Analysis on Conservation of Energy in Microwave Power Synthesizer

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Abstract

Instead of using the traditional method of scattering matrix analysis, this paper uses a method similar to optical waveguide propagation. The basic theory of electromagnetic waves is used to analyze the amplitude and power of microwaves. Proved Conservation of energy is satisfied in microwave power synthesizer. Meanwhile, explained the different electromagnetic wave propagation phenomenon between divider and synthesizer.

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1. Introduction

In recent years, with the rapid development of microwave communication, radar application system and electronic countermeasure technology, there is an increasing demand for the output power and bandwidth of the microwave transmitting system are becoming higher and higher. Therefore, figuring out how to achieve the maximum power output in the broadband is always one of the hot research issues in microwave and millimeter wave communication and electronic warfare system.

At present, the power amplifier used in engineering is divided into two types: one is the electric vacuum device, such as a traveling wave tube power amplifier. The other one is Solid-state devices. The solid-state device is small in size, good in stability, low in power supply voltage, and can be mass produced at low cost. However, due to the limitations of voltage breakdown and heat dissipation of semiconductor materials, the output power of the corresponding solid-state device is far less than that of the vacuum electronic device.

So, it is widely used that multiple solid-state devices combined to obtain higher output power. Therefore, the synthesis power technology is a key to microwave millimeter wave communication system.

The core components of power synthesis are power synthesizer (divider). It is used to be analyzed by scattering theory. Which makes the physical concept is not very clear, especially when it comes on the amplitude variation, electromagnetic wave superposition and energy distribution. This paper analyzes the mode of the electromagnetic wave, with the basic electromagnetic wave transmission theory, to get the energy before and after the power synthesizer (divider), verified whether the conservation of energy is satisfied in power synthesizer (divider).

2. Y-type power synthesizer (divider)

To make the problem much easy and clear, take the Y-type microstrip line power synthesizer to illustrate the problem, as shown in Figure 1. In the microstrip line circuit, the transmission microwave can be treated as TEM mode.

3. Zero angle power synthesizer

To obtain a concise physical concept, we found a simple model which assuming the synthesizer’s angle 0 is 0 and created a coordinate system like Fig. 2. The junction point of the three channels as the origin of the coordinate and X axis perpendicular to the paper(Y-Z plane).
3.1. As a power divider

When the device is used as a divider, see Fig.3. Assuming the section area of the three ports are $S_1, S_2, S_3$, and

$$S_1 = 2S_2 = 2S_3$$  \hspace{1cm} (1)

When the microwave from port 1 transmitted to $z=0$. According to the boundary condition\(^4\),

$$E_1 = E_2 = E_3$$ \hspace{1cm} (2)

Where $E_1, E_2, E_3$ represent the electric field intensity of channel 1, channel 2 and channel 3 respectively. At section $z=0$, the power densities are\(^4\)

$$W_1 = W_2 = W_3 = \frac{1}{2} \varepsilon |E_1|^2$$ \hspace{1cm} (3)

And the powers in channel 1, channel 2 and channel 3 are

$$P_1 = |\overline{W}_1 \times \overline{S}_1| = \frac{1}{2} \varepsilon |\overline{E}_1|^2 S_1$$ \hspace{1cm} (4)

$$P_2 = |\overline{W}_2 \times \overline{S}_2| = \frac{1}{2} \varepsilon |\overline{E}_2|^2 S_2 = \frac{1}{4} \varepsilon |\overline{E}_1|^2 S_1$$ \hspace{1cm} (5)

$$P_3 = |\overline{W}_3 \times \overline{S}_3| = \frac{1}{2} \varepsilon |\overline{E}_3|^2 S_3 = \frac{1}{4} \varepsilon |\overline{E}_1|^2 S_1$$ \hspace{1cm} (6)

We can find

$$P_1 = P_2 + P_3$$ \hspace{1cm} (7)

Satisfying energy conservation.

3.2. As a power synthesizer

When the device is used as a divider, see Fig.3. At this point, the microwave transmission from port 2 and port 3 to port 1.
Fig. 4  As a power synthesizer

Set the Electric field strength in channel 1, channel 2 and channel 3 are $E_1, E_2, E_3$, and $E_{21}$ means the electric field strength in channel 1 which transmitted from channel 2, $E_{31}$ means the electric field strength in channel 1 which transmitted from channel 3.

According to electric field principle of superposition

$$E_1 = E_{21} + E_{31} \quad (8)$$

Analysis port 2, assuming all the power can transmit from port 2 to port 1, according to energy conservation, satisfying:

$$\frac{1}{2} \varepsilon |E_{21}|^2 S_1 = \frac{1}{2} \varepsilon |E_2|^2 S_2 \quad (9)$$

From equation (1) $S_1 = 2S_2 = 2S_3$, we can find

$$\vec{E}_{21} = \frac{\sqrt{2}}{2} \vec{E}_2 \quad (10)$$

As the same way, we can also get

$$\vec{E}_{31} = \frac{\sqrt{2}}{2} \vec{E}_3 \quad (11)$$

According to electric field principle of superposition, if $E_{21}$ and $E_{31}$ have the same frequency, same phase, and same vibration direction, it will satisfy

$$E_1 = E_{21} + E_{31} \quad (12)$$

Assuming $E_3 = E_2$, then calculate the powers at port 1, port 2 and port 3:

$$P_1 = |\vec{W}_1 \times \vec{S}_1| = \frac{1}{2} \varepsilon (E_{21} + E_{31})^2 S_1 = \varepsilon E_2^2 S_1 = 2 \varepsilon E_2^2 S_2 \quad (13)$$

$$P_2 = |\vec{W}_2 \times \vec{S}_2| = \frac{1}{2} \varepsilon E_2^2 S_2 \quad (14)$$

$$P_3 = |\vec{W}_3 \times \vec{S}_3| = \frac{1}{2} \varepsilon E_3^2 S_3 = \frac{1}{2} \varepsilon E_2^2 S_2 \quad (15)$$

We can find

$$P_1 = 2(P_2 + P_3) \quad (16)$$

Which violate the principle of conservation of energy. So, where is the problem? In fact, here can not use
superposition in such a simple way. We can learn from the process of light transmission in the optical fiber. According to the coupling theory, there are two modes, include guided wave mode and radiation modes. The above analysis from equation (9) to equation (16), only consider guided wave mode guided wave mode and without the radiation modes. The conclusion \( \vec{E}_{21} = \frac{\sqrt{2}}{2} \vec{E}_2, \vec{E}_{31} = \frac{\sqrt{2}}{2} \vec{E}_3 \) are the maximum value of guided mode. At the same time, if we consider the radiation mode, the electric field should include the whole space, and not only in the microstrip line.

For the simple and straightforward description of the problem, the coupling effect of channel 2 and channel 3 is neglected in the area \( z > 0 \).

Assuming the two microwave in channel 2 and channel 3 respectively transmit to \( z = 0 \), and then began to superposition.

Express the electric field strength in the areas \( z > 0 \) by which in the area \( z < 0 \):

\[
\vec{E}_{2(x,y)} = C_2 \vec{E}_{12(x,y)} + \int_0^\infty C_\sigma \vec{E}_{12\sigma}(x,y) d\sigma \tag{17}
\]

\[
\vec{E}_{3(x,y)} = C_3 \vec{E}_{13(x,y)} + \int_0^\infty C_\sigma \vec{E}_{13\sigma}(x,y) d\sigma \tag{18}
\]

Where \( \vec{E}_{12(x,y)}, \vec{E}_{13(x,y)} \) are guided mode, \( \vec{E}_{12\sigma}(x,y), \vec{E}_{13\sigma}(x,y) \) are radiation modes, Satisfying \( C_2 = C_3 = C_0 \) because of symmetry, put them into equation (17) and (18), then plus the two equations:

\[
\vec{E}_{2(x,y)} + \vec{E}_{3(x,y)} = C_0 \vec{E}_{12(x,y)} + C_0 \vec{E}_{13(x,y)} + \int_0^\infty C_\sigma \vec{E}_{12\sigma}(x,y) d\sigma + \int_0^\infty C_\sigma \vec{E}_{13\sigma}(x,y) d\sigma \tag{19}
\]

Using the orthogonality of the characteristic mode, both sides multiply \( (\vec{E}_{12(x,y)} + \vec{E}_{13(x,y)})^* \), then integral:

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\vec{E}_{2(x,y)} + \vec{E}_{3(x,y)}) (\vec{E}_{12(x,y)} + \vec{E}_{13(x,y)})^* dxdy = C_0 (\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\vec{E}_{12(x,y)} + \vec{E}_{13(x,y)})^2 dxdy)^{1/2} \tag{20}
\]

Get the relative power after synthesis of two channels microwave at the field of \( z < 0 \),

\[
P_{z < 0} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\vec{E}_{2(x,y)} + \vec{E}_{3(x,y)})^2 dxdy = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\vec{E}_{2(x,y)} + \vec{E}_{3(x,y)}) (\vec{E}_{12(x,y)} + \vec{E}_{13(x,y)})^* dxdy}{|C_0|^2} \tag{21}
\]

According to the definition of coupling efficiency:

\[
\eta = \frac{|C_0|^2}{\int_{-\infty}^{\infty} \int_{0}^{\infty} |\vec{E}_{2(x,y)} + \vec{E}_{3(x,y)}|^2 dxdy} \tag{22}
\]

get

\[
|C_0|^2 = \eta \int_{-\infty}^{\infty} \int_{0}^{\infty} |\vec{E}_{2(x,y)} + \vec{E}_{3(x,y)}|^2 dxdy \tag{23}
\]

Put it into equation (21):

\[
P_{z < 0} = \int_{-\infty}^{\infty} \int_{0}^{\infty} (|\vec{E}_{12(x,y)}|^2 + |\vec{E}_{13(x,y)}|^2) dxdy \tag{24}
\]

The sum relative power of port 2 and port 3, which means the total relative power of the field \( z > 0 \), is

\[
P_{z > 0} = \int_{-\infty}^{\infty} \int_{0}^{\infty} (|\vec{E}_{2(x,y)}|^2 + |\vec{E}_{3(x,y)}|^2) dxdy \tag{25}
\]
Compared the two relative power:

$$\frac{P_{z<0}}{P_{z>0}} = \frac{\sqrt{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( |E_{2}(x,y) + E_{3}(x,y)|^2 + |E_{2}(x,y) + E_{3}(x,y)|^2 \right) dx dy}}{\eta \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( |E_{2}(x,y)|^2 + |E_{3}(x,y)|^2 \right) dx dy} \tag{26}$$

From equation (10) and (11), we know the max value of $E_{12}(x,y)$, $E_{12}(x,y)$ are

$$\vec{E}_{12}(x,y)_{\text{max}} = \frac{\sqrt{2}}{2} \vec{E}_{2}(x,y), \quad \vec{E}_{12}(x,y)_{\text{max}} = \frac{\sqrt{2}}{2} \vec{E}_{2}(x,y) \tag{27}$$

At this point the physical meaning is no radiation mode exist, that also means $\eta = 1$

Note, here $\eta = 1$ is only an assumption. Put (27) and $\eta = 1$ into equation (26),

$$\frac{P_{z<0}}{P_{z>0}} = \frac{\left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( |\vec{E}_{2}(x,y) + \vec{E}_{3}(x,y)|^2 + |\vec{E}_{2}(x,y) + \vec{E}_{3}(x,y)|^2 \right) dx dy \right]^2}{\frac{1}{2} \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( |\vec{E}_{2}(x,y)|^2 + |\vec{E}_{3}(x,y)|^2 \right) dx dy \right]^2} \tag{28}$$

The condition to get equality in this equation is $E_{2}(x,y) = \vec{E}_{3}(x,y)$. When $E_{2}(x,y) \neq \vec{E}_{3}(x,y)$, the power after synthesis will be less than before the synthesis, the difference is the synthetic loss. It also explains two-wave coupling will cause the overall dissipation even if no single beam wave dissipation ($\eta = 1$).

If

$$\eta < 1 \tag{29}$$

We can find

$$\vec{E}_{12}(x,y) = \frac{\sqrt{2}}{2} \sqrt{\eta} \vec{E}_{2}(x,y), \quad \vec{E}_{12}(x,y) = \frac{\sqrt{2}}{2} \sqrt{\eta} \vec{E}_{2}(x,y) \tag{30}$$

Put (30) into equation (28), the same result can be gotten.

4. Splitter angle is not zero

If the angle of the splitter is not zero, we can choose a very narrow area beside $z = 0$ as shown in Fig.5 marked as rectangular C. This part can be approximately regarded as its angle can be neglected, so it can be analyzed follow the angle is zero. We can get the similar results in this narrow area as above analysis. When it is far from the $z=0$ point, the situation will changed to much more complex, but the energy will still keep conserved because it will keep transmitting in each single channel.
5. Summary

Taking the Y-type power synthesizer as an example, based on the basic principle of electromagnetic wave propagation, using the theory similar to optical wave propagation in the fiber, analyzed the amplitude and power of dividing and synthesizing the two processes of the microwave power synthesizer. In order to avoid complicated mathematical operations, assuming the angle of the power synthesizer is zero, verified the Conservation of energy is satisfied in microwave power synthesizer. And the results of general situation can be extended from the angle of zero. Explained the phenomenon of electromagnetic wave transmission is not the same in the two processes of dividing and synthesizing.

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