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Lie group analysis of unsteady MHD three dimensional by natural convection from an inclined stretching surface saturated porous medium

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Abstract

Lie group method is investigated for solving the problem of heat transfer in an unsteady, three-dimensional, laminar, boundary-layer flow of a viscous, incompressible and electrically conducting fluid over inclined permeable surface embedded in porous medium in the presence of a uniform magnetic field and heat generation/absorption effects. A uniform magnetic field is applied in the y-direction and a generalized flow model is presented to include the effects of the macroscopic viscous term and the microscopic permeability of porous medium. The infinitesimal generators accepted by the equations are calculated and the extension of the Lie algebra for the problem is also presented. The restrictions imposed by the boundary conditions on the generators are calculated. The investigation of the three-independent-variable partial differential equations is converted into a two-independent-variable system by using one subgroup of the general group. The resulting equations are solved numerically with the perturbation solution for various times. Velocity, temperature and pressure profiles, surface shear stresses, and wall-heat transfer rate are discussed for various values of Prandtl number, Hartmann number, Darcy number, heat generation/absorption coefficient, and surface mass-transfer coefficient.

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Keywords: Lie group method; Unsteady viscous flow; Porous medium; Stretching surface; MHD

1. Introduction

The flow and heat transfer stirred up by a stretching surface entering the cooling viscous fluid through porous media is momentous in a number of practical engineering processes. For example, materials which are manufactured by extrusion processes and heat-treated substances proceeding between a feed roll and a wind-up roll can be classified as the continuously stretching surface. In order to acquire the top-grade property of the final product, the cooling procedure should be effectively controlled.

In the past few decades, the related investigation about the stretching surface has never been interrupted. Sakiadis [16,17] firstly put forward the very basic governing equation on this continuous moving solid surface problem.

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The numerical results of Sakiadis [16,17] were confirmed experimentally by Tsuo et al. [20] for continuously moving surface with a constant velocity. In addition, Chen and Strobel [5] and Jacobi [12] have reported results for uniform motion of the stretched surface. Gorla and Sidawi [9] have reported similarity transformations and numerical solutions for the problem of steady, three-dimensional free convection flow on a stretching surface with suction and blowing. Some work concerning hydromagnetic flows and heat transfer of electrically conducting fluids over a stretching surface can be found in the papers [6,4,21]. As mentioned by Vajravelu and Hadjinicolaou [21], the rate of cooling involved in these processes can greatly affect the properties of the end product. This rate of cooling has been shown to be controlled by the use of electrically conducting working fluids with applied magnetic fields. Chamkha [2,3] has investigated the problem of steady and unsteady states, respectively, of laminar, hydromagnetic, three-dimensional free convection flow over a vertical stretching surface in the presence of heat generation or absorption effects.

On the other hand, it is well known that the discovery of the integration theory of differential equations, by the Norwegian mathematician Sophus Lie at the beginning of the nineteenth century, has played a vital role in the investigations of different mathematical aspects of solution systems governed by continuous equations during the past few decades. The primary objective of the Lie symmetry analysis advocated by Sophus Lie is to find one- or several-parameter local continuous transformations leaving the equations invariant and then exploit them to obtain the so-called invariant or similarity solutions, invariants, integrals of motion, etc. [15,14,1,10]. Yurusoy and Pakdemirli [22] found Symmetry reductions of unsteady three-dimensional boundary layers of some non-Newtonian fluids. They [23] have obtained exact solutions of boundary layer equations of a special non-Newtonian fluid over a stretching sheet. Kalpakides and Balassas [13] studied the free convective boundary layer problem of an electrically conducting fluid over an elastic surface using group theoretic method. Ibrahim et al. [11] investigated the similarity reductions for problems of radiative and magnetic field effects on free convection and mass-transfer flow past a semi-infinite flat plate. Sivasankaran et al. [18,19] studied coupled heat and mass transfer fluid flow by natural convection past an inclined semi-infinite porous surface using Lie group analysis. EL-Hakiem et al. [7] presented group theoretic analysis of unsteady free convection flow over a continuous moving vertical plate embedded in a fluid-saturated porous medium in the presence of magnetic field effect. They [8] have applied Lie group method for solving the present problem with uniform heat flux in steady state.

In this paper, we present Lie group method to obtain a new class of symmetry groups admitted by a system of PDEs. A concept of our treatment is to carry out symmetry group analysis on particular special solutions. To illustrate this method, we apply our method to equations of heat transfer in an unsteady, three-dimensional, laminar, boundary-layer flow of a viscous, incompressible and electrically conducting fluid over inclined permeable surface saturated porous medium in the presence of a uniform magnetic field and heat generation/absorption effects. As a result, we obtain a new class of invariant solution which gives an account of a problem which was considered in [3] in the absence of permeability of porous medium effect. We have also solved the resulting equations using the perturbation technique for various times. Velocity, temperature and pressure profiles with surface shear stresses and wall-heat transfer rate are discussed and presented graphically for different values of governing physical parameters.

2. Mathematical formulation

Consider unsteady, laminar, three-dimensional natural convective boundary-layer flow of an electrically conducting and heat generation/absorbing fluid over a semi-infinite inclined permeable surface embedded in porous medium. The surface is assumed to be permeable and linearly stretched in the x-direction with a velocity bx. The y-direction makes an angle Ω with the horizontal line while the z-direction is normal to the plate surface. A uniform magnetic field is applied in the y-direction. This gives rise to magnetic effects in both the x- and z-directions. The application of the magnetic field in the y-direction is done so as to allow suppression of convective flow in these directions. This is important in terms of controlling the quality of the product being stretched (see [21]). In addition, uniform suction or injection is imposed at the plate surface in the z-direction. The coordinate system and flow model are shown in Fig. 1. All fluid properties are assumed constant except the density in the buoyancy terms of the x- and y-momentum equations. Assume that the edge effects are negligible, and the magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected. Also, the Hall effect of magneto-hydrodynamics, Joule heating, and the viscous dissipation are neglected. The thermophysical properties of the fluid and porous media are constant. All dependent variables will be independent of the y-direction; see Chamkha [3], including the viscous term, Darcy term and magnetic force, boundary-layer and

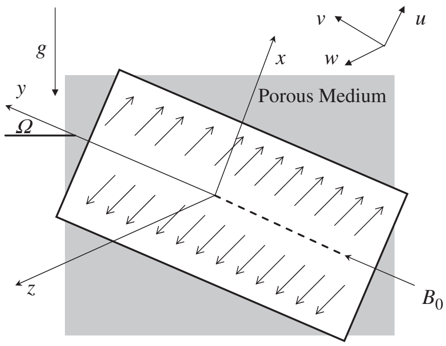

Fig. 1. Flow model and physical co-ordinate system.

Boussinesq approximations of the governing equations can be written as

$$\Delta_1 = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\Delta_2 = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} - \upsilon\frac{\partial^2 u}{\partial z^2} - g\beta(T - T_\infty)\cos\Omega + \frac{\sigma B_0^2}{\rho}u + \frac{\upsilon}{K}u = 0, \tag{2}$$

$$\Delta_3 = \frac{\partial v}{\partial t} + w\frac{\partial v}{\partial z} - \upsilon\frac{\partial^2 v}{\partial z^2} - g\beta(T - T_\infty)\sin\Omega + \frac{\upsilon}{K}v = 0, \tag{3}$$

$$\Delta_4 = \frac{\partial w}{\partial t} + w\frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} - \upsilon\frac{\partial^2 w}{\partial z^2} + \frac{\sigma B_0^2}{\rho}w + \frac{\upsilon}{K}w = 0, \tag{4}$$

$$\Delta_5 = \frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z} - \frac{\upsilon}{\Pr}\frac{\partial^2 T}{\partial z^2} - \frac{Q_0}{\rho C_p}(T - T_\infty) = 0, \tag{5}$$

where x, y, and z are the coordinate directions. u, v, w, p, and T are the fluid velocity components in the x-, y-, and z-directions, pressure and temperature, respectively. ρ, υ, C_p, and $Pr = \rho C_p \upsilon / k$ are the fluid density, kinematic viscosity, specific heat at constant pressure, and the effective Prandtl number, respectively. k, g, b, T_∞, and Ω are the thermal conductivity, gravitational acceleration, coefficient of thermal expansion, ambient temperature, and the inclination angle, respectively. σ, B_0, and Q_0 are the fluid electrical conductivity, magnetic induction, and dimensional heat generation/absorption coefficient, respectively. It is a known fact that in a physical application such as crystal growing the heat generation or absorption effect in the fluid is greatly dependent on temperature. Vajravelu and Hadjinicolaou [21] and Chamkha [3] have represented this dependence by a linear relationship. Following these authors, the heat generation or absorption term of Eq. (5) is assumed to vary linearly with the difference of the fluid temperature in the boundary layer and the ambient temperature.

The boundary conditions suggested by the physics of the problem are

$$u(x,0,t) = bx, \quad v(x,0,t) = 0, \quad w(x,0,t) = w_0, \quad \theta(x,0,t) = \theta_w(x,t), \tag{6.1}$$

$$u(x,\infty,t) = 0, \quad v(x,\infty,t) = 0, \quad w_z(x,0,t) = 0, \quad \theta(x,\infty,t) = 0, \tag{6.2}$$

where $\theta = T - T_\infty$. Also $\theta_w = T_w - T_\infty$ is a prescribed function along the boundary surface $z = 0$.

3. Determination of the symmetry groups

In this section we provide a complete symmetry analysis of the nonlinear differential equations (1)–(5). We obtain non-similar (up to symmetry transformations) subalgebras of the symmetry group. These subalgebras are used to construct distinct (up to symmetry transformations) classes of invariant solutions of (1)–(5).

3.1. Lie-point symmetries equations

Asymmetry of a differential equation is an inevitable transformation of the dependent and independent variables that maps the equation to itself. Amongst symmetries of differential equations, those depending continuously on a small parameter and forming a local one-parameter group of transformation can be calculated algorithmically thanks to a procedure due to Sophus Lie (see, for instance, [15,14,1,10]). One of the most useful and striking properties of symmetries is that they map solutions to solutions. For partial differentials, symmetries allow the reduction of the number of independent variables.

Consider the one-parameter Lie group of infinitesimal transformations in $(x, z, t, u, v, w, p, \theta)$ given by

$$
\begin{aligned}
x^* &= x + \varepsilon\xi^1(x, z, t, u, v, w, p, \theta) + \mathrm{O}(\varepsilon^2),\\
z^* &= z + \varepsilon\xi^2(x, z, t, u, v, w, p, \theta) + \mathrm{O}(\varepsilon^2),\\
t^* &= t + \varepsilon\xi^3(x, z, t, u, v, w, p, \theta) + \mathrm{O}(\varepsilon^2),\\
u^* &= u + \varepsilon\mu^1(x, z, t, u, v, w, p, \theta) + \mathrm{O}(\varepsilon^2),\\
v^* &= v + \varepsilon\mu^2(x, z, t, u, v, w, p, \theta) + \mathrm{O}(\varepsilon^2),\\
w^* &= w + \varepsilon\mu^3(x, z, t, u, v, w, p, \theta) + \mathrm{O}(\varepsilon^2),\\
p^* &= p + \varepsilon\mu^4(x, z, t, u, v, w, p, \theta) + \mathrm{O}(\varepsilon^2),\\
\theta^* &= \theta + \varepsilon\mu^5(x, z, t, u, v, w, p, \theta) + \mathrm{O}(\varepsilon^2),
\end{aligned}
\tag{7}
$$

where ε is the Lie group parameter. Requiring that Eqs. (1)–(5) are invariants under these transformations yields an over-determined, linear system of equations for infinitesimals $\xi^1(x, z, t, u, v, w, p, \theta)$, $\xi^2(x, z, t, u, v, w, p, \theta)$, $\xi^3(x, z, t, u, v, w, p, \theta)$, $\mu^1(x, z, t, u, v, w, p, \theta)$, $\mu^2(x, z, t, u, v, w, p, \theta)$, $\mu^3(x, z, t, u, v, w, p, \theta)$, $\mu^4(x, z, t, u, v, w, p, \theta)$ and $\mu^5(x, z, t, u, v, w, p, \theta)$. The associated Lie algebra of infinitesimal symmetries is the set of the vector field of the form

$$
\begin{aligned}
X &= \xi^1(x, z, t, u, v, w, p, \theta)\frac{\partial}{\partial x} + \xi^2(x, z, t, u, v, w, p, \theta)\frac{\partial}{\partial z} + \xi^3(x, z, t, u, v, w, p, \theta)\frac{\partial}{\partial t}\\
&\quad + \mu^1(x, z, t, u, v, w, p, \theta)\frac{\partial}{\partial u} + \mu^2(x, z, t, u, v, w, p, \theta)\frac{\partial}{\partial v} + \mu^3(x, z, t, u, v, w, p, \theta)\frac{\partial}{\partial w}\\
&\quad + \mu^4(x, z, t, u, v, w, p, \theta)\frac{\partial}{\partial p} + \mu^5(x, z, t, u, v, w, p, \theta)\frac{\partial}{\partial \theta}.
\end{aligned}
\tag{8}
$$

The action of X is extended to all derivatives appearing in (1)–(5) through the second prolongation

$$
\begin{aligned}
X^{(2)} &= X + \mu^1_x\frac{\partial}{\partial u_x} + \mu^1_z\frac{\partial}{\partial u_z} + \mu^1_t\frac{\partial}{\partial u_t} + \mu^2_z\frac{\partial}{\partial v_z} + \mu^2_t\frac{\partial}{\partial v_t} + \mu^3_z\frac{\partial}{\partial w_z} + \mu^3_t\frac{\partial}{\partial w_t}\\
&\quad + \mu^4_z\frac{\partial}{\partial p_z} + \mu^5_z\frac{\partial}{\partial \theta_z} + \mu^5_t\frac{\partial}{\partial \theta_t} + \mu^1_{zz}\frac{\partial}{\partial u_{zz}} + \mu^2_{zz}\frac{\partial}{\partial v_{zz}} + \mu^3_{zz}\frac{\partial}{\partial w_{zz}} + \mu^5_{zz}\frac{\partial}{\partial \theta_{zz}},
\end{aligned}
\tag{9}
$$

where

$$\begin{aligned}
\mu_x^1 &= \mathrm{D}_x(\mu^1) - u_x\mathrm{D}_x(\xi^1) - u_z\mathrm{D}_x(\xi^2) - u_t\mathrm{D}_x(\xi^3),\\
\mu_z^1 &= \mathrm{D}_z(\mu^1) - u_x\mathrm{D}_z(\xi^1) - u_z\mathrm{D}_z(\xi^2) - u_t\mathrm{D}_z(\xi^3),\\
\mu_t^1 &= \mathrm{D}_t(\mu^1) - u_x\mathrm{D}_t(\xi^1) - u_z\mathrm{D}_t(\xi^2) - u_t\mathrm{D}_t(\xi^3),\\
\mu_z^2 &= \mathrm{D}_z(\mu^2) - \upsilon_x\mathrm{D}_z(\xi^1) - \upsilon_z\mathrm{D}_z(\xi^2) - \upsilon_t\mathrm{D}_z(\xi^3),\\
\mu_t^2 &= \mathrm{D}_t(\mu^2) - \upsilon_x\mathrm{D}_t(\xi^1) - \upsilon_z\mathrm{D}_t(\xi^2) - \upsilon_t\mathrm{D}_t(\xi^3),\\
\mu_z^3 &= \mathrm{D}_z(\mu^3) - w_x\mathrm{D}_z(\xi^1) - w_z\mathrm{D}_z(\xi^2) - w_t\mathrm{D}_z(\xi^3),\\
\mu_t^3 &= \mathrm{D}_t(\mu^3) - w_x\mathrm{D}_t(\xi^1) - w_z\mathrm{D}_t(\xi^2) - w_t\mathrm{D}_t(\xi^3),\\
\mu_z^4 &= \mathrm{D}_z(\mu^4) - p_x\mathrm{D}_z(\xi^1) - p_z\mathrm{D}_z(\xi^2) - p_t\mathrm{D}_z(\xi^3),\\
\mu_z^5 &= \mathrm{D}_z(\mu^5) - \theta_x\mathrm{D}_z(\xi^1) - \theta_z\mathrm{D}_z(\xi^2) - \theta_t\mathrm{D}_z(\xi^3),\\
\mu_t^5 &= \mathrm{D}_t(\mu^5) - \theta_x\mathrm{D}_t(\xi^1) - \theta_z\mathrm{D}_t(\xi^2) - \theta_t\mathrm{D}_t(\xi^3),\\
\mu_{zz}^1 &= \mathrm{D}_z(\mu_z^1) - u_{xz}\mathrm{D}_z(\xi^1) - u_{zz}\mathrm{D}_z(\xi^2) - u_{zt}\mathrm{D}_z(\xi^3),\\
\mu_{zz}^2 &= \mathrm{D}_z(\mu_z^2) - \upsilon_{xz}\mathrm{D}_z(\xi^1) - \upsilon_{zz}\mathrm{D}_z(\xi^2) - \upsilon_{zt}\mathrm{D}_z(\xi^3),\\
\mu_{zz}^3 &= \mathrm{D}_z(\mu_z^3) - w_{xz}\mathrm{D}_z(\xi^1) - w_{zz}\mathrm{D}_z(\xi^2) - w_{zt}\mathrm{D}_z(\xi^3),\\
\mu_{zz}^5 &= \mathrm{D}_z(\mu_z^5) - \theta_{xz}\mathrm{D}_z(\xi^1) - \theta_{zz}\mathrm{D}_z(\xi^2) - \theta_{zt}\mathrm{D}_z(\xi^3)
\end{aligned} \tag{10}$$

and D_x, D_z and D_t are the operators of total differentiation with respect to x, z and t, respectively. The operator X is a point symmetry of (1)–(5) if

$$X^{(2)}(\Delta_j)|_{\Delta_j=0} = 0, \quad j = 1, 2, 3, 4, 5. \tag{11}$$

Since the coefficients of X do not involve derivatives, we can separate (11) with respect to derivatives and solve the resulting over a determined system of linear homogeneous partial differential equations known as the determining equations.

Eq. (11) with $i = 1, 2, 3, 4, 5$ implies that

$$\mu_x^1 + \mu_z^3 = 0, \tag{12}$$

$$\mu_t^1 + u_x\mu^1 + u\mu_x^1 + u_z\mu^3 + w\mu_z^1 - \upsilon\mu_{zz}^1 - g\beta\mu^4\cos\alpha\left(\frac{\sigma B_0^2}{\rho} + \frac{\upsilon}{K}\right)\mu^1 = 0, \tag{13}$$

$$\mu_t^2 + \upsilon_z\mu^3 + w\mu_z^2 - \upsilon\mu_{zz}^2 - g\beta\mu^4\sin\Omega + \frac{\upsilon}{K}\mu^2 = 0, \tag{14}$$

$$\mu_t^3 + w\mu_z^3 + w_z\mu^3 + \frac{1}{\rho}\mu_z^4 - \upsilon\mu_{zz}^3 + \left(\frac{\sigma B_0^2}{\rho} + \frac{\upsilon}{K}\right)\mu^3 = 0, \tag{15}$$

$$\mu_t^5 + w\mu_z^5 + \theta_z\mu^3 - \frac{\upsilon}{Pr}\mu_{zz}^5 - \frac{Q_0}{\rho C_p}\mu^5 = 0. \tag{16}$$

Carrying out a straightforward and tedious algebra, we finally obtain the form of the infinitesimals as

$$\begin{aligned}
\xi^1 &= 2C_1x + h_1(t),\\
\xi^2 &= C_2x + \frac{C_1}{2}z + h_2(t),\\
\xi^3 &= C_1t + C_3,\\
\mu^1 &= C_1u + h_1'(t) + h_3(x, z, t),\\
\mu^2 &= C_1\upsilon + h_4(x, z, t),\\
\mu^3 &= C_2u - \frac{C_1}{2}w + h_2'(t) + h_5(x, z, t),\\
\mu^4 &= C_1p + h_6(x, z, t),\\
\mu^5 &= C_1\theta + h_7(x, z, t),
\end{aligned} \tag{17}$$

where C_1, C_2 and C_3 are arbitrary constants, and $h_1, h_2, h_3, h_4, h_5, h_6$ and h_7 are arbitrary functions depending on x, z, t that will be determined later. Imposing the restrictions from boundaries and from the boundary conditions on the infinitesimals, we hence obtain the following form for Eqs. (17):

$$\xi^1 = 2C_1x, \quad \xi^2 = C_2x + \frac{C_1}{2}z, \quad \xi^3 = C_1t + C_3, \quad \mu^1 = C_1u + h_3,$$
$$\mu^2 = C_1v + h_4, \quad \mu^3 = C_2u - \frac{C_1}{2}w + h_5, \quad \mu^4 = C_1p + h_6, \quad \mu^5 = C_1\theta + h_7. \tag{18}$$

What remains is to require invariance of the data which must be held on the boundary surfaces. This requirement means

$$\begin{aligned}
&X^{(1)}[u - \Psi_1(x)] = 0 \quad \text{when } u(x,0,t) = \Psi_1(x) = bx,\\
&X^{(1)}[v - \Psi_2(x)] = 0 \quad \text{when } v(x,0,t) = \Psi_2(x) = 0,\\
&X^{(1)}[-w - \Psi_3(t)] = 0 \quad \text{when } w(x,0,t) = -\Psi_3(t) = w_0,\\
&X^{(1)}[p - \Psi_4(x)] = 0 \quad \text{when } p(x,0,t) = \Psi_4(x) = 0,\\
&X^{(1)}[\theta - \theta_w(x)] = 0 \quad \text{when } \theta(x,0,t) = \theta_w(x) = (T_w(x) - T_\infty).
\end{aligned} \tag{19}$$

Examining the above conditions, we obtain the following differential equations:

$$\begin{aligned}
&2C_1x\Psi_1' - C_1\Psi_1 = h_3,\\
&2C_1x\Psi_2' - C_1\Psi_2 = h_4,\\
&(C_1t + C_3)\Psi_3' - (C_1/2)\Psi_3 + C_2u = -h_5,\\
&2C_1x\Psi_4' - C_1\Psi_4 = h_6,\\
&2C_1x\theta_w' - C_1\theta_w = h_7,
\end{aligned} \tag{20}$$

which directly give the admissible form for the functions $\psi_1, \psi_2, \psi_3, \psi_4, \theta_w$,

$$\begin{aligned}
&\Psi_1(x) = k_1|C_1x|^{1/2} - h_3/C_1 = bx,\\
&\Psi_2(x) = k_2|C_1x|^{1/2} - h_4/C_1 = 0,\\
&\Psi_3(t) = k_3|(C_1t + C_4)|^{1/2} + 2(C_2u + h_5)/C_1 = -w_0,\\
&\Psi_4(x) = k_4|C_1x|^{1/2} - h_6/C_1 = 0,\\
&\theta_w(x) = k_5|C_1x|^{1/2} - h_7/C_1 = 0,
\end{aligned} \tag{21}$$

where k_1, k_2, k_3, k_4 and k_5 are arbitrary constants that must be satisfied its equations. Consequently, a set of boundary conditions conformed to symmetries (18) should be of the form

$$\begin{aligned}
&z = 0, \quad u(x,0,t) = k_1|C_1x|^{1/2} - h_3/C_1,\\
&v(x,0,t) = k_2|C_1x|^{1/2} - h_4/C_1, \quad x, z \succ 0,\\
&w(x,0,t) = -k_3|(C_1t + C_4)|^{1/2} - 2(C_2u(x,0,t) + h_5)/C_1,\\
&p(x,0,t) = k_4|C_1x|^{1/2} - h_6(x,0,t)/C_1, \quad x, z \succ 0,\\
&\theta(x,0,t) = k_5|C_1x|^{1/2} - h_7(x,0,t)/C_1
\end{aligned} \tag{22}$$

and

$$\begin{aligned}
&z \to \infty: u(x,\infty,t) = 0,\ v(x,\infty,t) = 0,\\
&w(x,\infty,t) = 0, \quad \theta(x,\infty,t) = 0.
\end{aligned} \tag{23}$$

3.2. Proposition

The boundary value problem, described by Eqs. (1)–(5), and the data (6), admits the following multi-parameter group of symmetries:

$$
\begin{aligned}
&x^* = x + \varepsilon(2C_1x) + O(\varepsilon^2),\\
&z^* = z + \varepsilon(C_2x + (C_1/2)z) + O(\varepsilon^2),\\
&t^* = t + \varepsilon(C_1t + C_3) + O(\varepsilon^2),\\
&u^* = u + \varepsilon(C_1u + h_3) + O(\varepsilon^2),\\
&v^* = v + \varepsilon(C_1v + h_4) + O(\varepsilon^2),\\
&w^* = w + \varepsilon(C_2u - (C_1/2)w + h_5) + O(\varepsilon^2),\\
&p^* = p + \varepsilon(C_1p + h_6) + O(\varepsilon^2),\\
&\theta^* = \theta + \varepsilon(C_1\theta + h_7) + O(\varepsilon^2).
\end{aligned}
\tag{24}
$$

Moreover, the admissible form of the data on the boundaries should be of the form given by Eq. (21). Looking at the transformation equations (18), one can recognize the kind of symmetry corresponding to the problem already studied numerically in [3]. Neglecting the parameters C_2 and C_3, the scaling group is parameterized by C_1.

4. Group-invariant solutions

The next question is whether the symmetry group we have obtained in the last section gives any of the so-called group-invariant solutions. A group-invariant solution is nothing else but a solution of the boundary value problem (1)–(6), which is also invariant under the group (24), see [14,1]. Suppose (u, v, w, p, θ) is a solution of the problem (1)–(6). In order for this solution to be invariant under the transformation group (24), the following system of partial differential equations must hold:

$$
\xi^1\frac{\partial u}{\partial x} + \xi^2\frac{\partial u}{\partial z} + \xi^3\frac{\partial u}{\partial t} = \mu^1, \quad \xi^1\frac{\partial v}{\partial x} + \xi^2\frac{\partial v}{\partial z} + \xi^3\frac{\partial v}{\partial t} = \mu^2,
$$
$$
\xi^1\frac{\partial w}{\partial x} + \xi^2\frac{\partial w}{\partial z} + \xi^3\frac{\partial w}{\partial t} = \mu^3 \quad \text{and} \quad \xi^1\frac{\partial \theta}{\partial x} + \xi^2\frac{\partial \theta}{\partial z} + \xi^3\frac{\partial \theta}{\partial t} = \mu^4,
$$

or

$$2C_1x\frac{\partial u}{\partial x} + (C_2x + (C_1/2)z)\frac{\partial u}{\partial z} + (C_1t + C_3)\frac{\partial u}{\partial t} = C_1u + h_3, \tag{25.1}$$

$$2C_1x\frac{\partial v}{\partial x} + (C_2x + (C_1/2)z)\frac{\partial v}{\partial z} + (C_1t + C_3)\frac{\partial v}{\partial t} = C_1v + h_4, \tag{25.2}$$

$$2C_1x\frac{\partial w}{\partial x} + (C_2x + (C_1/2)z)\frac{\partial w}{\partial z} + (C_1t + C_3)\frac{\partial w}{\partial t} = C_2u - (C_1/2)w + h_5, \tag{25.3}$$

$$2C_1x\frac{\partial p}{\partial x} + (C_2x + (C_1/2)z)\frac{\partial p}{\partial z} + (C_1t + C_3)\frac{\partial p}{\partial t} = C_1p + h_6, \tag{25.4}$$

$$2C_1x\frac{\partial \theta}{\partial x} + (C_2x + (C_1/2)z)\frac{\partial \theta}{\partial z} + (C_1t + C_3)\frac{\partial \theta}{\partial t} = C_1\theta + h_7. \tag{25.5}$$

Using the method of characteristics, we can solve systems (25.1)–(25.5):

$$u(x, z, t) = k_1|C_1x|^{1/2}\bar{F}(\tau, \eta) + k_1^*M(\tau, \eta), \tag{26.1}$$

$$v(x, z, t) = k_2N(\tau, \eta), \tag{26.2}$$

$$w(x, z, t) = -k_3|C_1t + C_3|^{1/2}F(\tau, \eta), \tag{26.3}$$

$$p(x, z, t) = k_4G(\tau, \eta), \tag{26.4}$$

$$\theta(x, z, t) = k_5H(\tau, \eta), \tag{26.5}$$

where F, M, N, G, H are arbitrary functions and k_1, k_2, k_3, k_4 and k_5 are integral constants, respectively, and is the similarity variable η and non-similar variable are given, respectively, by the relations:

$$\eta = k_6 z|t|^{-1/2}, \quad \tau = t, \tag{26.6}$$

Moreover, in our problem to satisfy continuity balance in Eq. (1), without loss of generality, $\bar{F}(\tau, \eta)$ is expressed as

$$\bar{F}(\tau, \eta) = x^{1/2} F'(\tau, \eta). \tag{26.7}$$

Also, arbitrary functions of the above solutions of Eqs. (26) can be defined as follows:

$$h_3(x, z, t) = (C_2 k_2 x/\tau^{1/2})\lfloor k_1 x^{1/2} \bar{F}'(\tau, \eta) + k_1^* M'(\tau, \eta)\rfloor - (C_3\eta/2\tau)\lfloor k_1 x^{1/2} \bar{F}'(\tau, \eta) + k_1^* M'(\tau, \eta)\rfloor + (C_1\tau + C_3)\lfloor x^{1/2} \bar{F}_\tau(\tau, \eta) + M_\tau(\tau, \eta)\rfloor, \tag{27.1}$$

$$h_4(x, z, t) = N'(\tau, \eta)((C_2 k_2 k_6 x/\tau^{1/2}) - (C_3 k_2 \eta/2\tau)) + C_1(\tau N_\tau(\tau, \eta) - N(\tau, \eta)) + C_3 N_\tau(\tau, \eta), \tag{27.2}$$

$$h_5(x, z, t) = F'(\tau, \eta)((-2C_2 k_3 k_6 x/\tau^{1/2}) - (C_3 k_4 \eta/2\tau)) + (C_1\tau + C_3) F_\tau(\tau, \eta), \tag{27.3}$$

$$h_6(x, z, t) = G'(\tau, \eta)((C_2 k_4 k_6 x/\tau^{1/2}) - (C_3 k_4 \eta/2\tau)) + C_1(\tau G_\tau(\tau, \eta) - G(\tau, \eta)) + C_3 G_\tau(\tau, \eta), \tag{27.4}$$

$$h_7(x, z, t) = H'(\tau, \eta)((C_2 k_5 k_6 x/\tau^{1/2}) - (C_3 k_5 \eta/2\tau)) + C_1(\tau H_\tau(\tau, \eta) - H(\tau, \eta)) + C_3 H_\tau(\tau, \eta). \tag{27.5}$$

Eqs. (26.1)–(26.7), describe the new form for any group-invariant solution of our problem. The interesting point here is that such a solution has the property to reduce the number of the independent variables of the problem. Thus, the initial boundary value problem of PDEs has been transformed into a boundary value problem of non-similar transient equations which is generally easier to be solved by some numerical method.

5. Scaling symmetry

In this section, we proceeded further to numerical results, we are confined to the case of scaling symmetry, consequently we choose $C_1 = 1$ and $C_2 = C_3 = 0$. Furthermore, we examine a case corresponding to the problem already presented in [3] in the absence of permeability of porous medium effect.

In this case, we examine the case in which the transformation equations are of the following form:

$$x^* = e^{2\varepsilon} x, \quad z^* = e^{\varepsilon/2} z, \quad t^* = e^{\varepsilon} t, \quad u^* = e^{\varepsilon} u, \quad v^* = e^{\varepsilon} v,$$
$$w^* = e^{-\varepsilon/2} w, \quad p^* = e^{\varepsilon} p, \quad \theta^* = e^{\varepsilon}\theta. \tag{28}$$

Also, under this choice of the parameters, the self-similar solutions (26.1)–(26.7) take the form:

$$u(x, z, t) = k_1 x F'(\tau, \eta) + k_1^* M(\tau, \eta),$$
$$v(x, z, t) = k_2 N(\tau, \eta),$$
$$w(x, z, t) = -k_3 \sqrt{t} F(\tau, \eta),$$
$$p(x, z, t) = k_4 G(\tau, \eta),$$
$$\theta(x, z, t) = k_5 H(\tau, \eta),$$
$$\eta = k_6 z/\sqrt{t}, \quad \tau = t. \tag{29}$$

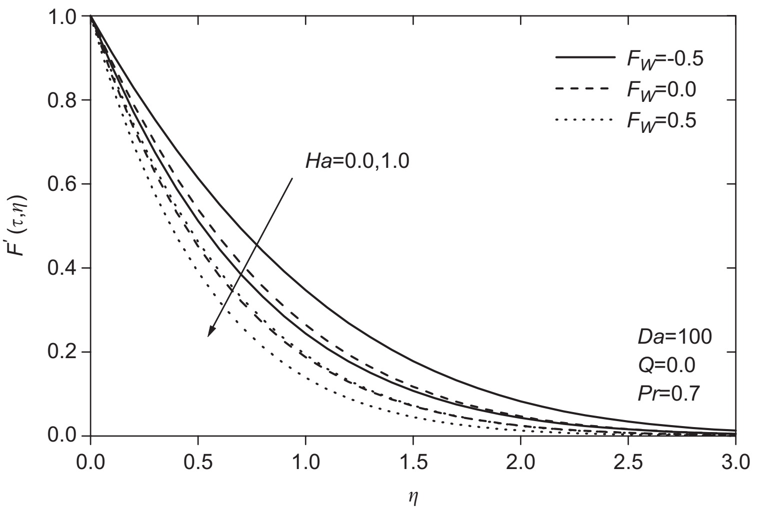

Fig. 2. Effects of Hartmann number *Ha* and blowing/suction parameter F_W on the fluid velocity of *x*-direction (F').

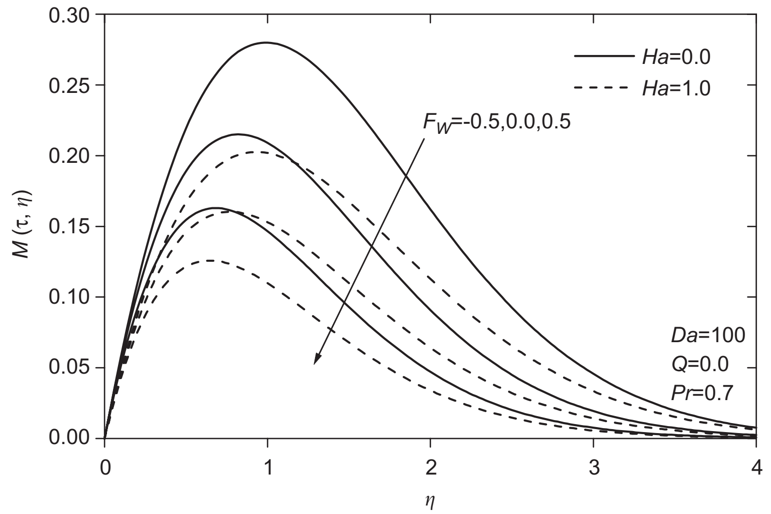

Fig. 3. Effects of Hartmann number *Ha* and blowing/suction parameter F_W on the fluid velocity of *x*-direction (M).

Substituting Eqs. (29) into Eqs. (1)–(6) reduces the number of independent variables by one and produces the following non-similar transient equations:

$$F''' + \frac{\eta}{2}F'' + b\tau\left(FF'' - F'^2 - \left(Ha^2 + \frac{1}{Da}\right)F'\right) - \tau\frac{\partial F'}{\partial \tau} = 0, \tag{30}$$

$$M'' + \frac{\eta}{2}M' + b\tau\left(FM' - MF' - \left(Ha^2 + \frac{1}{Da}\right)M + H\right) - \tau\frac{\partial M}{\partial \tau} = 0, \tag{31}$$

$$N'' + \frac{\eta}{2}N' + b\tau\left(FN' - \frac{1}{Da}N + H\right) - \tau\frac{\partial N}{\partial \tau} = 0, \tag{32}$$

$$G' + F'' + \frac{\eta}{2}F' + b\tau\left(FF' - \left(Ha^2 + \frac{1}{Da}\right)F\right) - \tau\frac{\partial F}{\partial \tau} = 0, \tag{33}$$

$$H'' + Pr\frac{\eta}{2}H' + Pr\,b\tau(FH' + QH) - Pr\,\tau\frac{\partial H}{\partial \tau} = 0. \tag{34}$$

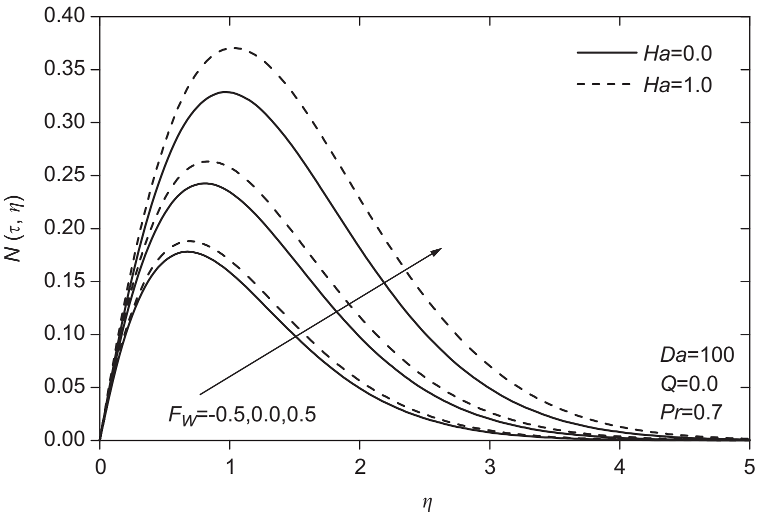

Fig. 4. Effects of Hartmann number *Ha* and blowing/suction parameter F_W on the fluid velocity of *y*-direction (N).

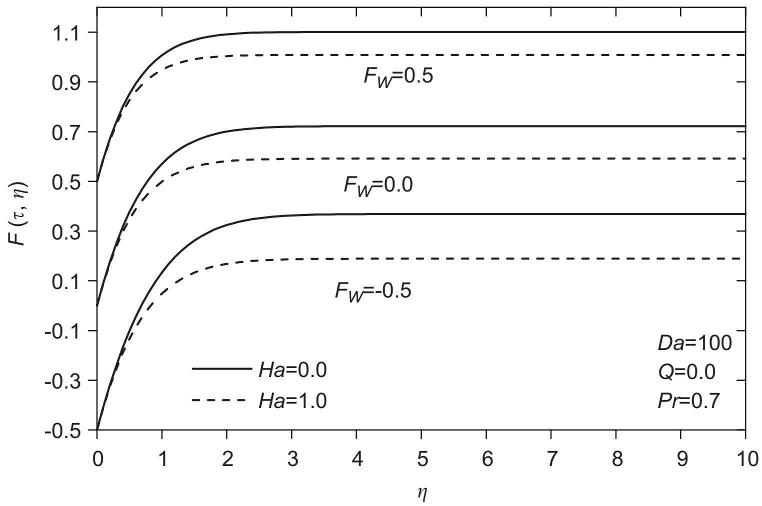

Fig. 5. Effects of Hartmann number *Ha* and blowing/suction parameter F_W on the fluid velocity of *z*-direction (F).

Also, the transformed boundary conditions associated with this choice of parameters become

$$\begin{aligned}&F(\tau,0)=F_w,\quad F'(\tau,0)=1,\quad F'(\tau,\infty)=0,\quad M(\tau,0)=0,\quad M(\tau,\infty)=0,\\&N(\tau,0)=0,\quad N(\tau,\infty)=0,\quad G(\tau,0)=0,\quad \theta(\tau,0)=1,\quad \theta(\tau,\infty)=0.\end{aligned} \tag{35}$$

To avoid the fluid properties appearing explicitly in the coefficients of the above equations, we have introduced four new appropriate arbitrary constants as follows:

$$\begin{aligned}&k_1=b,\quad k_1^*=(g\beta\theta_w\cos\Omega/b),\quad k_2=(g\beta\theta_w\sin\Omega/b),\quad k_3=(b/\upsilon^{1/2}),\\&k_4=\rho b\upsilon,\quad k_5=\theta_w,\quad k_6=\upsilon^{-1/2}.\end{aligned} \tag{36}$$

Also, a prime denotes partial differentiation with respect to η and $Ha^2=\sigma B_0^2/\rho b$, $Da^{-1}=\upsilon/bK$ are the square of the magnetic Hartmann number and the inverse Darcy number, respectively. $Q=Q_0/\rho C_p b$, $F_W=-w_0/\sqrt{b\upsilon}$ are the dimensionless heat generation/absorption coefficient, and wall mass transfer coefficient, respectively. It should be

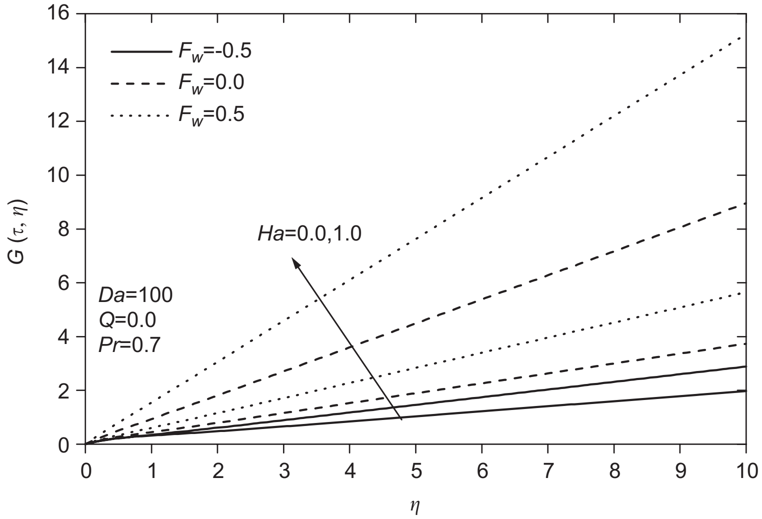

Fig. 6. Effects of Hartmann number *Ha* and blowing/suction parameter F_W on the pressure profiles (G).

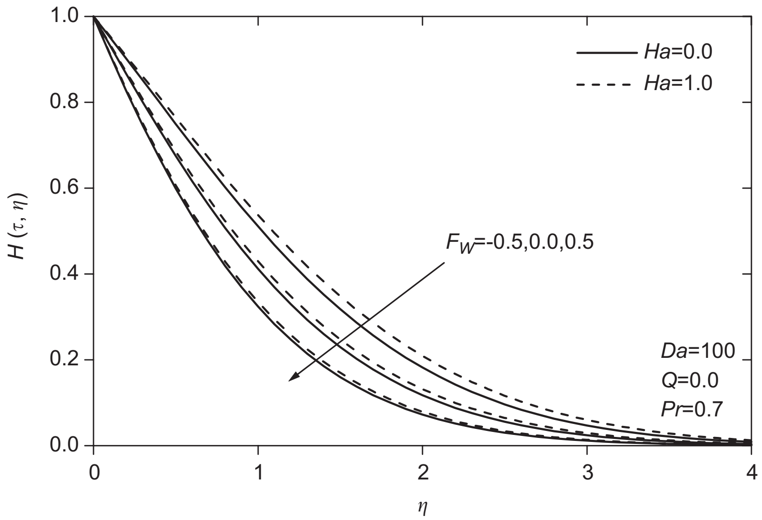

Fig. 7. Effects of Hartmann number *Ha* and blowing/suction parameter F_W on the fluid temperature (H).

noted that positive values of F_W indicate fluid suction at the plate surface while negative values of F_W indicate fluid blowing or injection at the wall.

Important physical parameters for this flow and heat transfer situation are the skin-friction coefficients in the *x*- and *y*-directions and the local Nusselt number. The shear stresses at the stretching surface are given by

$$\tau_{zx} = \mu \frac{\partial u}{\partial z}(x, 0, t) = \frac{\mu}{\sqrt{\upsilon t}}(bxF''(\tau, 0) + (g\beta\theta_w \cos\Omega/b)M'(\tau, 0)), \tag{37}$$

$$\tau_{zy} = \mu \frac{\partial v}{\partial z}(x, 0, t) = \frac{\mu}{\sqrt{\upsilon t}}(g\beta\theta_w \sin\Omega/b)N'(\tau, 0), \tag{38}$$

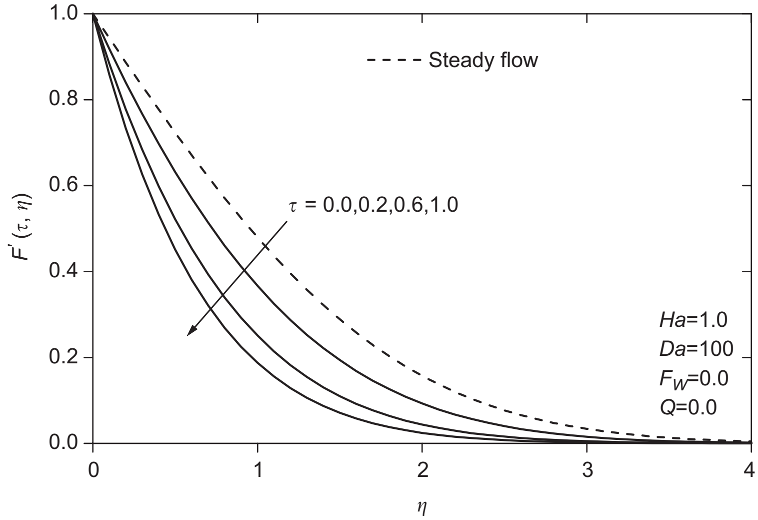

Fig. 8. Effects of Prandtl number Pr and the transient parameter τ on the fluid velocity of x-direction (F').

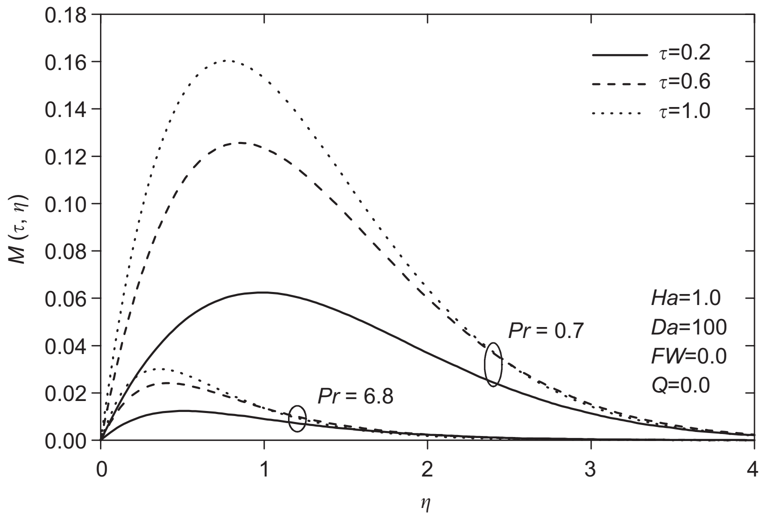

Fig. 9. Effects of Prandtl number Pr and the transient parameter τ on the fluid velocity of x-direction (M).

where $\mu = (\rho\upsilon)$ is the dynamic viscosity of the fluid. Upon quantities of τ_{zx} and τ_{zy} by $\mu = \rho(bx)^2/2$, the following respective expressions for the skin-friction coefficients in the x- and y-directions result:

$$C_{fx} = \frac{2x}{Re_x\sqrt{\upsilon t}}\left(F''(\tau, 0) + \frac{Gr_x}{Re_x^2}\cos\Omega M'(\tau, 0)\right), \tag{39}$$

$$C_{fy} = \frac{2xGr_x}{Re_x^3\sqrt{\upsilon t}}\sin\Omega N'(\tau, 0), \tag{40}$$

where $Gr_x = g\beta\theta_w x^3/\upsilon^2$ and $Re_x = bx^2/\upsilon$ are the local Grashof and Reynolds numbers, respectively.

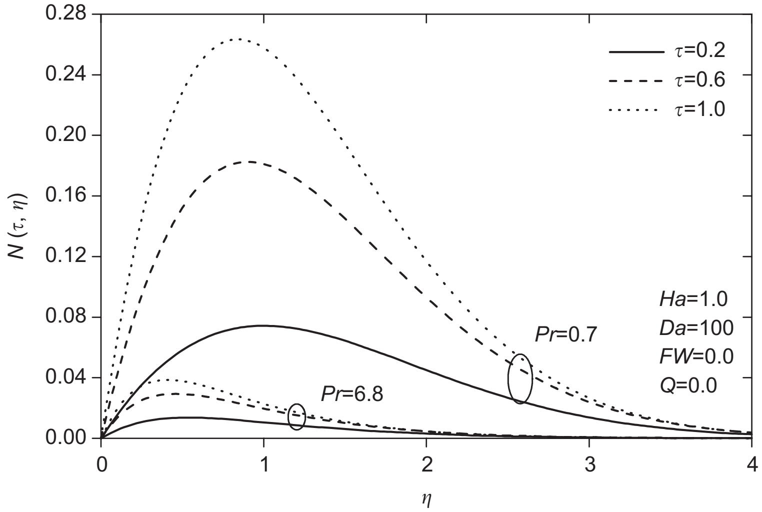

Fig. 10. Effects of Prandtl number Pr and the transient parameter τ on the fluid velocity of y-direction (N).

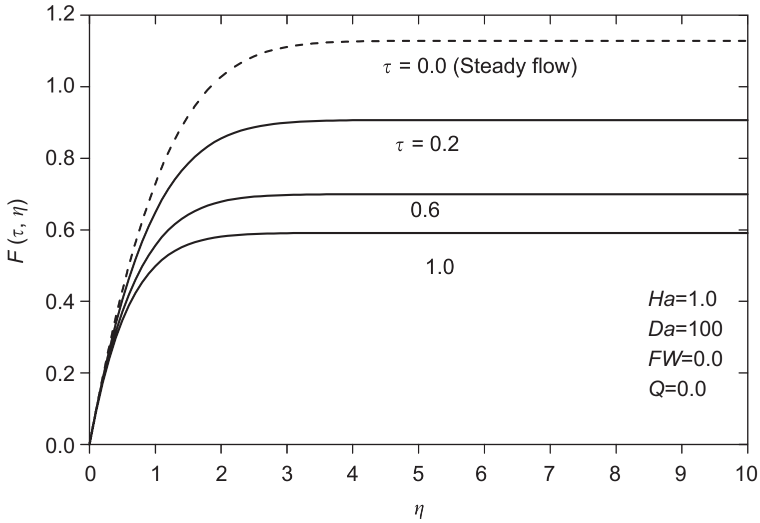

Fig. 11. Effects of Prandtl number Pr and the transient parameter τ on the fluid velocity of z-direction (F).

The wall heat transfer is given by Fourier's law of conduction as follows:

$$q_w = -k\frac{\partial T}{\partial z}(x, 0, t) = -\frac{k\theta_w}{\sqrt{\upsilon t}}\theta'(\tau, 0). \tag{41}$$

The local Nusselt number for this situation can then be defined as

$$Nu_x = \frac{hx}{k} = \frac{q_w x}{k\theta_w} = -\frac{x}{\sqrt{\upsilon t}}\theta'(\tau, 0), \tag{42}$$

where h is the local heat transfer coefficient.

6. Numerical scheme

It should be intimated that the problem under consideration is susceptible to perturbation analysis. It is of interest then to assess the value of approximate series representations in forecasting the essential physical parameters of the

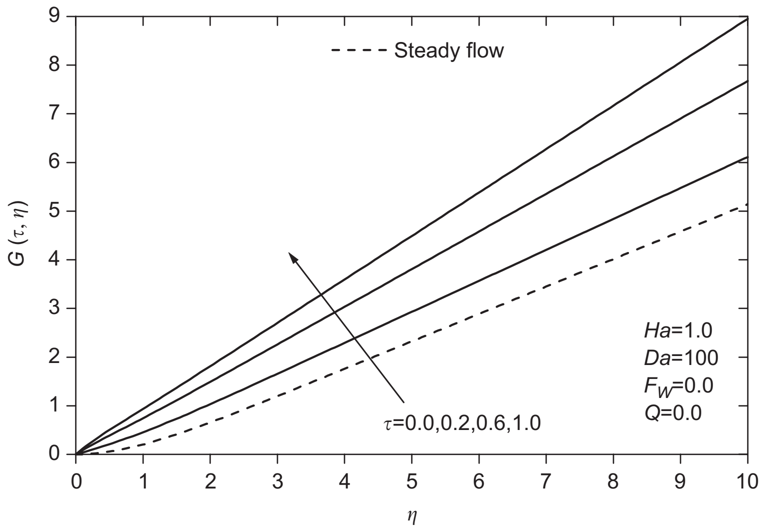

Fig. 12. Effects of Prandtl number Pr and the transient parameter τ on the pressure profiles (G).

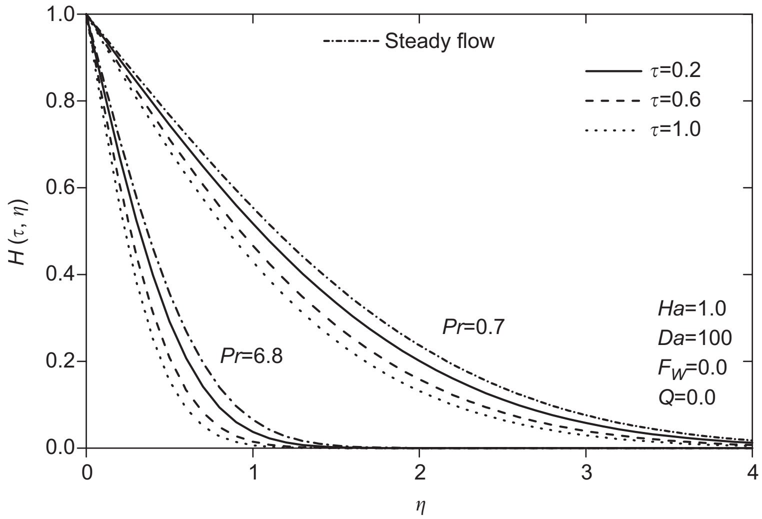

Fig. 13. Effects of Prandtl number Pr and the transient parameter τ on the fluid temperature (H).

problem, namely, the local skin-friction coefficient and the local Nusselt number. The series solution is valid in the region $0 \leqslant \tau \leqslant 1$, for sufficiently small values of time. Accordingly, the above functions can be expanded in a power series in τ as follows:

$$F(\tau,\eta) = F_0(\eta) + \tau F_1(\eta) + \tau^2 F_2(\eta) + \cdots, \tag{43.1}$$

$$M(\tau,\eta) = M_0(\eta) + \tau M_1(\eta) + \tau^2 M_2(\eta) + \cdots, \tag{43.2}$$

$$N(\tau,\eta) = N_0(\eta) + \tau N_1(\eta) + \tau^2 N_2(\eta) + \cdots, \tag{43.3}$$

$$G(\tau,\eta) = G_0(\eta) + \tau G_1(\eta) + \tau^2 G_2(\eta) + \cdots, \tag{43.4}$$

$$H(\tau,\eta) = H_0(\eta) + \tau H_1(\eta) + \tau^2 H_2(\eta) + \cdots. \tag{43.5}$$

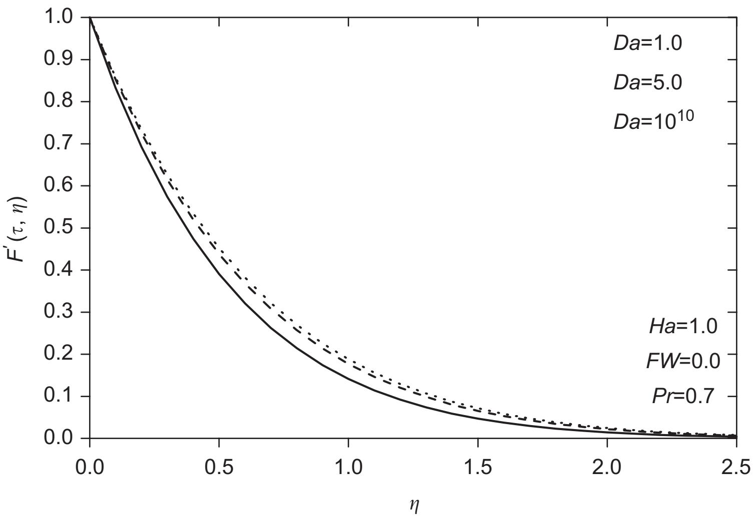

Fig. 14. Effects of Darcy number *Da* and the heat generation/absorption coefficient *Q* on the fluid velocity of *x*-direction (F').

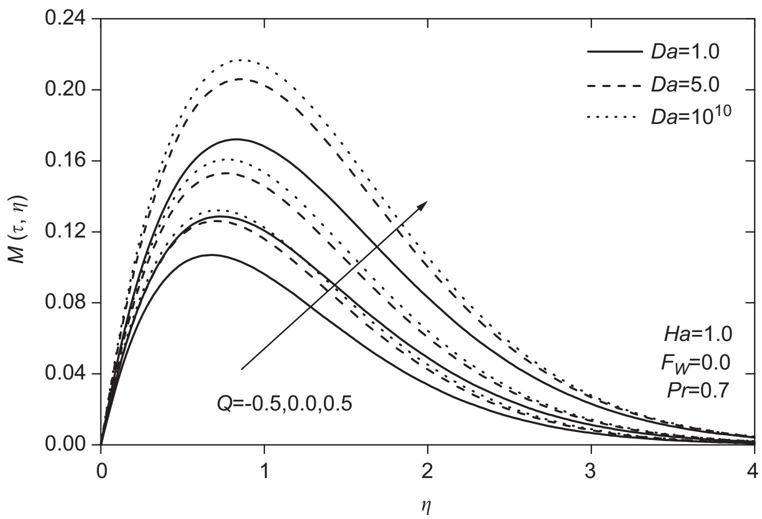

Fig. 15. Effects of Darcy number *Da* and the heat generation/absorption coefficient *Q* on the fluid velocity of *x*-direction (M).

Substituting the above expansion into (30)–(35) and equating the coefficients of skin friction we get the following equations:

$$F_0''' + \frac{\eta}{2}F_0'' = 0, \tag{44.1}$$

$$M_0'' + \frac{\eta}{2}M_0' = 0, \tag{44.2}$$

$$N_0'' + \frac{\eta}{2}N_0' = 0, \tag{44.3}$$

$$G_0' + F_0'' + \frac{\eta}{2}F_0' = 0, \tag{44.4}$$

$$H_0'' + Pr\frac{\eta}{2}H_0' = 0 \tag{44.5}$$

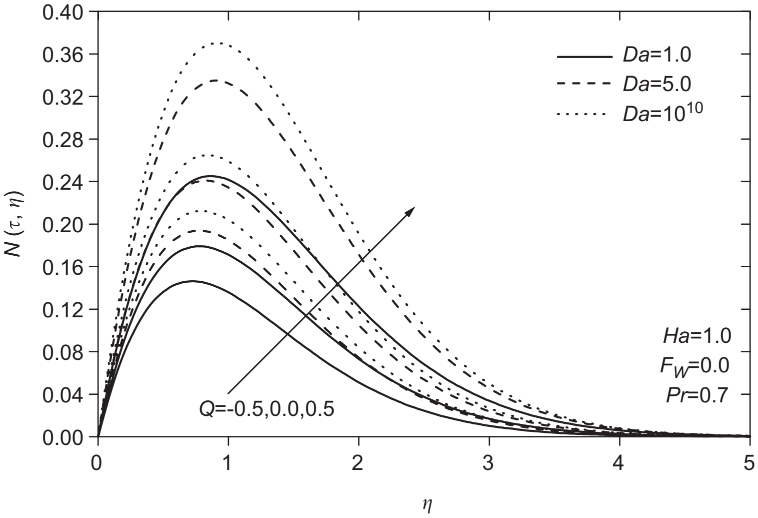

Fig. 16. Effects of Darcy number *Da* and the heat generation/absorption coefficient *Q* on the fluid velocity of *y*-direction (N).

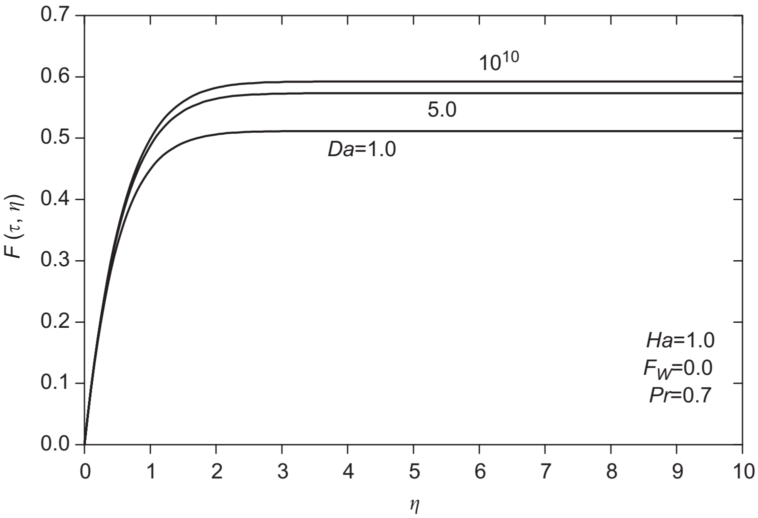

Fig. 17. Effects of Darcy number *Da* and the heat generation/absorption coefficient *Q* on the fluid velocity of *z*-direction (F).

subject to $F_0(0) = F_w$, $F_0'(0) = 1$, $F_0'(\infty) = 0$, $M_0(0) = M_0(\infty) = 0$

$$N_0(0) = N_0(\infty) = G_0(0) = 0, \quad \theta_0(0) = 1, \quad \theta_0(\infty) = 0, \tag{44.6}$$

$$F_1''' + \frac{\eta}{2}F_1'' + b\left(F_0F_0'' - F'^2 - \left(Ha^2 + \frac{1}{Da}\right)F_0'\right) - F_1' = 0, \tag{45.1}$$

$$M_1'' + \frac{\eta}{2}M_1' + b\left(F_0M_0' - M_0F_0' - \left(Ha^2 + \frac{1}{Da}\right)M_0 + H_0\right) - M_1 = 0, \tag{45.2}$$

$$N_1'' + \frac{\eta}{2}N_1' + b\left(F_0N_0' - \frac{1}{Da}N_0 + H_0\right) - N_1 = 0, \tag{45.3}$$

$$G_1' + F_1'' + \frac{\eta}{2}F_1' + b\left(F_0F_0' - \left(Ha^2 + \frac{1}{Da}\right)F_0\right) - F_1 = 0, \tag{45.4}$$

$$H_1'' + Pr\frac{\eta}{2}H_1' + Pr\, b(F_0H_0' + QH_0) - Pr\, H_1 = 0, \tag{45.5}$$

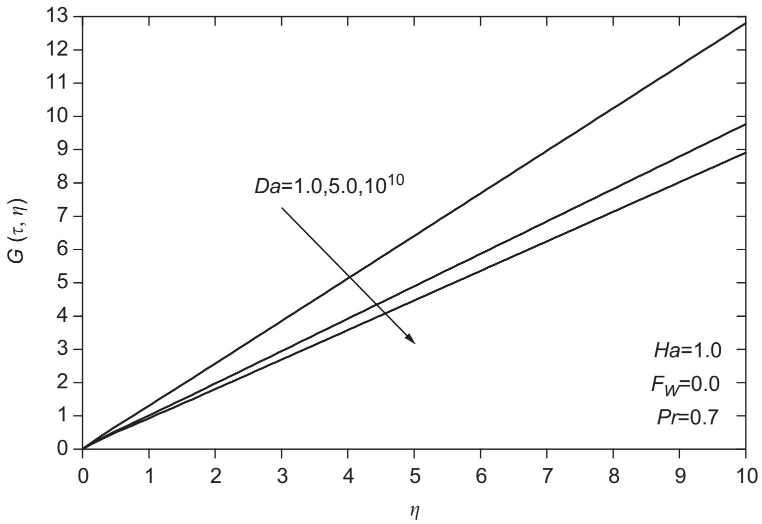

Fig. 18. Effects of Darcy number Da and the heat generation/absorption coefficient Q on the pressure profiles (G).

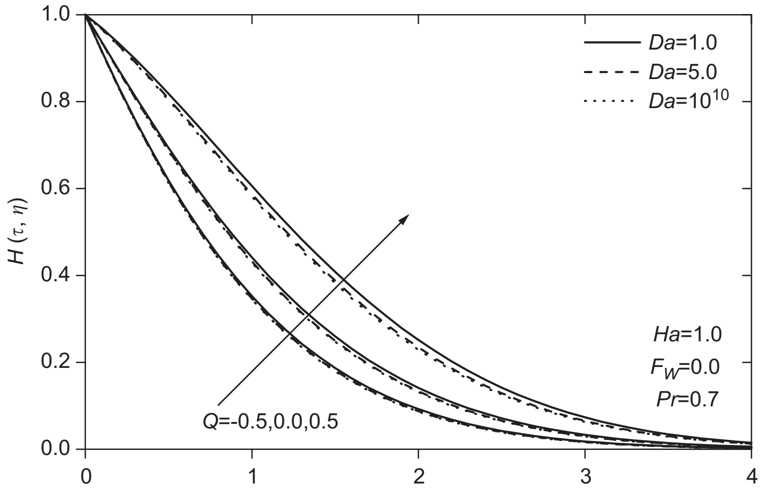

Fig. 19. Effects of Darcy number Da and the heat generation/absorption coefficient Q on the fluid temperature (H).

$$F_1(0) = F_1'(0) = F_1'(\infty) = 0, \quad M_1(0) = M_1(\infty) = 0,$$
$$N_1(0) = N_1(\infty) = G_1(0) = 0, \quad \theta_1(0) = \theta_1(\infty) = 0, \tag{45.6}$$

$$F_2''' + \frac{\eta}{2}F_2'' + b\left(F_0F_1'' + F_1F_0'' - 2F_0'F_1' - \left(Ha^2 + \frac{1}{Da}\right)F_1'\right) - 2F_2' = 0, \tag{46.1}$$

$$M_2'' + \frac{\eta}{2}M_2' + b\left(F_0M_1' + F_1M_0' - M_0F_1' - M_1F_0' - \left(Ha^2 + \frac{1}{Da}\right)M_1 + H_1\right) - 2M_2 = 0, \tag{46.2}$$

$$N_2'' + \frac{\eta}{2}N_2' + b\left(F_0N_1' + F_1N_0' - \frac{1}{Da}N_1 + H_1\right) - 2N_2 = 0, \tag{46.3}$$

$$G_2' + F_2'' + \frac{\eta}{2}F_2' + b\left(F_0F_1' + F_1F_0' - \left(Ha^2 + \frac{1}{Da}\right)F_1\right) - 2F_2 = 0, \tag{46.4}$$

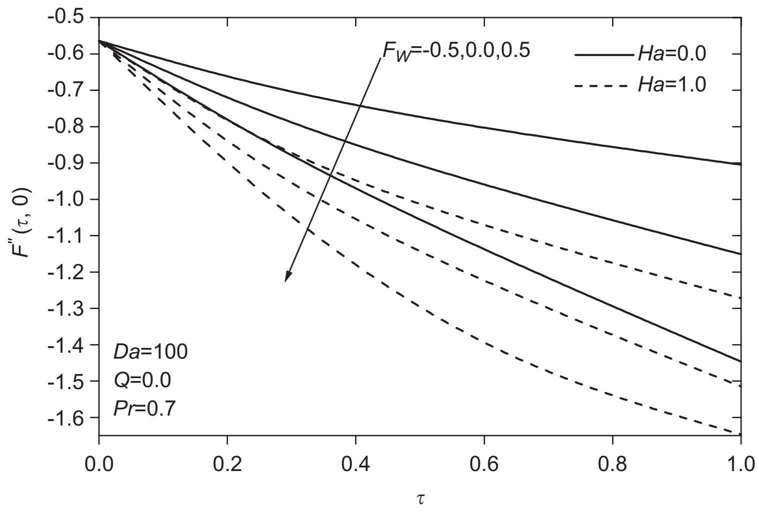

Fig. 20. Effects of Hartmann number Ha and blowing/suction parameter F_W on the skin-friction coefficient in the x-direction $F''(\tau, 0)$.

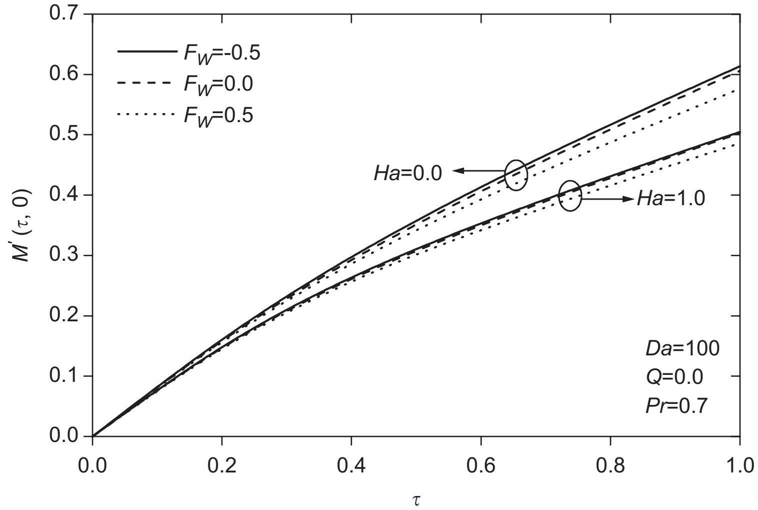

Fig. 21. Effects of Hartmann number Ha and blowing/suction parameter F_W on the skin-friction coefficient in the x-direction $M'(\tau, 0)$.

$$H_2'' + Pr\frac{\eta}{2}H_2' + Pr\,b(F_0H_1' + F_1H_0' + QH_1) - 2Pr\,H_2 = 0, \tag{46.5}$$

$$F_2(0) = F_2'(0) = F_2'(\infty) = 0, \quad M_2(0) = M_2(\infty) = 0,$$
$$N_2(0) = N_2(\infty) = G_2(0) = 0, \quad \theta_2(0) = \theta_2(\infty) = 0. \tag{46.6}$$

7. Results and discussion

For this present system of non-similar transient equations (30)–(34), subject to boundary conditions (35) numerical computations have been carried out by employing a fourth-order Runge–Kutta scheme for the perturbation series solution method for different values of τ. The step size is $\Delta\eta = 0.05$ while obtaining the numerical solution with $\eta_\infty = 10$ and five-decimal accuracy as the criterion convergence. The results in Figs. 2–27 present typical profiles for the variables of the fluid's x-component of velocity F' and M, y-component of velocity N, z-component of velocity F, pressure G,

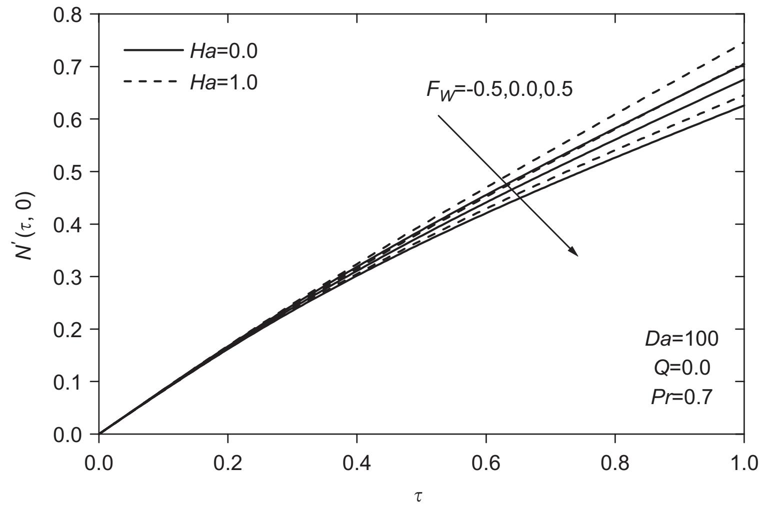

Fig. 22. Effects of Hartmann number Ha and blowing/suction parameter F_W on the skin-friction coefficient in the y-direction $N'(\tau, 0)$.

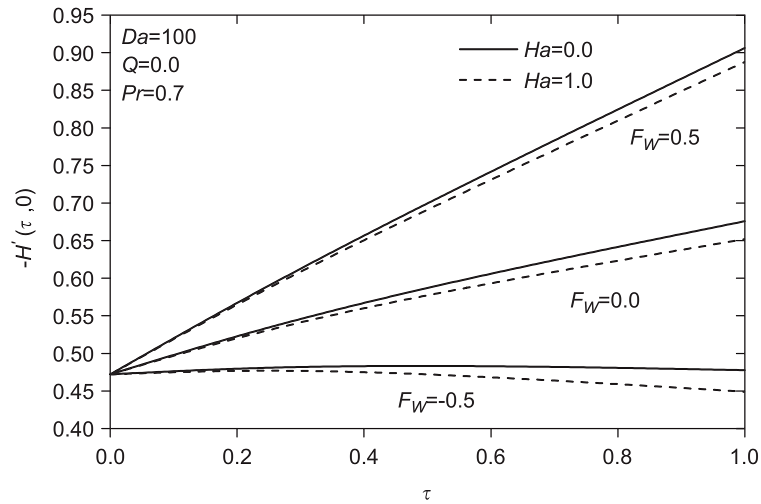

Fig. 23. Effects of Hartmann number Ha and blowing/suction parameter F_W on the wall heat transfer $-H'(\tau, 0)$.

and the temperature H for various values of the magnetic Hartmann number Ha, blowing/injection parameter F_W, Prandtl number Pr, Darcy number Da and heat generation/absorption coefficient Q at $\tau = 1$, respectively. Effects of the magnetic Hartmann number Ha, blowing/injection parameter F_W on components of velocities flow, fluid temperature and pressure profiles are depicted in Figs. 2–7. Application of a magnetic field normal to the flow in the y-direction gives rise to a resistive drag-like force acting in a direction opposite to that of flow (i.e., this force will be present in both x- and z-directions). This has a tendency to reduce the fluid velocities of x-direction (F' and M) and z-direction F and increase y-direction component N and fluid temperature. Also, imposition of fluid suction $F_W \succ 0$ at a surface reduces the region of viscous domination close to the wall, which causes a decrease in the fluid's velocity components in the x- and y-directions and fluid temperature profile, whereas an increase in its velocity component in the z-direction and the fluid pressure profile G. On the other hand, blowing fluid $F_W \prec 0$ from the porous surface into the main stream of the flow produces the opposite effect, that is, increases in the x and y velocity components and decreases in the velocity component in the z-direction.

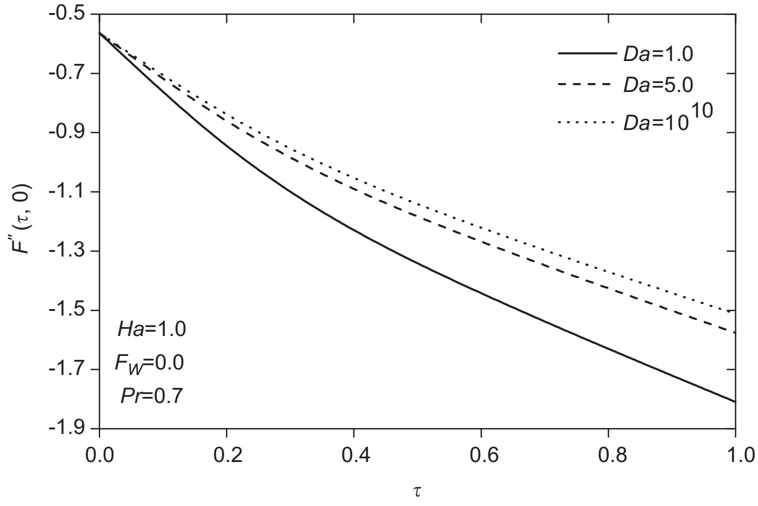

Fig. 24. Effects of Darcy number *Da* and heat generation/absorption coefficient *Q* on the skin-friction coefficient in the *x*-direction $F''(\tau, 0)$.

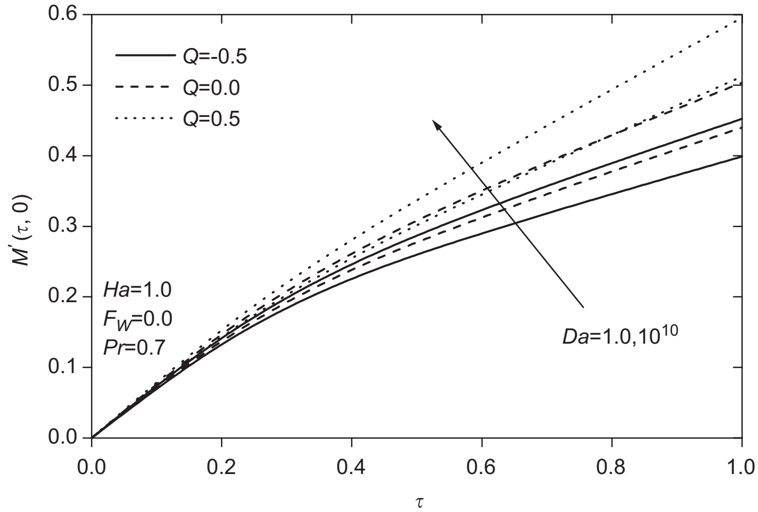

Fig. 25. Effects of Darcy number *Da* and heat generation/absorption coefficient *Q* on the skin-friction coefficient in the *x*-direction $M'(\tau, 0)$.

Figs. 8–13 display the components of velocity flow, fluid temperature and pressure profiles for various values of Prandtl number *Pr* and time parameter τ, respectively. Obviously, an increase of Prandtl number decreased the fluid's velocity component in the *x*-direction *M* and the fluid temperature *H*, hence, decreased thermal boundary layer flow along the stretching surface. Also, the other components of velocities flow and pressure profile are unaffected by various values of *Pr*; that is because Eqs. (30) and (33) governing F', *F* and *G* are uncoupled from other equations. However, *x*-velocity component *M*, and *y*-velocity component *N* increase with an increase of time parameter τ, whereas the opposite effect is observed with *x*-velocity component F', *z*-velocity component *F* and fluid temperature profiles.

From Figs. 14–19, it should be mentioned that, increasing the Darcy number has a tendency to increase the *z*-velocity component *F* as well as *y*-velocity component *N*, *x*-velocity components F' and *M*, whereas it reduces the fluid temperature component *H* and pressure profiles *G*. This is due to the increased restriction resulting from the decreasing porosity of porous medium. Also, it can be seen that *x*-velocity component *M*, *y*-velocity component *N* and the fluid temperature *H* increases as heat generation/absorption coefficient increases. This is expected since heat generation

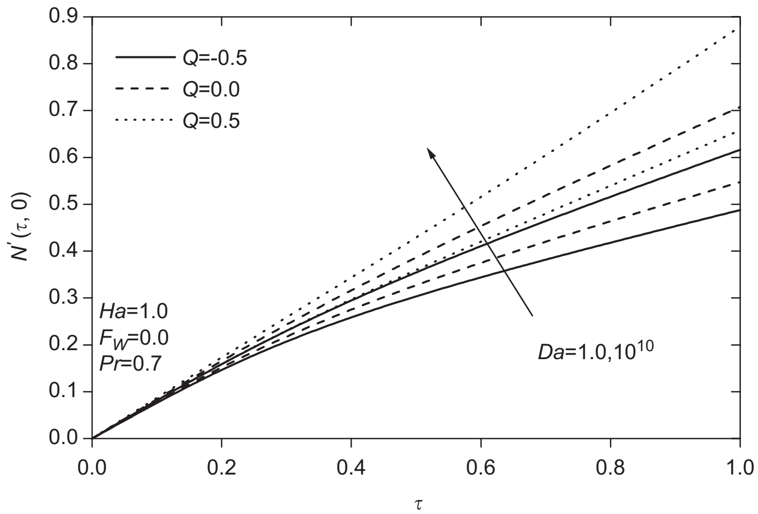

Fig. 26. Effects of Darcy number *Da* and heat generation/absorption coefficient *Q* on the skin-friction coefficient in the *y*-direction $N'(\tau, 0)$.

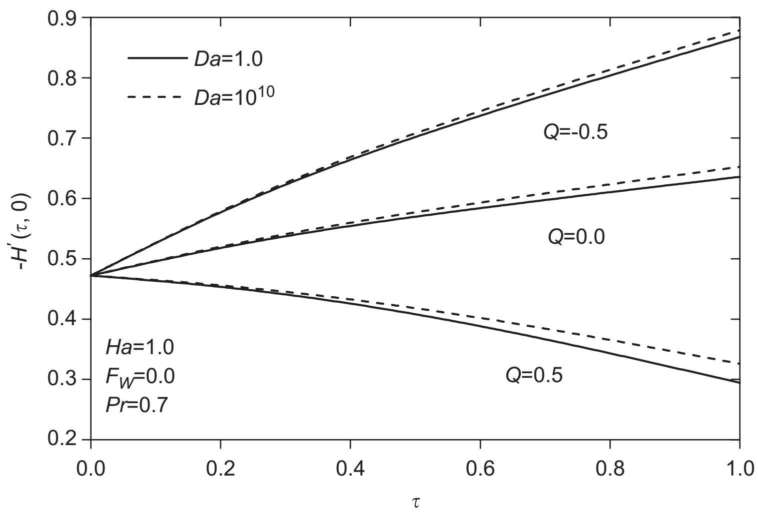

Fig. 27. Effects of Darcy number *Da* and heat generation/absorption coefficient *Q* on the wall heat transfer $-H'(\tau, 0)$.

$(Q > 0)$ causes the thermal boundary layer to become thicker and the temperature of the fluid to increase; this increase in temperature produces an increase in the flow field due to the buoyancy effect, which couples the flow and thermal problems, whereas it has the opposite effect with heat absorption $(Q < 0)$, reducing the temperature of the fluid and the thermal buoyancy effect, and reducing the fluid flow field. In addition, heat generation/absorption coefficient unaffected on other velocities flow and pressure profiles, for the same cause is mentioned above.

Figs. 20–27 present the effects of the governing physical parameters on the skin-friction coefficients of *x*-direction F'', M', *y*-direction N', respectively, and also transient wall heat transfer rate $-H'$ with different values of transient-parameter τ. From Figs. 20–23, it can be seen that, as the strength of magnetic field *Ha* increases, both the skin-friction coefficients of *x*-direction F'', M' and the local Nusselt number $-H'$ decrease while the skin-friction coefficient in the *y*-direction N' increases. On the other hand, it is seen that an increasing suction/injection F_W increased the wall heat transfer rate and decreased the skin-friction coefficients in the *x*- and *y*-directions.

Also, from Figs. 24–27, it can be noted that, on increasing the Darcy number *Da*, all of wall shear stress in directions of *x* and *y*, respectively (i.e., F'', M' and N'), and the local Nusselt number $-H'$ increased. Also, the skin-friction coefficient in y-direction N' increases, whereas the local Nusselt number $-H'$decreases as heat generation/absorption Q increases. As mentioned before, Q has no effect on the fluid flow field in the x-direction, and, therefore, it has no effect on the skin-friction coefficients in the z-direction.

8. Concluding remarks

The unsteady, three-dimensional, laminar, boundary-layer flow of a viscous, incompressible and electrically conducting fluid over an inclined permeable surface embedded in porous medium in the presence of a uniform magnetic field and heat generation/absorption effects are treated. Using Lie group method, we have presented the transformation groups for the problem, apart from the scaling group; the system admits a group of translations, as well, concerning the group of scaling and the associated self-similar solutions. Moreover, due to the generality of our procedure and the lack of unnecessary assumptions, we have obtained the general form of the functions involved in the boundary conditions. Finally, the application of three-independent-variable partial differential equations transformed to two-independent-variable system by using one subgroup of the general group. The resulting system of governing equations is solved numerically using perturbation technique for various values of physical parameters.

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