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The Jarzynski identity and the AdS/CFT duality

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ABSTRACT

We point out a remarkable analogy between the Jarzynski identity from non-equilibrium statistical physics and the AdS/CFT duality. We apply the logic that leads to the Jarzynski identity to renormalization group (RG) flows of quantum field theories and then argue for the natural connection with the AdS/CFT duality formula. This application can be in principle checked in Monte Carlo simulations of RG flows. Given the existing generalizations of the Jarzynski identity in non-equilibrium statistical physics, and the analogy between the Jarzynski identity and the AdS/CFT duality, we are led to suggest natural but novel generalizations of the AdS/CFT dictionary.

In this communication we wish to point out a deep analogy between the Jarzynski identity [1,2], one of the most remarkable results in the recent history of non-equilibrium statistical physics, and the AdS/CFT duality [3–5], one of the most astonishing developments in the recent history of quantum field theory and string theory.

The Jarzynski identity has been tested in many experimental situations in non-equilibrium systems [6–8] and it has been also theoretically generalized [9–12]. On the other hand, the AdS/CFT duality has been used in fields as diverse as quantum gravity, quantum chromodynamics, nuclear physics, and condensed matter physics [13–18]. The relationship between non-equilibrium statistical physics and AdS/CFT duality has been recently discussed in the context of aging in systems far from equilibrium [19,20]. The present Letter aims to establishing a closer connection between these two fields of physics.

The Jarzynski identity [1,2] gives the *exact* relation between the thermodynamic free energy differences ΔG and the irreversible work W

$$\langle \exp(-\beta W) \rangle = \exp(-\beta \Delta G) \qquad (1)$$

where $\beta^{-1} = k_B T$ with k_B denoting the Boltzmann constant and T the temperature. The average $\langle \ldots \rangle$ is over all trajectories that take the system from an initial to the final equilibrium states. Note that this exact equality extends the well-known inequality between work and change in free energy, $W \geqslant \Delta G$, which follows from the second law of thermodynamics. The relation $W \geqslant \Delta G$ is implied by the Jarzynski identity and Jensen's inequality $\langle e^A \rangle \geqslant e^{\langle A \rangle}$.

In the AdS/CFT correspondence, one computes the on-shell bulk action S_{bulk} and relates it to the appropriate boundary correlators. The *conjecture* [3–5] is then that the generating functional of the vacuum correlators of the operator O for a d-dimensional conformal field theory (CFT) is given by the partition function $Z(\phi)$ in (anti-de-Sitter) AdS_{d+1} space

$$\left\langle \exp\left(\int J O \right) \right\rangle = Z(\phi) \to \exp\left[-S_{bulk}(g, \phi, \ldots) \right] \qquad (2)$$

where in the semiclassical limit the partition function $Z = \exp(-S_{bulk})$. Here g denotes the metric of the AdS_{d+1} space, and the boundary values of the bulk field ϕ are given by the sources J of the boundary CFT.[1] Note that we have written here a semi-classical expression for the correspondence, which is what is essentially used in many tests of this remarkable conjecture [13–18].

Obviously, there exists a naive formal similarity between the expressions (1) and (2), given the fact that $\int JO$ formally corresponds to generalized "work". What we wish to argue in this Letter is that this naive similarity is actually deeper and points to a profound analogy between the two relations. Given the fact that (1) is exact (under certain assumptions) and (2) is still regarded as conjectural, but extremely profound and technically powerful, this analogy might point a way for a formal "proof" of (2). Also, we will

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[1] Essentially, in the language of the second part of this Letter, here one reinterprets the RG flow of the boundary non-gravitational theory in terms of bulk gravitational equations of motion, and then rewrites the generating functional of vacuum correlators of the boundary theory in terms of a semi-classical wave function of the bulk "universe" with specific boundary conditions.

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argue that this analogy points to some novel views on the RG flows of quantum field theories as well as to natural generalizations of the AdS/CFT dictionary.

We begin by stating clearly that we are making two points in this note: (a) first, after reviewing the proof of Jarzynski's identity in statistical physics, we propose an analogous identity in the context of the Wilsonian renormalization group approach, by applying the same logic applied to the original Jarzynski argument, and by clearly emphasizing the difference in the respective physics in these two cases, and then (b) based on the expected intuitive relation between the Wilsonian renormalization group formalism in the boundary quantum field theory and the holographic renormalization in the bulk gravitational theory, we propose to use this new form of the Jarzynski identity in the context of AdS/CFT-like duality (with some explicitly stated caveats). We emphasize that the first point (the RG Jarzynski identity) is very precise, and that the second point concerning the AdS/CFT duality, is essentially heuristic.

First, we start with a path-integral proof of the Jarzynski identity as presented by Hummer and Szabo [21] because we take this formalism to be most appropriate for the AdS/CFT context. What Hummer and Szabo pointed out [21] is that the Jarzynski identity follows from the Feynman–Kac theorem for path integrals [22, 23]. In what follows we summarize the argument of Hummer and Szabo and then we transcribe it to the case of renormalization group (RG) flows of Euclidean quantum field theories.

Consider a system whose phase space ($\vec{x}$) density $f(\vec{x},t)$ evolves according to the canonical Liouville equation

$$\frac{\partial f(\vec{x},t)}{\partial t} = L_t f(\vec{x},t). \tag{3}$$

The Liouville operator L_t explicitly depends on time t and the Boltzmann distribution is its stationary solution

$$L_t e^{-\beta H(\vec{x},t)} = 0. \tag{4}$$

Next, Hummer and Szabo consider the unnormalized Boltzmann distribution at time t

$$p(\vec{x},t) = \frac{e^{-\beta H(\vec{x},t)}}{\int d\vec{y}\, e^{-\beta H(\vec{y},0)}}. \tag{5}$$

This distribution is stationary, i.e. $L_t p(\vec{x},t) = 0$, and it also obviously satisfies

$$\frac{\partial p}{\partial t} = -\beta \left(\frac{\partial H}{\partial t}\right) p. \tag{6}$$

Therefore, the above distribution $p(\vec{x},t)$ is a solution of the following "sink" equation, which is of a Fokker–Planck type with a "sink" term,

$$\frac{\partial p}{\partial t} = L_t p - \beta \left(\frac{\partial H}{\partial t}\right) p. \tag{7}$$

Next, Hummer and Szabo point out that the solution of this "sink" equation, starting from an equilibrium distribution at time $t = 0$, can be expressed as a path integral, using the Feynman–Kac theorem [22,23], i.e.

$$p(\vec{x},t) = \left\langle \delta(\vec{x} - \vec{x}_t) \exp\left[-\beta \int_0^t \frac{\partial H}{\partial t'}(\vec{x}'_t, t')\, dt'\right]\right\rangle \tag{8}$$

where the average $\langle \cdots \rangle$ is over an ensemble of trajectories starting from the equilibrium distribution at $t = 0$ and evolving according to the Liouville equation. Each trajectory is weighted with the Boltzmann factor of the external work W_t done on the system

$$W_t = \int_0^t \frac{\partial H}{\partial t'}(\vec{x}'_t, t')\, dt'. \tag{9}$$

By remembering that the exponent of the free energy difference is given by definition as

$$e^{-\beta \Delta G} \equiv \frac{\int d\vec{x}\, e^{-\beta H(\vec{x},t)}}{\int d\vec{y}\, e^{-\beta H(\vec{y},0)}} \tag{10}$$

we are thus lead to the Jarzynski identity

$$\exp(-\beta \Delta G) \equiv \frac{\int d\vec{x}\, e^{-\beta H(\vec{x},t)}}{\int d\vec{y}\, e^{-\beta H(\vec{y},0)}} = \langle \exp(-\beta W_t)\rangle. \tag{11}$$

In what follows, we will repeat this derivation, step by step, by utilizing the well-known formal dictionary between the Hamiltonian H of a dynamical system in phase space and the action S of a Euclidean quantum field theory [24,25]

$$\beta H(\vec{x},t) \to S(\varphi, \Lambda) \tag{12}$$

where we have also introduced the cut-off Λ at which the action of the quantum field theory is evaluated according to the dynamical RG equation [24,25]. The RG evolution parameter ("RG time") is given by the fact that the operation of rescaling formally corresponds to the "temporal" evolution

$$\Lambda \frac{\partial}{\partial \Lambda} \to \frac{\partial}{\partial \tau}. \tag{13}$$

We wish to be very clear: in the discussion of the Jarzynski-like identity in the context of the renormalization group we follow the proof just outlined for the case of the Liouville dynamics, but we point out: (i) that the renormalization group dynamics is *not* the Liouville dynamics, because of its fundamental irreversible nature and yet (ii) the *logic* applied to the context of the Liouville dynamics can be used in the context of the renormalization group in order to arrive at the statement of the Jarzynski-like identity! The important point here is that in what follows in the context of the Wilsonian renormalization group one ultimately gets a stochastic like equation which is then solved by averaging over the renormalization group trajectories for the appropriate expressions involving the "free energy" and "work". This then leads to a new Jarzynski-like identity involving averages over ensembles of RG (and not dynamical) trajectories! Here one should emphasize that both the RG "free energy" and "work" introduced below are defined with respect to the renormalization group formalism and are fully covariant. Also, this proposed Jarzynski-like identity can be now tested in numerical RG experiments which are being planned at the moment.

Thus we can repeat the steps of Hummer and Szabo by replacing $\beta H \to S$ and $t \to \tau$. Therefore, consider a Euclidean quantum field theory (and for simplicity, the scalar field theory in 4 space–time dimensions) whose exponent of the effective action $e^{-S(\varphi,\tau)}$ evolves according to the canonical RG equation [24,25]

$$\frac{\partial e^{-S_I(\varphi,\tau)}}{\partial \tau} = L_\tau e^{-S_I(\varphi,\tau)} \tag{14}$$

where the exact RG equation [24,25] states that

$$L_\tau = -\frac{1}{2} \int d^4p\, (2\pi)^4 \left(p^2 + m^2\right)^{-1} \frac{\partial K}{\partial \tau} \frac{\delta^2}{\delta\varphi(-p)\delta\varphi(p)}. \tag{15}$$

Here $K(\frac{p^2}{\Lambda^2})$ is the cut-off function in 4-momentum (p) space, and the total action $S = S_0 + S_I$, where S_I is the interacting part and the free part S_0 is purely quadratic

$$S_0 = \frac{1}{2}\int d^4p\,(2\pi)^{-4}\varphi(p)\varphi(-p)\big(p^2+m^2\big)K^{-1}\left(\frac{p^2}{\Lambda^2}\right). \tag{16}$$

The exact RG equation comes from the requirement that the generating functional of the vacuum correlators $Z[J]_\tau = \int D\varphi\, e^{-S+\int J\varphi} \equiv \langle e^{-\int J\varphi}\rangle_\tau$ is τ-independent,

$$\frac{\partial Z[J]_\tau}{\partial\tau} = 0. \tag{17}$$

Note that the exact RG equation is equivalent to the fundamental Schrödinger equation for quantum field theory, or equivalently, to the knowledge of its kernel, the path integral, as clearly pointed out in the classic review by Wilson and Kogut [24, pp. 154, 155]. Thus the RG equation can be precisely viewed as a functional Fokker–Planck equation for the probability density given by e^{-S_I}. Note that a Fokker–Planck equation with a "sink" term is precisely what we have encountered in our previous analysis of the Liouville dynamics, even though the RG and the Liouville operators, and the RG and Liouville dynamics are entirely different! The crucial point is that in both cases we have Fokker–Planck equations, even though generated by different operators. Thus, we can draw a precise analogy between an equilibrium distribution and the conformal fixed point, described by a conformal field theory for which

$$L_\tau e^{-S_I(\varphi,\tau)} = 0. \tag{18}$$

Thus in the case of the RG flow, we will consider all trajectories that connect one conformal fixed point to another one. This is in complete analogy with all non-equilibrium paths that connect two equilibrium states, as in the case of the proof of the Jarzynski identity. Therefore we are led to consider (we use S for S_I)

$$P(\varphi,\tau) = \frac{e^{-S(\varphi,\tau)}}{\int D\psi\, e^{-S(\psi,\tau_0)}}. \tag{19}$$

This distribution is stationary by construction, i.e. $L_\tau P = 0$, and it also obviously satisfies

$$\frac{\partial P}{\partial\tau} = -\left(\frac{\partial S}{\partial\tau}\right)P. \tag{20}$$

Therefore, the above distribution P is a solution of the following "sink" equation

$$\frac{\partial P}{\partial\tau} = L_\tau P - \left(\frac{\partial S}{\partial\tau}\right)P. \tag{21}$$

For clarity let us stress once again that this is a Fokker–Planck-type equation, as was the case with the analogous discussion for the Liouville dynamics, even though the nature of the RG and the Liouville operators that feature in the corresponding equations is entirely different. The crucial point is that both equations can be solved using the same mathematical techniques. Again, the solution of this "sink" equation, starting from an equilibrium distribution at time $t=0$ can be expressed as a path integral, using the Feynman–Kac theorem which applies to any linear stochastic equation (a good reference to the stochastic equations and the general Feynman–Kac formula is the book by Oeksendal [26] as well as the papers by Kac [27] and Feynman [28]):

$$P(\varphi,\tau) = \left\langle \delta(\varphi-\varphi_\tau)\exp\left[-\int_0^\tau \frac{\partial S}{\partial\tau'}\big(\varphi_{\tau'},\tau'\big)\,d\tau'\right]\right\rangle \tag{22}$$

where the average $\langle\cdots\rangle$ is over an ensemble of RG trajectories starting from one "equilibrium distribution", i.e. one conformal fixed point, which evolves according to the RG equation to another conformal fixed point. The reason why we average over an ensemble of RG trajectories is that they are defined by the RG equation and the evolution parameter τ, even though the nature of the RG trajectories is completely different from the ordinary dynamical trajectories – in particular the RG flow is not reversible, in contrast to the usual Liouville dynamics. Still, the above solution is correct because it follows from the general properties of linear stochastic equations and the Feynman–Kac theorem. Note that in the final result only the "sink" term enters, which was also true in the discussion of the Liouville dynamics. Each RG trajectory is weighted with the appropriate "Boltzmann factor" of the "external work" W_τ done on the system

$$W_\tau = \int_0^\tau \frac{\partial S}{\partial\tau'}\big(\varphi_{\tau'},\tau'\big)\,d\tau'. \tag{23}$$

Also, the "free energy" difference is given by definition as

$$e^{-\Delta G} \equiv \frac{\int D\varphi\, e^{-S(\varphi,\tau)}}{\int D\psi\, e^{-S(\psi,\tau_0)}} \tag{24}$$

where τ_0 corresponds to the initial cutoff Λ_0. We are thus lead to the RG form of the Jarzynski identity

$$\exp(-\Delta G) \equiv \frac{\int D\varphi\, e^{-S(\varphi,\tau)}}{\int D\psi\, e^{-S(\psi,\tau_0)}} = \big\langle\exp(-W_\tau)\big\rangle. \tag{25}$$

This is the equation that should be tested in numerical RG experiments. This equations has not been considered in the literature before, even though it is of an exact form, presumably because the physical construction that leads to the relevant linear stochastic equation which implies this exact equality, is not really motivated without thinking about the original Jarzynski equality.

We expect that there exists a natural interpretation of the various quantities involved in the above discussion of the RG Jarzynski identity, such as the stochastic trajectories between the conformal fixed points and the work done, if one thinks of the specific case of 2d RG flows. What we have in mind is that the difference between free energy and work (as defined in terms of the Euclidean action for the QFT) should define the natural entropy for the 2d flows, which in turn should be precisely interpreted from the point of view of the c-theorem. In view of our discussion of the RG Jarzynski identity such an interpretation should not be limited to 2d, and should be applicable to higher-dimensional QFTs. We also expect that the numerical computation should be more efficient for the higher-dimensional QFTs as compared to the direct computation. However, due to their special properties 2d QFTs might be exceptional in this regard.

Next we turn to the relation of the RG formalism and the AdS/CFT correspondence. In this context, the bulk dynamics that is being rewritten in terms of the so-called holographic renormalization group formalism is Hamiltonian (i.e. it can be recast in terms of the Hamiltonian formulation of the bulk gravity), and thus could be understood, in detail, as a Liouville dynamics! On the other hand, the dual field theory is understood in the context of the usual Wilsonian RG. The relation between the two is still on the level of an intuitive, but progressively deeper, understanding [29]. In the previous part of this Letter we have already applied the logic of the original Jarzynski argument to the Wilsonian RG and given a proposal for a Jarzynski like identity in that context. But given the nature of the bulk gravitational dynamics in the AdS/CFT correspondence the connection with the Liouville dynamics of the original Jarzynski argument is much more appropriate. Hence, given our proposal for a Jarzynski-like identity for the Wilsonian RG flows, and the intuitive relation of Wilsonian RG flows and the holographic flows, as well as the Hamiltonian nature of the latter, we naturally extrapolate the RG Jarzynski identity for the vacuum averages which then directly relate to the fundamental AdS/CFT formula, provided one takes into account the caveats listed below. We emphasize that our discussion of the AdS/CFT duality is more speculative than the application of the Jarzynski identity to the RG. Our arguments regarding the AdS/CFT-like duality are of a heuristic nature, as opposed to the precise statement of the RG Jarzynski identity. In particular, to approach the AdS/CFT dictionary from the RG Jarzynski identity we envision an infinitesimal proximity of the initial and final conformal points, with an infinitesimal set of stochastic trajectories connecting them. Then in principle we should have only one CFT to work with, and the averages over the infinitesimal set of stochastic trajectories should be the natural averages pertaining to that CFT, i.e. the averages defined by the path integral of that CFT. That is why we expect that the vacuum average should replace the average over RG trajectories.

Next, we equate the work W_τ with the work done by the external source. This can be understood very simply by invoking the conjugate relation between the sources and fields with respect to the covariant action. The relation of this type defines generalized forces (in this cases, sources) and thus

$$W_\tau \equiv -\int J\varphi \tag{26}$$

can be understood as a generalized work (where the integral is over space). We think that this substitution is natural given the covariant nature of the RG Jarzynski identity, and the fact that in the case of vacuum averages, which we have argued replace the average over the RG trajectories, the only "covariant work" is done by sourcing the vacuum. We do not see any other natural candidate for such "covariant work". We also identify the initial and the final conformal fixed point and apply the above proposal for the Jarzynski-like identity in the renormalization group context and then we are led to an AdS/CFT-like relation

$$\left\langle \exp\left(\int J\varphi\right)\right\rangle = \exp(-\Delta G). \tag{27}$$

Note that we have treated φ as the fundamental field. The same reasoning can be applied to any general operator O in the above Euclidean quantum field theory. Now, we would like to appeal to the extra dimension τ to argue that this formula can be rewritten as the actual AdS/CFT relation provided:

(1) We assume a geometrization of assumed conformal invariance in the τ direction, so that the metric in the τ direction has the isometries of the conformal group associated with the assumed initial and final conformal fixed points. This leads us to asymptotically AdS metrics $ds^2 = dr^2 + e^{Ar}\,ds^2_{CFT}$, where $\tau = Ar$ in the flat coordinate system (A determining the size of the bulk space) and where ds^2_{CFT} is the natural flat metric of the boundary CFT.

(2) We assume a map between the choice of RG scheme to the choice of coordinates in the τ extra-dimensional space, thus effectively inducing gravitational interactions in this AdS space. This is reasonable from what we know about perturbative string theory and its relation to the Wilsonian RG [30–32], as well as from what we know about holographic RG in the context of AdS/CFT [33–39]. Nevertheless, this might be harder to justify than the first assumption.

As another general caveat we note that the field theories for which we expect holographic duals are gauge theories for which we do not have a nice Wilsonian RG because a cutoff corresponding to a physical length scale typically breaks gauge invariance, and a cutoff for the gauge theory that could be geometrized is not known at present. Also, most field theories do not have a semiclassical gravity dual and thus AdS/CFT should work only for a limited number of quantum field theories. Most probably, the theories for which this duality works can be obtained from the open string sector, in which case AdS/CFT is really an open/closed string duality of a very specific kind (the gravity dual coming from the closed string sector). A nice discussion of this point is given by I. Heemskerk and J. Polchinski in the first reference of [29].

Finally, we recall that gravity is a very special interaction whose energy is given in term of boundary data [40–42], or symbolically

$$\Delta G = \Delta S_{bulk} \tag{28}$$

and thus the RG Jarzynski identity, with above assumptions, becomes the canonical AdS/CFT formula. Note that the semiclassical limit has to come in here, if the expression for the change of the free energy defined in the context of the RG Jarzynski identity is used so that the relative partition function is expanded in some appropriate WKB limit. That WKB expression for the relative partition function will necessarily involve an exponent of some effective action, which could be interpreted as an on-shell "bulk" action. Of course, the reason for the fundamental appearance of gravity is obscure in this heuristic argument. As we have mentioned before, presumably the true origin of the AdS/CFT duality should be sought in the open/closed string duality, which would then make the appearance of gravity more palatable.

To conclude: the RG Jarzynski identity is not identical to the usual Jarzynski identity and the AdS/CFT relation to the RG is illuminated using the RG Jarzynski identity provided the RG ensemble is replaced by vacuum averages and provided some natural caveats are met. Nevertheless, the statement of the RG Jarzynski identity is very precise, and the connection to the AdS/CFT duality is at this point only heuristic.

This heuristic analogy between the Jarzynski identity and the AdS/CFT duality has many potential applications: For example, we can envision tests of the RG Jarzynski identity (25) that closely mirror the existing single-molecule tests of the original Jarzynski relation [6–8]. In these experiments a single molecule is repeatedly stretched mechanically and the work is recorded. These non-equilibrium work fluctuations are then used in order to reconstruct the free-energy landscape of the molecule. A similar "pulling" test of the RG Jarzynski identity can be designed where RG flows are simulated using Renormalization Group Monte Carlo techniques [43,44]. In that case one would measure the generalized work due to the change of the Euclidean action along the RG trajectory and from that reconstruct the partition function for the Euclidean quantum field theory.

Next, we can try to apply the generalizations of Jarzynski's identity [9] in order to generalize AdS/CFT-like dualities. In that case we do not need to assume conformally invariant fixed points. On the side of non-equilibrium physics [9], this would correspond to the situation where one has a probability distribution of a steady state (ss) with some parameter α, $\rho_{ss}(x;\alpha)$, with the corresponding (negative) "entropy" (in the sense of Boltzmann's definition)

$$\Phi(x;\alpha) = -\log\rho_{ss}(x;\alpha). \tag{29}$$

Given the general properties of probability distributions one can assert the following mathematical identity [9] (for a discrete time evolution, labeled by $i = 1, 2, \ldots, N$)

$$\left\langle \prod_{i=0}^{N-1} \frac{\rho_{ss}(x_{i+1};\alpha_{i+1})}{\rho_{ss}(x_{i+1};\alpha_i)}\right\rangle = 1 \tag{30}$$

that implies in the limit $N \to \infty$ [9] the generalized Jarzynski identity

$$\left\langle \exp\left(-\int_0^t dt' \frac{d\alpha}{dt'} \frac{\partial \Phi(x;\alpha)}{\partial \alpha}\right)\right\rangle = 1. \tag{31}$$

The usual Jarzynski identity follows when $\Phi = -\beta(G - W)$. Given our dictionary between time and the logarithm of the cut-off Λ ($t \to \tau$) we can obviously translate this general Jarzynski formula into a general AdS/CFT-like formula,

$$\left\langle \exp\left(-\int_0^\tau d\tau' \frac{d\alpha}{d\tau'} \frac{\partial \tilde{\Phi}(x;\alpha)}{\partial \alpha}\right)\right\rangle = 1 \tag{32}$$

which, curiously, involves the gradient of "entropy" $\frac{\partial \tilde{\Phi}}{\partial \alpha}$. (In the usual AdS/CFT case $\tilde{\Phi} = -(S_{bulk} + \int JO)$.) This gradient of "entropy" corresponds to some kind of "entropic force", a concept that has recently been invoked in the context of the holographic treatment of gravity [45]. Thus, it is quite plausible that the concept of entropic force does play a very precise, albeit hidden, role in the AdS/CFT-like dualities. Such a generalized AdS/CFT formula should be useful in illuminating the puzzling duals of cosmological backgrounds or pure (non-conformal) Yang–Mills theory, or various condensed matter systems.

We conclude this Letter with the following set of questions: We have briefly invoked the generalized Jarzynski identity and what this could imply for the AdS/CFT duality. Conversely, it is natural to ask: Does the full AdS/CFT duality, in which one uses the full bulk partition function instead of $e^{-S_{bulk}}$, say something about even more generalized versions of Jarzynski's identity? In our translation of the Jarzynski identity to the language of RG evolutions of conformal field theories we have encountered the concept of free energy. On the other hand, the concept of the c-function is somewhat analogous to free energy. Thus, it is natural to ask: What is the connection of the free energy change ΔG and the holographic c-function [46]? Similarly, given the current activity concerning the application of AdS/CFT duality to many-body physics, one could wonder whether this unexpected relation with the Jarzynski identity illuminates the uses of AdS/CFT in the condensed matter settings [13–18]? Finally, on a more ambitious and speculative level: Does the topic of this Letter point to a more general relation between quantum gravity and non-equilibrium statistical physics [47–52]? We leave these and many other questions for future work.

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