



Axion inflation in F-theory

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ABSTRACT

We study the dynamics of axion-like fields in F-theory and suggest that they can serve as inflatons in models of natural inflation. The axions arise from harmonic three-forms on the F-theory compactification space and parameterize a complex torus that varies over the geometric moduli space. In particular, this implies that the axion decay constants depend on the complex structure moduli that can be fixed by background fluxes. This might allow tuning them to be super-Planckian in a controlled way and allow for interesting single field inflationary models. We argue that this requires a localization of the three-forms near regions of strong string coupling, analogously to the reasoning that GUT physics requires the use of F-theory. These models can admit a tensor to scalar ratio $r > 0.1$.

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1. Introduction and summary

Inflation was proposed to solve several cosmological puzzles, including the homogeneity, isotropy, and flatness of the universe, as well as the absence of relic monopoles [1–3]. The simplest models of inflation are driven by slowly rolling scalar fields [4]. Inflationary models also predict small inhomogeneities in the cosmic microwave background that can be used to test the inflationary paradigm. Recently, the BICEP2-experiment announced the discovery of a non-zero signal of primordial gravitational waves in the B-mode power spectrum [5]. The measured B-mode spectrum was well-fitted with a lensed Λ -CDM model with a tensor to scalar ratio $r = 0.20^{+0.07}_{-0.05}$. If this result survives further tests it places remarkably strong constraints on inflationary models. In particular, it suggests that large field models in which the inflaton rolls over super-Planckian distances are favored. To control such a scenario within an effective field theory, an embedding into a theory of quantum gravity such as string theory is desired. Such a UV completion allows to examine the flatness of the potential for such large field ranges.

There have been various suggestions how to realize an inflation dynamics within string theory [4] and only a few can accommodate a large tensor to scalar ratio as has now been observed. One way to construct a well-controlled large field inflationary model is to postulate that the scalar inflaton admits a classical shift symmetry that is preserved perturbatively. Such a Peccei–Quinn symmetry can naturally protect the two scales of inflation necessary to roll over a long distance in a flat potential [6].

Many candidate axions with these properties arise in string theory as zero modes of the R-R and NS-NS form fields. The ex-

PLICIT value of the axion decay constants in string theory has been examined in various string compactifications. A systematic study appeared, for example, in [7]. Already earlier, it was claimed that axion decay constants are always sub-Planckian in string theory. More precisely, it was argued that in a controlled compactification one has to be at large volume and small string coupling, at least in some dual frame, which naturally leads to a suppression of the axion decay constants. In this work we argue that F-theory provides a natural set of axions that can admit large axion decay constants. More precisely, we claim that two crucial points about inflation in string theory can be successfully addressed: (1) We identify axions with perturbative shift-symmetries that are the lightest fields during inflation in a controlled setting generalizing [8]; (2) We argue that in F-theory field excursions can be potentially super-Planckian. The axion decay constants crucially depend on a calculable holomorphic function of the complex structure moduli that can be evaluated also for geometries with strong string coupling regions. A non-perturbatively induced scalar potential is periodic in the axions, but also depends on the aforementioned holomorphic function.

It was suggested in [8] that axions arising from the R-R two-form in Type IIB orientifold compactifications with minimal four-dimensional supersymmetry might serve as candidate inflatons. To realize such a scenario it was important that moduli stabilization using background fluxes and non-perturbative effects [9–11] ensures that these axions can be the lightest fields during inflation. Furthermore, they combine with the NS-NS two-form axions into complex fields G^a and are therefore not directly linked to the geometric moduli of the compactification space. In fact, the G^a parameterize a complex torus with metric depending on the

geometric moduli of the compactification space and the string coupling. The arguments of [12] state that in a controlled region of moduli space, i.e. at weak string coupling and large volume, the metric for axions encoding the axion decay constants is always sub-Planckian. Therefore, [8] implemented the N -flation scenario [13,14], in which inflation is driven by N axions using an assistance effect [15,16].¹

In this work we want to propose a scenario that might circumvent the no-go results of [12] by using an F-theory background. It should be stressed, however, that we do not expect that one can tune the axion decay constants arbitrarily large without ruining the inflationary potential. Instead, we suggest that the limits from the weakly coupled Type IIB analysis will be weakened and a wider range of possibilities for model building is accessible in F-theory. The precise upper limits on the axion decay constants will depend on the form of the scalar potential, which we discuss in more detail in Sections 2.2 and 3.3. We will also not comment on the impact of further corrections to the Kähler potential, such as α' -corrections. While they might be crucial for such an extreme scenario with large axion decay constants their calculation is beyond the scope of this work. We will therefore work in the moderately large volume limit of the geometry and focus on pure g_s corrections encoded by the F-theory compactification geometry.

1.1. Description of the scenario: a landscape of computable axion decay constants

In order to obtain large axion decay constants, we propose to consider an F-theory setup. We first note that the axion decay constants of the R-R two-form axions are proportional to the string coupling. While suppressed at weak string coupling, there can be an enhancement in backgrounds with strong string coupling regions. F-theory allows to geometrically describe Type IIB backgrounds in which the complexified string coupling $\tau = C_0 + i/g_s$ varies over the compact six-dimensional space [18]. More precisely, one interprets τ as the complex structure modulus of an auxiliary two-torus, and encodes the background by an elliptically fibered geometry with two additional real dimensions. Keeping $\mathcal{N} = 1$ supersymmetry in four-dimensions requires this space to be an elliptically fibered Calabi–Yau fourfold Y_4 with a base B_3 . The space–time filling seven-branes are located at the singularities of the elliptic fiber, i.e. when the torus pinches. Therefore, one finds that general F-theory backgrounds will admit regions of strong string coupling. Let us recall, that one cannot globally employ the $Sl(2, \mathbb{Z})$ -symmetry of Type IIB to exchange strong and weak coupling. While there exist seven-brane configurations that admit a genuine weak coupling limit [19], this is generally not the case. In fact, when aiming to embed Grand Unified Theories (GUTs) into F-theory, an inherently non-perturbative configuration is required [20,21]. Let us also stress that such F-theory backgrounds automatically account for certain instanton corrections becoming relevant at strong coupling. The effective theory, at least for the moduli sector relevant in this work, can nevertheless be reliably calculated [22].

In the F-theory background the considered axions arise from harmonic three-forms on the Calabi–Yau fourfold Y_4 that admit one leg in the elliptic fiber. There are N such forms, where $2N$ is the number of harmonic three-forms on Y_4 minus the number of harmonic three-forms on the base B_3 . Indeed, these axions correspond in Type IIB to R-R and NS-NS two-form axions as well as Wilson line moduli on seven-branes. The three-forms on Y_4 parameterize a complex torus $\mathbb{T}_\mathbb{C}^{2N}$, defined in (24), with metric

depending on the complex structure moduli and Kähler moduli of Y_4 . This metric determines the axion decay constants and takes the simple form

$$f_{ab}^2 = \frac{i}{\mathcal{V}} \int_{Y_4} J \wedge \bar{\Psi}_a \wedge \Psi_b, \quad (1)$$

where Ψ_a are $(2, 1)$ -forms depending on the complex structure chosen on Y_4 , J is the Kähler form, and \mathcal{V} is the volume of Y_4 . A more detailed discussion of (1) can be found in Section 3. The complex structure dependence of Ψ_a allows to compute the axion decay constants in various regions in moduli space. Furthermore, the stabilization of complex structure moduli by fluxes has been investigated intensively [9–11]. In fact, the counting of flux vacua suggests that there are a vast number of vacua in complex structure moduli space near strong curvature regions [23,24]. A variety of computable axion decay constants therefore seems attainable even at controllably large volume. Depending on the value of the axion decay constants, one can implement either models of single axion natural inflation or N -flation. Clearly, also axion monodromy models [25–32] should be attainable in F-theory.

The functional dependence of the axion decay constants on the complex structure moduli can be computed for specific Calabi–Yau fourfolds using methods known from the computation of periods.² We will not attempt to perform such a computation here. Instead, we will employ a local picture and motivate that large values can indeed arise if the axions localize in the internal space near certain seven-brane configurations.

1.2. Single axion model and GUTs on seven-branes

The simplest models one can consider arise from geometries with $N = 1$. In order to obtain inflationary dynamics in such a setup, it is necessary to either engineer super-Planckian axion decay constants f^2 , or to roll over several periods of the axion. The single axion now lives on a two-torus $\mathbb{T}_\mathbb{C}^2$ with complex structure induced by the complex structure of the Calabi–Yau fourfold Y_4 . As an elliptic curve the complex structure of this $\mathbb{T}_\mathbb{C}^2$ can be encoded by a single function h varying holomorphically over the complex structure moduli space of Y_4 . For the axion decay constant f^2 we find the relation $f^2 \propto (\text{Im } h)^{-1}$. While $\text{Im } h \approx \text{Im } \tau = 1/g_s \gg 1$ at weak string coupling, we claim that $\text{Im } h$ can be small when localizing the axion also near a strong coupling seven-brane. In order to see this, we will motivate that in a patch $\mathcal{B} \subset B_3$ one can write

$$f^2 \propto \frac{1}{\mathcal{V}_\mathcal{B}} \int_{\mathcal{B}} (\text{Im } \tau)^{-1} J_\mathcal{B} \wedge \tilde{\omega}^2, \quad (2)$$

where $\tilde{\omega}^2$ is localizing the axion in the patch \mathcal{B} , $J_\mathcal{B}$ is the Kähler form, and $\mathcal{V}_\mathcal{B}$ is the volume of B_3 . Let us stress that the fact that the axions arise from three-forms with one leg in the elliptic fiber allows to localize the physics of these fields in the base B_3 . The claim is that (2) can pick up large contributions from strong string coupling regions.

Strong coupling regions in an F-theory compactification are generic, but are particularly crucial for constructing F-theory GUTs. In these constructions the GUT group arises from a seven-brane stack on a four-cycle S in B_3 . In order to obtain the necessary Yukawa structure, the geometry has to have points on S where the local gauge symmetry enhances to an exceptional group E_6, E_7 , or E_8 [20,21]. One can envision that the axion is localized near such an exceptional point as depicted in Fig. 1.

¹ See also [17] for a different attempt to realize N -flation in string theory.

² See, for example, Refs. [33,34] for the study of fourfold periods.

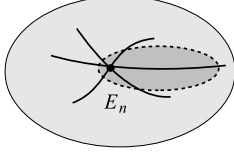


Fig. 1. Localization of the axion support (dashed line) near a strong coupling point.

Furthermore, in such a case one will also find that the complex field G containing the inflating axion $\text{Re} G$ will correct the GUT gauge coupling function as

$$\alpha_{\text{GUT}}^{-1} = \text{Vol}(S) + \kappa (\text{Im} h)^{-1} (\text{Im} G)^2, \quad (3)$$

where κ is a model-dependent number encoding the intersection number with S . This coupling has to be fixed at the GUT scale of 3×10^{16} GeV. Crucially, this will also choose a frame for the $Sl(2, \mathbb{Z})$ symmetry of \mathbb{T}_c^2 and render the notion of having large $(\text{Im} h)^{-1}$ well-defined. In other words, as we discuss in more detail in Section 3.3, moduli stabilization of the Kähler sector is linked to the value of f^2 in two ways: (1) f^2 directly depends on the vacuum value of the Kähler form J_b , (2) the correct definition of the $\mathcal{N} = 1$ coordinates of the volume moduli is modified by G and has non-trivial monodromies on \mathbb{T}_c^2 .

2. Inflation driven by axions

In the following we review some basics on natural inflation [6,35]. We introduce the basic constructions in Section 2.1. A candidate supergravity embedding is described in Section 2.2. We will argue in the next section that such a supergravity theory can arise in F-theory.

2.1. Brief review of natural inflation

Let us start with a set of axion-like scalars c^a , $a = 1, \dots, N$. By definition these admit a perturbatively preserved Peccei–Quinn shift symmetry

$$c^a \rightarrow c^a + \lambda^a, \quad (4)$$

where λ^a are constants. The Lagrangian for these fields takes the form

$$\mathcal{L} = \frac{1}{2} f_{ab}^2 \partial_\mu c^a \partial^\mu c^b - V. \quad (5)$$

The f_{ab}^2 might depend on other scalar fields of the theory, but are perturbatively independent of c^a . Only non-perturbative effects can induce a subleading c^a dependence.

To briefly discuss the phenomenological properties of such a model we assume that all fields determining f_{ab}^2 have been fixed to their minimum and diagonalize $f_{aa}^2 = f_a^2$. Canonically normalized scalars θ^a are obtained as

$$\theta^a = c^a f_a, \quad (6)$$

where no sum is performed. If the axions c^a are periodic with period 2π the accessible field ranges are

$$-\pi < c^a \leq \pi, \quad -f_a \pi < \theta^a \leq f_a \pi. \quad (7)$$

Since the shift symmetry (4) protects the theory from perturbative corrections in c^a , a scalar potential can only be induced by non-perturbative effects. Schematically, neglecting all cross couplings, the potential for the normalized axions θ^a takes the form

$$V(\theta^a) = \Lambda_0^4 + \sum_{n^a} \Lambda_{n^a}^4 (1 - \cos[n_a \theta^a / f_a]), \quad (8)$$

where Λ_0 is the cosmological constant at the vacuum $\theta^a = 0$, $\{n_a\}$ is a model dependent set of integers, and $\Lambda_{n_a}^4$ are the scales at which the Peccei–Quinn symmetries (4) are broken. One observes that the continuous symmetry (4) is broken by V to a discrete subgroup determined by the set $\{n^a\}$.

A theory with axions c^a and scalar potential (8) allows for models of natural inflation [6] or chaotic inflation for small θ^a [36]. Let us introduce the slow roll parameters for a separable potential V as in (8). In this case the slow roll parameters are given by

$$\epsilon = \frac{M_P^2}{2} \sum_a \left(\frac{V_{,a}}{V} \right)^2, \quad \eta = M_P^2 \min_a \left(\frac{V_{,aa}}{V} \right), \quad (9)$$

where $V_{,a} \equiv \partial_{\theta^a} V$ and $V_{,aa} = \partial_{\theta^a}^2 V$, and $M_P = 2.436 \times 10^{18}$ GeV is the reduced Planck mass. The slow roll conditions read $\epsilon < 1$ and $|\eta| < 1$ and define a multi-dimensional subspace in the fields θ^a where inflation takes place. In this inflationary region of the field-space the relevant physical observables can be defined as a function of the potential V and its derivatives only. For example, the tensor to scalar ratio r , is given by

$$r = 16\epsilon. \quad (10)$$

Therefore, it is straightforward to evaluate r for the scalar potential (8). As was noted in [15,16,13,14], even if each of the axions θ^a rolls over a distance smaller than M_P , an assistance effect for N axions can ensure a sufficient number of e-folds and a large r .

Of particular importance in this work will be the case of one axion $\theta = c \cdot f$. This yields the simple and elegant model of natural or chaotic inflation. In a general string compactification one might encounter numerous axions that are counted by the topological numbers of the compactification space. However, the masses of these axions can differ significantly during inflation, such that effectively only one field should be viewed as the inflaton. In such a scenario, however, the axion has to roll over a distance larger than M_P . In particular, calculating r one finds

$$r = 8 \left(\frac{M_P}{f} \frac{\sin(\theta/f)}{1 - \cos(\theta/f)} \right)^2, \quad (11)$$

where we have set $\Lambda_0 \approx 0$ and $f_1 \equiv f$, $n^1 = 1$ in (8). This simple formula implies that in order to have $0.1 < r < 0.2$ one has to have a super-Planckian axion decay constant f [35]. Furthermore, in a single field model the width of the potential must exceed the Lyth bound [37]

$$\Delta\theta \geq M_P \sqrt{2r}, \quad (12)$$

where $\Delta\theta$ is the field excursion during inflation.

The value of r is also related to the height of the potential determining the scale of inflation. Concretely, one finds

$$V_0 = (1.94 \times 10^{16} \text{ GeV})^4 \frac{r}{0.12}. \quad (13)$$

This implies that for $r > 0.1$ the scale of inflation is at least 10^{16} GeV, which is also the GUT scale at which gauge coupling unification occurs. Compatibility of natural inflation with the claimed BICEP2-results has been recently discussed in [38].

2.2. Embedding into supergravity

Before discussing axion inflation in F-theory it will be instructive to consider the encountered embedding into a purely four-dimensional supergravity setup. On the one hand, this will allow us to already comment on the properties of the appearing functions. On the other hand, by comparing with the Type IIB weak

coupling setup [8], we find that by requiring large axion decay constants, one is naturally led to consider F-theory setups.

Four-dimensional $\mathcal{N} = 1$ supergravity theories require the specification of a Kähler potential K and superpotential W to encode the relevant parts of the action. The scalar potential is then given by

$$V = e^K (K^{A\bar{B}} D_A W \overline{D_B W} - 3|W|^2), \quad (14)$$

where $K^{A\bar{B}}$ is the inverse Kähler metric and D_A is the Kähler-covariant derivative with respect to the complex Kähler coordinates.

The axions are complexified into fields $G^a = c^a + id^a$, with d^a real scalars in the same $\mathcal{N} = 1$ multiplet. In order that the shift symmetry is preserved perturbatively, the Kähler potential should only depend on $G^a - \bar{G}^a$. Indeed, in our F-theory setting with one Kähler structure modulus T , the Kähler potential takes the form

$$K = K_{\text{CS}} - 2 \log \mathcal{V}_b, \\ (6\mathcal{V}_b)^{3/2} \equiv T + \bar{T} + \frac{1}{4} C_{ab}^{\text{CS}} (G^a - \bar{G}^a)(G^b - \bar{G}^b), \quad (15)$$

where K_{CS} and C_{ab}^{CS} generally depend on a number of additional complex fields z^k , the complex structure moduli. In F-theory we will find that C_{ab}^{CS} can in fact be written as

$$C_{ab}^{\text{CS}} = (\text{Im } h^{ab})^{-1}, \quad (16)$$

where $h^{ab}(z)$ is a holomorphic function of the z^k . The axion decay constants are trivially computed from this Kähler potential for small $\text{Re } G^a$ to be

$$f_{ab}^2 = 2 \partial_{G^a} \partial_{\bar{G}^b} K = 3 \frac{(\text{Im } h^{ab})^{-1}}{T + \bar{T}}. \quad (17)$$

Therefore, both the vacuum value of $\text{Re } T > 0$ and $\text{Im } h^{ab} > 0$ will crucially influence the size of the axion decay constants in this setup.

The superpotential can contain a classical piece $W_{\text{flux}}(z)$ only depending on the z^k and a non-perturbative contribution W_{np} :

$$W = W_{\text{flux}}(z) + W_{\text{np}}(z, G, T). \quad (18)$$

One notes that the non-perturbative part depending on G^a is always suppressed by $\text{Re } T$ and hence should take the form

$$W_{\text{np}} = \Theta(z, G) e^{-T}. \quad (19)$$

Within string theory this is not surprising, since $\text{Re } T \rightarrow \infty$ corresponds to decompactification to a higher-dimensional theory without superpotential [39–42]. We will discuss the form of $\Theta(z, G)$ in Section 3.3, and only make some general remarks in the following. Since $\text{Im } G^a$ has a Peccei–Quinn shift symmetry, it should only depend on $e^{i n_a G^a}$, i.e. arise from non-perturbative effects. This implies that after fixing z^k and T to their vacuum values, one encounters an effective non-perturbative superpotential

$$W_{\text{eff}}(G^a) = \sum_{n_a} \Lambda_{n_a}^2 e^{i n_a G^a}, \quad (20)$$

where one sums as over a set of integer vectors $\{n^a\}$. The effective superpotential (20) allows to induce the scalar potential (8) for the axions $c^a = \text{Re } G^a$. In order to realize a simple natural inflation or N -flation model it is crucial that the $\Lambda_{n_a}^2$ and the exponential $e^{-n_a \text{Im } G^a}$ sufficiently suppress higher harmonics for the c^a .

Before turning to the construction of the described supergravity theory in F-theory, it is worth making the following simple observation. As we will see in the next section, the effective theory is

only valid for sufficiently large $\text{Re } T$, but it can equally well be applied for all values of $\text{Im } h^{ab} > 0$. An interesting regime to consider is therefore

$$\text{Im } h^{ab} \ll 1, \quad (21)$$

since it allows the axion decay constants (17) to be large, in fact, potentially super-Planckian. If this regime can be reliably approached in a string theory construction, a single axion model can implement inflation with a large r given in (11). However, deriving the above data in weakly coupled Type IIB string theory, one finds $\text{Im } h^{ab} \propto 1/g_s$, where g_s is the string coupling constant. Therefore, one has to leave the weak coupling regime to allow for (21). Here F-theory comes into the game and we suggest that $f_{ab}^2 > M_p^2$ implies that one has to use a full-fledged F-theory setting. It is interesting to remark that this is analogous to the fact that Grand Unified Theories cannot be studied in weakly coupled Type IIB string theory.

3. Inflation from F-theory axions

Let us consider F-theory compactified on an elliptically fibered Calabi–Yau fourfold Y_4 with base threefold B_3 . This theory corresponds to Type IIB string theory on B_3 with seven-branes located at the singularities of the elliptic fibration. The four-dimensional effective theory is minimally supersymmetric, such that its specifying data include a Kähler potential and superpotential. For the analysis of the moduli action it will be sufficient to apply the results of [22].

3.1. F-theory axions and their decay constants

Crucial for our F-theory models is the fact that the effective four-dimensional theory arising from such a reduction admits

$$N = h^{2,1}(Y_4) - h^{2,1}(B_3) \quad (22)$$

complex scalars G^a [11,22]. Here $h^{p,q}(Y_4)$, $h^{p,q}(B_3)$ are the Hodge numbers of the manifolds Y_4 , B_3 . To simplify our analysis we consider base manifolds B_3 with $h^{2,1}(B_3) = 0$ in the following. Furthermore, recall that $h^{3,0}(Y_4) = 0$ for every Calabi–Yau fourfold that yields an $\mathcal{N} = 1$ effective theory in four space–time dimensions. To define the G^a we use the dual M-theory picture, and expand the M-theory three-form C_3 as

$$C_3 = i G^a \bar{\Psi}_a - i \bar{G}^a \Psi_a, \quad (23)$$

where the Ψ_a , $a = 1, \dots, N$ form a basis of $H^{2,1}(Y_4)$. Since the Ψ_a have one leg in the elliptic fiber, performing the M-theory to F-theory limit shows that the fields G^a correspond to modes of the R-R and NS-NS two-forms and Wilson line moduli of seven-branes [22].

From (23) one realizes that the G^a are coordinates on a complex N -dimensional torus

$$\mathbb{T}_c^{2N} = H^{2,1}(Y_4)/H^3(Y_4, \mathbb{Z}), \quad (24)$$

which varies over the geometric moduli space of the manifold Y_4 . The complex structure on \mathbb{T}_c^{2N} will be induced by the complex structure on Y_4 and hence will vary over the space of complex structure deformations of Y_4 . The metric G_{ab} on \mathbb{T}_c^{2N} takes the form

$$G_{ab} = \frac{1}{2\mathcal{V}} \int_{Y_4} \bar{\Psi}_a \wedge * \Psi_b = \frac{i}{2\mathcal{V}} \int_{Y_4} J \wedge \bar{\Psi}_a \wedge \Psi_b. \quad (25)$$

where $\mathcal{V} = \int_{Y_4} J^4$ is the volume and J is the Kähler form of Y_4 . This metric encodes the four-dimensional kinetic terms of G^a as

$\mathcal{L}_{kin} = G_{ab} \partial_\mu G^a \partial^\mu \bar{G}^b$. One notes that G_{ab} depends on the complex structure deformations through Ψ_a and the Kähler structure deformations through the appearance of J and \mathcal{V} . Importantly, one expects that one can thus follow \mathbb{T}_c^{2N} into the interior of the complex structure moduli space of Y_4 .

The metric (25) can be derived from a Kähler potential [22], which for the simple case of having $h^{1,1}(B_3) = 1$ is of the form (15). In this case the Kähler form J on the base B_3 is proportional to the single harmonic form ω_b . Comparing the metric (25) with (17) and (15) one finds

$$C_{ab}^{cs} = 2i \int_{Y_4} \omega_b \wedge \bar{\Psi}_a \wedge \Psi_b. \tag{26}$$

The complex structure dependence of C_{ab}^{cs} therefore arises from the fact that the notion of Ψ_a being a $(2, 1)$ -form depends on the choice of complex structure on Y_4 .

The form (16) of C_{ab}^{cs} can now be inferred as follows. Let us introduce a real symplectic basis (α_a, β^b) on $H^3(Y_4, \mathbb{Z})$ satisfying

$$\int_{Y_4} \omega_b \wedge \beta^b \wedge \alpha_a = \delta_a^b, \tag{27}$$

$$\int_{Y_4} \omega_b \wedge \alpha_a \wedge \alpha_b = \int_{Y_4} \omega_b \wedge \beta^a \wedge \beta^b = 0.$$

The $(2, 1)$ -forms Ψ_a can be expanded in this basis as

$$\Psi_a = \frac{1}{2} (\text{Im } h^{ab})^{-1} (\beta^b - h^{bc} \alpha_c), \tag{28}$$

where h^{ab} is a symmetric matrix that depends holomorphically on the $h^{3,1}(Y_4)$ complex structure deformations z^k . This can be justified by the fact that $H^{2,1}(Y_4)$ can be chosen to vary holomorphically over the space of complex structure deformations. Indeed, setting

$$\psi^a = 2 \text{Im } h^{ab} \Psi_b = \beta^b - h^{bc} \alpha_c, \tag{29}$$

one finds $\partial_{z^k} \psi^a = 0$, for a holomorphic $h^{bc}(z)$. Inserting the expansion (28) into (26) one evaluates by using (27) that $C_{ab}^{cs} = (\text{Im } h^{ab})^{-1}$, just as claimed in (16).

In order to implement axion inflation one crucially has to show that the classical Kähler potential is independent of the axions $\text{Re } G^a = c^a$. This is true in the above setup, since $\text{Im } \Psi_a$ is independent of the complex structure moduli. Expanding the M-theory three-form $C_3 = \tilde{b}_a \beta^a - \tilde{c}^a \alpha_a$ in the basis (27) one finds by comparison with (23) and using (28) that the complex coordinates are defined as

$$G^a = \tilde{c}^a - h^{ab} \tilde{b}_b, \tag{30}$$

and the shift symmetry of C_3 translates to a shift-symmetry of $c^a = \tilde{c}^a - \text{Re } h^{ab} \tilde{b}_b$. This justifies the simple form of the Kähler potential (15). A detailed derivation can be found in [22].

Let us note that using the axions c^a was first suggested in the weakly coupled Type IIB N -fibration scenario of [8], and later used in axion monodromy inflation [25,27–30]. At weak coupling the axions c^a can be the zero-modes of the R-R two-forms C_2 , and the coordinates G^a are given by $G^a = \tilde{c}^a - \tau \tilde{b}^a$. In this case one finds

$$h^{ab} = \tau \delta^{ab}, \tag{31}$$

where $\tau = C_0 + ie^{-\phi}$ is a four-dimensional complex scalar field comprising the R-R zero-form C_0 and the dilaton ϕ . As stressed before, at weak coupling $\text{Im } \tau \gg 1$ and the axion decay constants are naturally sub-Planckian. Inflation then requires the use of an

assistance effect [15,16], as suggested for the N -fibration scenario of [13,14].

Let us close this section by noting that $h^{ab}(z)$ can be computed explicitly for a given Calabi–Yau fourfold example. The required techniques are similar to the ones for computing the complex structure dependent periods of the $(4, 0)$ -form Ω on Y_4 . In fact, by considering the variation of Hodge-structures, a basis ψ_a of $H^{2,1}(Y_4)$ varying holomorphically over the complex structure moduli space is expected to satisfy a second order homogeneous differential equation in the complex structure moduli z^k . This should allow to derive the moduli dependence of ψ_a explicitly. While it would be interesting to do that, we will take a more local route in the following to infer properties about h^{ab} . This will also hint to the fact that Y_4 should admit certain singularities that lead to four-dimensional gauge groups.

3.2. Decay constant in a one axion model

Let us next discuss the simplest possible model and assume that the geometry satisfies

$$h^{2,1}(Y_4) = 1, \quad h^{1,1}(B_3) = 1, \tag{32}$$

with $h^{2,1}(B_3) = 0$ as above. In this case one finds a single axion $c \equiv c^1$ as the real part of a single complex field G .

The complex field G parameterizes a complex one-dimensional torus \mathbb{T}_c^2 as defined (24). Recall the basic fact that such a torus can be mapped to an elliptic curve:

$$\mathbb{T}_c^2: \quad \tilde{y}^2 = 4\tilde{x}^3 - g_2(z)\tilde{x} - g_3(z), \tag{33}$$

by using the Weierstrass \wp -function $(\tilde{x}, \tilde{y}) = (\wp, \wp')$. The coefficient functions g_2, g_3 depend on the complex structure induced on \mathbb{T}_c^2 , and hence on the complex structure moduli z^k of Y_4 . To relate the $g_2(z), g_3(z)$ to the holomorphic function $h(z)$ introduced in (28), we note that ψ defined in (29) is given by $\psi = \beta - h\alpha$, where (α, β) span the lattice $H^3(Y_4, \mathbb{Z})$. Therefore, again applying a standard fact about elliptic curves, one finds the relation

$$j(h) = 1728 \frac{g_2^3}{g_3^2 - 27g_2^2}, \tag{34}$$

where j is the j -invariant of the elliptic curve. The axion decay constant for the single axion is given by

$$f^2 = 3 \frac{(\text{Im } h)^{-1}}{T + \bar{T}}. \tag{35}$$

The key question is whether the function $\text{Im } h$ of the complex structure moduli of Y_4 can be small such that f becomes large.

In order to proceed we aim to relate the elliptic curve (33) to the elliptic fiber of the Calabi–Yau manifold Y_4 . Let us first recall some basic facts about elliptic fibrations. The equation describing Y_4 can be brought into the Weierstrass form

$$Y_4: \quad y^2 = 4x^3 - f(u, z)x - g(u, z). \tag{36}$$

In contrast to (33), however, the coefficient functions depend both on the coordinates u on B_3 and on the complex structure moduli z^k . The complex structure τ of the elliptic fiber is given by $j(\tau)$ expressed as a function of f, g as in (34) with $g_2 \rightarrow f$ and $g_3 \rightarrow g$. Clearly, also τ depends both on the coordinates u on B_3 and the complex structure moduli z^k .

In order to relate τ and h we employ a local picture. The $(2, 1)$ -form Ψ can be locally written as

$$\Psi = \frac{1}{2} (\text{Im } \tau)^{-1} \tilde{\omega} \wedge (dx - \tau dy) \tag{37}$$

where $\tilde{\omega}$ is a two-form supported on a patch \mathcal{B} in the base B_3 . Both $\tilde{\omega}$ and τ depend on the location on B_3 and the integral (25) reduces on \mathcal{B} as

$$f^2 = \frac{1}{\mathcal{V}_b} \int_{\mathcal{B}} (\text{Im } \tau)^{-1} J_b \wedge \tilde{\omega}^2, \tag{38}$$

where we have integrated over the torus fiber with normalized volume form $dx \wedge dy$ and localized to \mathcal{B} . In (38) we further use that the base Kähler form J_b and volume \mathcal{V}_b arise when taking the M-theory to F-theory limit $\mathcal{V}^{-1} J|_{B_3} \rightarrow \mathcal{V}_b^{-1} J_b$ [22]. One concludes that the axion decay constants can indeed gain large contributions near strong coupling regions in the base \mathcal{B} . Interestingly, this is a local configuration and we believe that an explicit computation for an appropriate seven-brane configuration will confirm this result.

Let us observe that (38) at first seems ill-defined, since τ is only defined up to an overall $Sl(2, \mathbb{Z})$ monodromy of the elliptic fiber. In fact, such a monodromy can exchange a D7-brane into a general (p, q) -seven-brane. This symmetry translates into the $Sl(2, \mathbb{Z})$ symmetry of \mathbb{T}_c^{2N} . In the vacuum, however, a frame for this symmetry is chosen when considering a concrete moduli stabilization scenario. Indeed, the definitions of the $h^{1,1}(B_3)$ complex fields T_α containing the cycle volumes of Y_4 are non-trivially shifted by G^a [22]. For $h^{1,1}(B_3) = 1$ one finds

$$T = \mathcal{V}_b^{2/3} + i\rho - \frac{1}{4} C_{ab}^{cs} G^a (G^b - \bar{G}^b), \tag{39}$$

where ρ is the R-R four-form axion. Hence, a general $Sp(2N, \mathbb{Z})$ symmetry transformation of \mathbb{T}_c^{2N} will non-trivially shift the Kähler coordinates T_α . Recall that also the GUT gauge coupling is given by the combination $n_{\text{GUT}}^\alpha \text{Re } T_\alpha$, where the constant vector n_{GUT}^α determines the location of the GUT-brane in B_3 . Hence, fixing the gauge coupling on the observable brane will equally choose a frame for the symmetry of \mathbb{T}_c^{2N} . In the final subsection we will collect further comments on moduli stabilization and the generation of a potential for G^a .

3.3. Remarks on moduli stabilization

In the final part of this letter we comment on moduli stabilization and the generation of an axion potential. To begin with, note that the complex structure moduli of Y_4 can be stabilized using background four-form fluxes G_4 on Y_4 . In the four-dimensional effective theory the scalar potential arises from a superpotential [45]

$$W(z) = \int_{Y_4} \Omega(z) \wedge G_4 \tag{40}$$

where Ω is the $(4, 0)$ -form on Y_4 depending on the complex structure deformations z^k . Moduli stabilization with fluxes was studied intensively [9,10]. In particular, it was argued in [23,24] that a large fraction of the vacua derived from (40) are in the interior of the complex structure moduli space.

Clearly, in addition to stabilizing the complex structure moduli z^k using (40) one also needs to find a potential for the Kähler structure moduli T_α . Following the suggestion of [46], one can stabilize these fields using a non-perturbatively generated superpotential

$$W(T) = \sum_{n_\alpha} \Theta_{n_\alpha} e^{-n_\alpha T_\alpha}, \tag{41}$$

where the sum runs over a model dependent set of integers n_α classifying allowed non-perturbative configurations in F-theory [47].

The coefficient functions Θ_{n_α} can, in general, holomorphically depend on all other complex scalar fields of the theory. In partic-

ular, they can be a non-trivial functions of the complex structure deformations z^k and the fields G^a . The precise definition of T_α , which arises from the world-volume action of a brane instanton, suggest that the $\Theta_{n_\alpha}(z, G)$ are theta-functions on the torus \mathbb{T}_c^{2N} [39–41]. For example, for a single Kähler modulus $\Theta(z, G)$ takes the form

$$\Theta(z, G) = f(z) \sum_{n_a \in \Gamma} e^{\frac{1}{2} i h^{ab} n_a n_b + i n_a G^a}, \tag{42}$$

where Γ is some model dependent integer lattice. For instance, Γ can be the lattice of supersymmetric fluxes supported on the brane instanton, as discussed e.g. in [42–44].

In order to obtain the potential for the fields G^a we make the following observations. There are two contributions to the scalar potential for $\text{Im } G^a$. Firstly, there is a term arising from expanding the prefactor e^K in the $\mathcal{N} = 1$ scalar potential (14) as

$$e^K = e^{\langle K \rangle} (1 + 2 f_{ab}^2 \text{Im } G^a \text{Im } G^b + \dots), \tag{43}$$

where we have inserted the Kähler potential (15) and f_{ab}^2 given in (17), and expanded for small $\text{Im } G^a$. One observes that the arising term can give a significant contribution to the mass of $\text{Im } G^a$ when f_{ab}^2 is large. Secondly, the dependence of the superpotential (41) with (42) on the fields $\text{Im } G^a$ induces additional contributions to the scalar potential.

In contrast, the Kähler potential is independent of $c^a = \text{Re } G^a$. Therefore, a periodic potential of the form (8) is induced by (41) with (42). In order to build models of natural inflation or N -flation it will be crucial to ensure that higher harmonics for the c^a are suppressed. Investigating simple toy examples, one realizes that this is challenging for large $f_{ab}^2 \propto (\text{Im } h^{ab})^{-1}$ in (42), and that there will be a model-dependent upper bound on f_{ab}^2 with suppressed higher harmonics.

4. Conclusions

In this letter we have studied models of natural inflation realized in F-theory. The inflatons were chosen to be axions arising from harmonic three-forms with one leg in the elliptic fiber of the compactification Calabi–Yau fourfold. In the four-dimensional $\mathcal{N} = 1$ effective theory the inflating axions are the real parts of complex fields G^a . These are singled out, since only $\text{Im } G^a$ appear in the Kähler potential of the theory. The axions correspond to R-R or NS-NS two-form axions, and seven-brane Wilson line axions in Type IIB string theory. The three-forms span a complex N -dimensional torus \mathbb{T}_c^{2N} with complex structure and metric depending on the geometric moduli of Y_4 . In contrast to weakly coupled Type IIB compactifications, the F-theory axion decay constants are non-trivial functions over the complex structure moduli space of Y_4 . It is conceivable that a systematic fixing of complex structure moduli using the well-known flux superpotential will allow for a rich value distribution of axion decay constants. It would be desirable to compute the complex structure dependence explicitly for specific Calabi–Yau fourfold examples. While this is expected to be possible for Calabi–Yau fourfolds with few complex structure moduli, it will be challenging to perform such computations for fourfolds with GUT singularities. For such situations a local approach would be desirable at first.

An estimation or computation of additional corrections, such as α' -corrections, to the axion decay constants would be also desirable. It should be stressed that our main focus in this work was on the purely complex structure dependent part of this coupling. It might be possible that additional corrections depending on the Kähler moduli are also important, since one can only work at moderately large volume to not additionally suppress the value of the axion decay constants.

The main claim of this paper is that F-theory provides the opportunity to access possibly super-Planckian axion decay constants in a controlled way including g_s -dependent instanton corrections. We argued that large contributions to the axion decay constants arise when the axions are localized in strong coupling regions on the base space B_3 . Strong coupling effects are necessary to realize GUTs in F-theory, but could also occur near a hidden seven-brane. To establish an invariant notion of strong coupling it was also crucial to fix the symmetries of \mathbb{T}_c^{2N} that we argued to be linked to the overall $SI(2, \mathbb{Z})$ symmetry of the F-theory setup. The latter symmetry can be used to locally transform strong into weak string coupling. If the axions are localized near complicated seven-brane configurations, however, this symmetry might only permute which brane-regions contribute dominantly to f^2 . Furthermore, a frame for the symmetries of \mathbb{T}_c^{2N} is chosen once moduli stabilization is implemented. In particular, expanding the non-perturbative superpotential and keeping only the leading terms in the Kähler structure moduli T_α and the G^a will break the symmetries.

Clearly, it remains to be an important open task to find concrete examples with large axion decay constants f_{ab}^2 and a sufficiently flat scalar potential. One challenging part in this endeavour will be to identify models with scalar potentials arising from instanton effects that sufficiently suppress higher harmonics for the inflations c^a . The success will depend on the flux-lattice Γ appearing in (42) and the vacuum expectation values for the stabilized moduli including $\text{Im } G^a$.

Let us end with a final comment concerning reheating in these setups. At the end of natural inflation the axions oscillate around their minimum and decay into various coupled modes [6,35,38]. In particular, a coupling $m_a c^a \text{Tr}(F \wedge F)$ can yield decays into the GUT or a hidden sector gauge fields. In our setting m_a is a seven-brane flux. An appropriate choice of m_a therefore allows to control the size of the decays in this channel, similar to the discussion of [31]. It would be interesting to study reheating in this F-theory setting in detail.

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