitive to the unitarity violation in the sector of light leptons. Instead, the other one $\mu \to ee\bar{e}$ proceeding through tree level FCNC can set a stringent bound on it. In this scenario, the best one can expect for the decays is Br$(\mu \to e\gamma) \sim 5 \times 10^{-13}$ and Br$(\mu \to ee\bar{e}) \sim 10^{-12}$.

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