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Stable Habitual Domains: Existence and Implications

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Conditions for the number of elements in habitual domains and for the activation propensity of each element in the habitual domains to become stabilized are described. The formation of stable states implies enduring personality and attitudes and conditioned or programmed behavior. Some important implications for decision analysis, high-stake decision problems, optimality, gaming and conflict resolution, and career management are also discussed.

1. INTRODUCTION

High-stake decision problems involving such things as the introduction of a new product into a competitive market, an important capital investment, corporate long-range strategy, or the resolution of a national or international crisis/conflict do not happen everyday. However, they are quite important and frequently occur in business and social organizations. Solving successfully the high-stake problem is vital for the survival of the organization.

High-stake decision problems are, in general, characterized by a high level of uncertainty, great impact on future welfare, processing unfamiliar information, and rapid cognitive change before reaching the final solution. Usually the set of alternatives is not fixed. It is generated as needed. There are many conflicting criteria/attributes involved and their importance varies with time and situation variables. (See [9, 10] for further details.) Thus the setting of high-stake decision problems seems to be beyond the assumptions of traditional optimization theories (including mathematical programming, optimal control, game theory, and differential games) because optimization theories usually assume that the sets of alternatives and criteria are fixed, the perceived outcomes of decisions are clearly specified, and the preference for possible outcomes is known, simple, and unchanging. This observation implies that if one tries to understand and make good decisions in high-stake decision problems, a broader comprehension of human psychology and behavior than pure mathematical

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description and optimization is necessary. An attempt toward this end can be found in [2, 11] and citations therein. Another important question raised by the above observation is: Are traditional optimization theories applicable to high-stake decision problems? The results of this article can answer, at least partially, the above question affirmatively. Simply put, the results, using the concept of habitual domains [11] (to be briefly discussed at the end of this section), show that although decision elements (alternatives, criteria. perception of decision outcomes, and preference) can vary markedly over time in high-stake decision problems, they eventually become stabilized and, therefore, can be analyzed by optimization theory. Although this stabilizing phenomenon has been observed empirically [2], it requires formal mathematical study and proof.

The objective of this article is to demonstrate the existence of stable habitual domains. This will allow illustration of implications on optimality, high-stake decision problems, gaming and conflict dissolution, and career management. Specifically, Sections 2 and 3 are devoted to deriving two existence conditions for two kinds of stable habitual domains. Section 2 utilizes convergence theory of sequences and Section 3 relies heavily on Grossberg's theory [4]. Section 4 is devoted to a discussion of the implications of the existence theorems. Further topics of research are briefly suggested in Section 5.

As a convenience to the reader we shall briefly sketch the concept of habitual domains in the remaining part of this section. Details of the concepts can be found in [11].

The brain is the center of information processing for each human being. With an enormous number (estimated to be at least 15 billion) of neural cells, the brain processes the information by the sequence of pattern circuits of the *excited* cells. Similar to a computer, but far more sophisticated, the brain encodes, stores, and retrieves ideas, thoughts, and actions. In studying how a particular person (decision maker) makes his choice or decision, it is important to understand and to know what ideas and thoughts have been encoded and stored in his brain (or memory) and what ideas and thoughts can be activated by some particular events, stimuli, or by his own self-suggestion. The concept of habitual domain (HD) is thus introduced [11] as follows:

By the *habitual domain at time t*, denoted by HD_t, we mean the collection of ideas and actions that can *potentially* be activated at time t. The *actually* activated set of ideas and actions is called *actual domain*, denoted by AD_t. Similar to the relation between a sample space and a realized sample value of a random variable, HD_t indicates those that can possibly be activated, while AD_t those that actually are. One can then assign a probability or confidence measure, denoted by P_t, for $\mathrm{AD}_t \subset \mathrm{HD}_t$ to occure.

Note that HD_t and P_t are dynamic over time and complicated, but they may be approximated. When HD_t and P_t reach a stable or steady state, one could expect habitual ways of thinking and behavior. Conditioned or programmed behavior, for instance, see [8], will then occur.

In [11] the existence of stable HD_t based on a set of hypotheses is described. Roughly, as each human being learns, his HD_t grows with time, but at a decreasing rate because the probability for an arrival idea to be new with respect to HD_t becomes smaller as HD_t gets larger. Thus, unless unexpected extraordinary events arrive, HD_t will reach its stable state. If extraordinary events do not arrive very often, habitual ways of thinking and action will prevail most of the time. This observation is the main motivation to use "habitual" as the adjective. As a complement to [2, 11], this paper is devoted to describing rigorously the conditions under which HD_t and P_t can reach their stable states.

2. Stability Theories on the Number of Elements in Habitual Domains

Theoretically our mind is capable of almost unlimited expansion (imagine what can be done with a 15-billion "bit" computer) [8], and with sufficient effort one could learn almost anything new over time. Although the amount of knowledge or ideas that exists in one's mind may increase with time, the rate of increment tends to decrease as time goes by. As mentioned before, this may be due to the fact that the probability of learning new ideas or actions becomes lower as the number of ideas or actions in the HD becomes larger. These observations enable us to show that the number of ideas in one's HD_t converges when suitable conditions are met.

First let us introduce the following notations:

(i) Let a_t be the number of *additional* new ideas or concepts acquired during the period $(t-1, t]$. Note that the time scale can be in seconds, minutes, hours, or days, etc. Assume that $a_t \geqslant 0$, and that once an idea is registered or learned, it will not be erased from memory, regardless of whether it can be retrieved easily or not. This assumption is consistent with the *Law of Mass Action* (for instance, see [8]), which states that once an idea is learned it will be registered, though scattered throughout the brain, and it is almost impossible to erase the idea completely. When a particular event is emphasized, a_t designates the *additional* new ideas or concepts acquired during $(t-1, t]$ concerning that event.

(ii) For convenience denote the sequence of a_t throughout a period of time by $\{a_t\}$. Note that due to the biophysical and environmental condition of the individual, $\{a_t\}$ is not necessarily monotonic. It can go up or down and is subject to certain fluctuation. For instance, people may function better and more effectively in the morning than at night. Consequently, the a_t in the morning will be higher than that at night. Also observe that $\{a_t\}$ may display a pattern of periodicity (day/night, for instance) which is unique for each individual. The periodicity can be a result of biophysical rhythms or rhythms of the environment.

The following can be proved readily by applying the Ratio Test of Power series.

THEOREM 2.1. *Suppose there exists T such that whenever $t > T$, $a_{t+1}/a_t \leqslant r < 1$. Then as $t \to \infty$, $\sum_{t=0}^{\infty} a_t$ converges.*

Remark 2.1. Theorem 2.1 suggests that if the number of ideas or amount of learning in one's HD is nondecreasing and the rate of increase is strictly decreasing with bounded ratio in time $t > T$, then as t gets larger the total number of ideas or amount of knowledge will approach a stable state. Note that $a_{t+1}/a_t \geqslant 0$ because each $a_t \geqslant 0$.

The assumption of monotonicity of a_t in Theorem 2.1 can be relaxed as follows:

THEOREM 2.2. *Assume that* (i) *there exists time index s, periodicity constant $m > 0$, and constants D and M, such that $\sum_{n=0}^{\infty} a_{s+nm} \leqslant D$, where $\{a_{s+nm}\}$ is a subsequence of $\{a_t\}$ with periodicity m, and* (ii) *for any period n, $\sum_{i=1}^{m} a_{s+nm+i}/ma_{s+nm} \leqslant M$. Then $\sum_{t=0}^{\infty} a_t$ converges.*

Proof. By the assumption (ii) we have

$$\sum_{i=1}^{m} a_{s+nm+i} \leqslant Mm\, a_{s+nm} \tag{1}$$

$$\begin{aligned}\sum_{t=0}^{\infty} a_t &= \sum_{t=0}^{s} a_t + \sum_{i=1}^{m} a_{s+i} + \sum_{i=1}^{m} a_{s+m+i} + \cdots + \sum_{i=1}^{m} a_{s+km+i} + \cdots \\ &\leqslant \sum_{t=0}^{s} a_t + Mma_s + Mma_{s+m} + \cdots + Mma_{s+km} + \cdots \\ &\leqslant \sum_{t=0}^{s} a_t + MmD < \infty \qquad \text{as desired.}\end{aligned}$$

(Note that the inequalities come from (1) and assumption (i).)

Note that for HD to converge, Theorem 2.2 does not require a_t to be monotonically decreasing as required in Theorem 2.1. As long as there exists a convergent subsequence, and the sum of a_t within a time period of length m is bounded, then a_t can fluctuate up and down without affecting the convergence of HD_t. Thus the assumptions in Theorem 2.2 are a step closer to reality than those in Theorem 2.1.

Remark 2.2. Theorems 2.1 and 2.2 imply that if the subsequence $\{a_{s+nm}\}$ is monotonically decreasing with a bounded ratio as $n \to \infty$, then $\sum_{t=0}^{\infty} a_t$ converges. This idea is summarized as Corollary 2.1.

COROLLARY 2.1. *Suppose that the assumption* (ii) *of Theorem* 2.2 *holds and that*

$$a_{s+nm}/a_{s+(n-1)m} \leqslant \gamma < 1.$$

Then $\sum_{t=0}^{\infty} a_t$ *converges.*

Remark 2.3. The regular periodicity assumption in Theorem 2.2 can be removed and a more general assumption substituted resulting in the *Imbedding Theory* of Theorem 2.3.

THEOREM 2.3. *Assume that* (i) *there exists a subsequence* $\{a_{s_k} | k = 1, 2, \ldots\} \subset \{a_t\}$, *and constants D and M, such that* $\sum_k a_{s_k} \to D$ *and* (ii) $\sum_{i=1}^{s_{k+1}-s_k} a_{s_{k+i}} \leqslant M a_{s_k}$. *Then* $\sum_{t=0}^{\infty} a_t$ *converges.*

Proof. Note that

$$\begin{aligned}\sum_{t=0}^{\infty} a_t &= \sum_{t=0}^{s_1} a_t + \sum_{i=1}^{s_2-s_1} a_{s_1+i} + \sum_{i=1}^{s_3-s_2} a_{s_2+i} + \cdots \\ &\leqslant \sum_{t=0}^{s_1} a_t + M(a_{s_1} + a_{s_2} + \cdots) \\ &\leqslant \sum_{t=0}^{s_1} a_t + MD < \infty \qquad \text{as desired.}\end{aligned}$$

Observe that Theorem 2.2 assumes a regular periodicity m in $\{a_t\}$, which is replaced in Theorem 2.3 by a more general subsequence $\{a_{s_k}\}$, in which the length of periodicity $s_k - s_{k-1}$ can be varied for different k's. The HD_t still converges as $t \to \infty$ as long as the sum of subsequence a_{s_k} converges and the total rate of increase between $(s_{k-1}, s_k]$ is bounded as required.

Remark 2.4. (i) By allowing $s_k - s_{k-1}$ to vary for different k's, we actually generalize the single periodicity to multiple periodicities. This generalization makes it possible to cover a variety of fluctuations of $\{a_t\}$.

(ii) From Theorems 2.1 and 2.3 we see that if the subsequence $\{a_{s_k}\}$ is strictly decreasing in ratio as $k \to \infty$, then $\sum_{t=0}^{\infty} a_t$ converges. Precisely, we have Corollary 2.2.

COROLLARY 2.2. *Suppose that the assumption* (ii) *in Theorem* 2.3 *holds and* $a_{s_k}/a_{s_{k-1}} \leqslant \gamma < 1$. *Then* $\sum_{t=0}^{\infty} a_t$ *converges.*

3. The Stability of Habitual Domains in Terms of Activation Propensity

In this section the stability of the "strength" of the elements in HD_t to be activated, called *activation propensity*, is studied. Note that the activation propensity, like probability mass, is a measurement for P_t (discussed in Section 1). Define $x_i(t)$, $i \in HD_t$, to be the *activation propensity* of element i at time t. For simplicity let $HD_t = \{1, 2, ..., n\}$ and $x = (x_1, ..., x_n)$. Note that n, the number of elements in HD_t, can be very large. As $x_i(t)$ is a measurement of the force for idea i to be activated, we can assume that $x_i(t) \geqslant 0$. Note that $x_i(t) = 0$ means that idea i cannot be activated at time t. By assigning $x_i(t) = 0$ if necessary, we may assume that HD_t contains all possible ideas that may be acquired now and in the relevant future.

Similar to charge structures of Yu [11], $x_i(t)$ may be a measurement of charge or force for idea i to occupy the "attention" at time t. Note that $x_i(t)/\sum_j x_j(t)$ can be a measurement of the relative strength of idea i to be activated. If all $x_i(t)$ become stable after some time, the relative strength of each i to be activated will also be stable. In this case one may think that idea i can be activated with probability $x_i(t)/\sum_j x_j(t)$. We shall explore under what condition all $x_i(t)$ can become stable. In Section 3.1 we shall first formulate the dynamic change of $x(t)$ in terms of a system of differential equations by utilizing the results discovered in neuropsychology and mathematical psychology (see [3–5]). In Section 3.2 we shall prove a theorem of stability of $x(t)$. Our work has been motivated by Grossberg's works [3, 4].

3.1. *Formulation of the Dynamics of Activation Propensity*

It is known that some kind of "structural" or "chemical" change must take place in the brain when new information is acquired and stored permanently [8]. When external stimuli are presented and attended to or when one is actively engaged in the thinking process, a sequence of circuit patterns of activated neural cells will appear in the brain. This sequence is regarded as the interpreting and thinking process provoked by the stimuli or self-suggested thinking process [8, 11].

This observation and other empirical studies lead to the formation of the hypothesis concerning the neural representation of the elements in the HD, called the *Circuit Pattern Hypothesis*. The hypothesis is summarized as follows (for a detailed discussion, see [3, 8, 11]).

(i) Each element or idea in the HD is represented by a unique circuit pattern of the excited neural cells. The pattern can be imprinted in a number of sections of the brain. When the circuit pattern is activated by an appropriate stimulus, the corresponding idea will emerge.

(ii) The strength or the activation propensity $x_i(t)$ can be enhanced through its repeated activations. The stronger the circuit pattern and the more sections of the brain in which the circuit pattern is imprinted, the easier the corresponding idea or element can be retrieved or activated. Thus, $x_i(t)$ is an increasing function of the strength of the pattern and the number of sections of the brain in which circuit pattern i corresponding to idea i is imprinted.

Since idea i is represented by an activated circuit pattern of neural cells, the factors which influence the activation of the neural cells will also influence the activation propensity $x(t)$. According to the finding of neural psychology (see [3, 8], for instance), there are at least three factors which may influence the neural circuit pattern and $x_i(t)$:

(i) Spontaneous decay: When an idea or concept is not rehearsed, its propensity to activate tends to degenerate with time because (1) the relative strength of the circuit pattern against others may be reduced, and (2) according to the most efficient restructuring of memory (for instance, [8, 11]), the idea, if not activated, may be stored, perhaps gradually, in a relatively remote area and is relatively more difficult to be retrieved or activated.

(ii) Internal excitation and inhibition effects: When a circuit pattern i corresponding to element $i \in \mathrm{HD}_t$ is activated, the circuit pattern can send out both excitation and inhibition signals. The excitation signal activates the circuit pattern i imprinted in other sections of the brain, and at the same time imprints the circuit pattern i in new sections of the brain. While the activated circuit pattern i sends out excitation signal, it also sends out inhibition signal which inhibits or suppresses the excitation of the other circuit patterns.

(iii) External influence: External stimuli or information inputs acting as stimuli to the memory can affect various circuit patterns to be imprinted and stored in the brain. They have an impact on excitation or inhibition of various circuit patterns.

With the above observation and the work of neural cognition (for instance, see [3–5, 8]), we are ready to describe relevant assumptions and construct the dynamic evolution of $x(t)$ as follows. Refer to Fig. 1 for the interaction among ideas i, j, and k.

Assumption 3.1. Given $x(t)$ and sufficiently small time interval Δt, $x(t+\Delta t)-x(t)$ is given by

$$x_i(t+\Delta t)-x_i(t) = -\alpha_i x_i(t)\,\Delta t + (q_i - x_i(t))\,[h_i(x_i(t)) + I_i]\,\Delta t - x_i(t)\left[\sum_{k \neq i} g_k(x_k(t)) + J_i\right]\Delta t \tag{2}$$

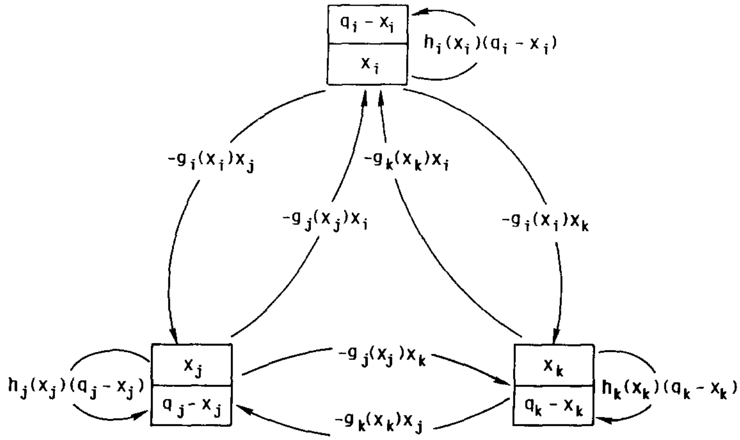

FIGURE 1

where

(i) $\alpha_i > 0$ is the instantaneous decaying factor for $x_i(t)$; thus $-\alpha_i x_i(t)\,\Delta t$ is the reduction of the propensity due to natural spontaneous decay.

(ii) $q_i > 0$ is a maximum potential propensity value for idea i; thus $q_i - x_i(t)$ is a measurement of "under potential," which gives the magnitude to which $x_i(t)$ can further be increased.

(iii) $h_i(x_i(t))$ is the self-excitation function for idea i so that during the time interval $(t, t+\Delta t]$, the propensity of idea i is increased by $(q_i - x_i(t))\,h(x_i(t))\,\Delta t$ due to the impact of $x_i(t)$.

(iv) $I_i \geqslant 0$ is the external rate of excitation by environment on idea i so that the propensity of idea i is increased by $(q_i - x_i(t))\,I_i\Delta t$ during the time interval $(t, t+\Delta t]$.

(v) $g_k(x_k)$ is the inhibition function of idea k so that the propensity reduction of idea i by $x_k(t)$ during $(t, t+\Delta t]$ is given by $g_k(x_k(t))\,x_i(t)\,\Delta t$.

(vi) $J_i \geqslant 0$ is the rate of inhibition of the external environment acting on idea i. Thus idea i's propensity is reduced by $x_i(t)J_i\,\Delta t$ during the time interval $(t, t+\Delta t]$.

Remark 3.1. Assumption 3.1 is an additive model which is similar to well-known on-center off-surround models for cognition patterns. Roughly speaking, when a cognition pattern is activated, it, on one hand, tends to enhance its strength for further activation and, on the other hand, tends to reduce the strength for activation of other cognitive patterns. For a more detailed discussion see [3–5, 8]. Note that the model proposed here is slightly different from that proposed in [3–5] in which it is assumed that the effective rates of excitation signals and inhibition signals are the same (i.e., $h_i(x_i)=g_i(x_i)$, for each $i=1,\dots,n$). We do not make such an assumption here.

Now, by taking

$$\lim_{\Delta t \to 0} \frac{x_i(t+\Delta t)-x_i(t)}{\Delta t},$$

from (2), one obtains

$$\begin{aligned}\dot{x}_i(t) = {} & -\alpha_i x_i(t) + (q_i - x_i(t))\,[h_i(x_i(t)) + I_i] \\ & -x_i(t)\left[\sum_{k \neq i} g_k(x_k(t)) + J_i\right]\end{aligned} \tag{3}$$

which can be written as

$$\begin{aligned}\dot{x}_i(t) &= x_i(t)\left\{-\alpha_i - J_i + g_i(x_i(t)) + \frac{q_i - x_i(t)}{x_i(t)}\,[h_i(x_i(t) + I_i] \right. \\ &\quad \left. - \sum_k g_k(x_k(t))\right\} \\ &= a_i(x(t))\,\{b_i(x_i(t)) - c(x(t))\}\end{aligned} \tag{4}$$

with

$$a_i(x(t)) = x_i(t) \tag{5}$$

$$b_i(x_i(t)) = -\alpha_i - J_i + g_i(x_i(t)) + \frac{q_i - x_i(t)}{x_i(t)}\,[h_i(x_i(t)) + I_i] \tag{6}$$

$$c(x(t)) = \sum_{k=1}^{n} g_k(x_k(t)). \tag{7}$$

Note that $b_i(x_i(t))$ depends on x_i only, while $c(x)$ depends on all x_i, $i=1,\dots,n$. Equation (4) has been extensively studied by Grossberg [4] for its stability.

We summarize the above discussion into:

THEOREM 3.1. *If Assumption* 3.1 *is satisfied, then the time derivative of* $x(t)$ *at time* t *satisfies Eqs.* (4)–(7).

3.2. *Stability of Activation Propensity* $x(t)$

In this subsection, conditions that guarantee that $\lim_{t\to\infty} x(t)$ exists will be derived. Note that when the limit exists, activation propensity $x(t)$ will reach its steady state. Toward this end, we shall first quote a known result of Grossberg [4] concerning the stability of the solution to Eq. (4).

THEOREM 3.2. *With respect to Eq.* (4), $\dot{x} = a_i(x)\,[b_i(x_i) - c(x)]$, *assume that the following conditions hold*:

(I) *Smoothness*:

(a) $a_i(x)$ *is continuous for* $x \geqslant 0$;

(b) $b_i(x_i)$ *is either continuous with piecewise derivatives for* $x_i \geqslant 0$, *or is continuous with piecewise derivatives for* $x_i > 0$ *and* $b_i(0) = \infty$;

(c) $c(x)$ *is continuous with piecewise derivatives for* $x \geqslant 0$.

(II) *Nonnegativity*:

$$a_i(x) > 0 \text{ if } x_i > 0 \quad \text{and} \quad x_j \geqslant 0,\ j \neq i$$
$$a_i(x) = 0 \text{ if } x_i = 0 \quad \text{and} \quad x_j \geqslant 0,\ j \neq i.$$

Moreover, there exists a function $\bar{a}_i(x_i)$ *such that, for sufficiently small* $\lambda > 0$, $\bar{a}_i(x_i) \geqslant a_i(x)$ *if* $x \in [0, \lambda]^n$ *and* $\int_0^\lambda (dw/\bar{a}_i(w)) = \infty$.

(III) *Boundedness*:

$$\limsup_{w \to \infty} b_i(w) < c(0,..., 0, \infty, 0,..., 0)$$

where "∞" *occurs in the* ith *entry*, $i = 1, 2,..., n$.

(IV) *Competition*:

$$\frac{\partial c}{\partial x_k} \geqslant 0, \qquad k = 1, 2,..., n.$$

Then, given any initial point $x(0) \geqslant 0$, *all the limits* $b_i(x_i(\infty)) = c(x(\infty))$ *exist* (*called* "*weak global stability*"). *Furthermore, if all* $b_i(x_i)$ *possess finitely many local maxima within the range of* x_i, *then given any* $x(0) \geqslant 0$, *all limits* $x_i(\infty) = \lim_{t \to \infty} x_i(t)$ *exist*, $i = 1,..., n$. (*This is called* "*strong global stability*.")

THEOREM 3.3. *Assume that* (i) *Assumption* 3.1 *is satisfied so that the evolution of* $x(t)$ *is described by Eqs.* (4)–(7) *and that for each* $i = 1,..., n$ *the inhibition function* $g_i(w)$ *and the self-activation function* $h_i(w)$, $i = 1,..., n$, *are bounded, nondecreasing, and continuous with piecewise derivative for* $w \geqslant 0$ *and* (ii) $g_i(0) = 0$, $h_i(0) = 0$. *Then given any initial point* $x(0) \geqslant 0$, *all the limits* $b_i(x_i(\infty)) = \lim_{t \to \infty} b_i(x_i(t))$ *exist and* $b_i(x_i(\infty)) = c(x(\infty))$ (*i.e., weak global stability is achieved*). *Furthermore, if all* $b_i(x_i)$ *defined in Eq.* (6) *possess finitely many local maxima within the range of* x_i, *then given any* $x(0) \geqslant 0$, *all limits* $x_i(\infty) = \lim_{t \to \infty} x_i(t)$ *exist*, $i = 1,..., n$ (*i.e., strong global stability is achieved*).

Proof. In view of Theorem 3.2, it suffices to verify that conditions (I)–(IV) are satisfied. That the (I) smoothness condition is satisfied is clear from Eqs. (5)–(7) and the assumption that each g_i and h_i is continuous with piecewise derivatives. That the (II) nonnegativity condition is satisfied is also clear by Eq. (5) and by selecting $\bar{a}_i(x_i) = x_i = a_i(x)$. To verify that the (III) boundedness condition is satisfied, note that by Eqs. (6) and (7)

$$\limsup_{w \to \infty} b_i(w) = -\alpha_i - J_i + g_i(\infty) - h_i(\infty) - I_i < c(0,..., 0, \infty, 0,.., 0) = g_i(\infty).$$

Note that since each $g_i(w)$ and $h_i(w)$ is nondecreasing and bounded, $g_i(\infty)$ and $h_i(\infty)$ do exist. The strict inequality holds because $\alpha_i > 0$. Finally, for (IV), by Eq. (7), $\partial c/\partial x_k = dg_k(x_k)/dx_k \geqslant 0$, whenever the derivative exists because each g_k is nondecreasing.

Remark 3.2. In the weak global stability, because $b_i(x_i(\infty)) = c(x(\infty))$, oscillations of $x_i(t)$, $i = 1,..., \infty$, that might occur become arbitrarily slow as $t \to \infty$. To ensure the strong global stability, we need that $b_i(x_i)$ possess finitely many local maxima within the range of x_i. This condition is usually satisfied in most applied problems. When all $g_i(x_i)$ and $h_i(x_i)$ are analytic functions, the condition will be satisfied.

Remark 3.3. The ideas of HD_t discussed in Theorem 3.3 may be regarded as "elementary" ideas (like "elementary event" in sampling space in statistics). An idea can be regarded as a collection of these elementary ideas. The similarity of two ideas may be represented by the intersection of the elementary ideas contained in the two ideas (or common elementary ideas). This definition may be used to study the concept that one idea may activate other ideas or association hypothesis [11]. The problem is very complex and beyond the scope of this paper.

4. Implications

In Sections 2 and 3 we have shown that under suitable conditions the number of elements in HD_t and the activation propensity of each element in HD_t will reach their stable or steady states. Under stable states we can expect habitual ways of thinking, response, and reaction to stimuli and events to occur. Thus, personality, attitude, and conditioned or programmed behavior will be formed for each individual. Such formation has great impact on decision making styles and optimization theories. We can sketch briefly some of the important implications of the existence of stable habitual domains as follows:

(i) *On high-stake decision problems.* Although the four decision elements, alternatives, criteria, perceived outcomes of decisions, and preference, can vary with time, information inputs, and psychological states of the decision maker, they can become stabilized (see [2] for an empirical study) and application of optimization theories to high-stake decision problems can become feasible. Before the stabilization, a formal analysis using optimization theory is not fruitful. During the transition period we might be better off to let our HD_t open and expand, allowing vigilant search for all relevant information on the four decision elements, making sure a "good" alternative is not overlooked.

(ii) *On optimal solutions.* As decision process depends on HD_t so does the resulting optimal solutions. Since HD_t can vary over time (even though it can reach its stable states most of the time), optimal solutions will change with time. This occurs when the set of alternatives, the set of criteria, dimension of alternatives, perception of outcomes, and preference change. This suggests that in dynamic settings "time-optimality" is important and that an alternative being perceived as optimal is valid only over a time horizon. See Yu [9, 10] for further details. Today's optimal solution does not imply that it will be optimal forever. As HD_t changes, it may become an inferior solution. Being aware of this fact we can prevent surprise over other people's (decision makers') "irrational" decisions. After all a decision is rational if it is consistent with the decision maker's HD_t. Everyone's HD_t is unique. What we perceive as irrational may be very rational from other people's point of view (HD_t).

(iii) *On gaming and conflict dissolution.* Each player has a unique HD_t. Understanding our own HD_t and the opponents' HD_t is essential to winning competitive games or resolving conflicts. If we know our own HD_t but do not know the opponents' HD_t, we cannot confidently construct winning strategy. Indeed we could lose the game entirely, like the Pearl Harbor disaster (see [7] for a detailed discussion of the mistake). If we do not know our own and the opponents' HD_t, very likely we would lose most games.

In partial cooperative and partial competitive games, like international trade or companies competing for market share and market volume with the same kinds of products, it might be desirable for the players to settle in peace and ensure that each one is benefited. To maintain some stability the settlement must allow each player to declare a victory, or the terms of agreement must be a time-optimal solution from the point of view of each player's HD_t. Certainly this is not an easy task. Proposing new alternatives, creating new conceptions of the criteria, and suggesting outcomes for the players to change their corresponding HD_t will become vital. Without a new set of compatible HD_t's, agreement can hardly be reached. Certainly, to successfully restructure HD_t we must first be aware of the existing HD_t of each player.

(iv) *On career management.* In a broader sense, each social organization (family, company, school, society, nation, etc.) can be regarded as a living entity and can have a habitual domain, HD_t. The habitual domain can also be stabilized. The persons within the organization performing a variety of functions also have their own HD_t. The match of these habitual domains is important for considering career success and happiness. If the HD's are compatible and if they enhance each other, we may expect a fairly happy individual in the organization; otherwise, conflict and frustration can occur and adjustment must be made in order to keep the individual with the organization.

If we regard human beings as adapting entities to their environments, then choosing an organization for association becomes an optimal match problem between his HD_t and the organization's HD_t. Are there *ideal* organization HD_t's to which an individual can adapt? Can he change the organization HD_t or is it easier to change his? Should he leave an organization for a new one which reveals a more compatible HD_t? These are some important questions that each individual must address for career success and so deserve a careful exploration. Some related literature can be found in Holland [6] and citations therein.

5. Conclusion

We have shown that under suitable conditions the number of elements in HD and the activation propensity of each element can reach its stable states. When the stable states are reached, one can expect habitual ways of thinking process and decision making. The implications are many as discussed in Section 4. We are especially excited about the implications on high-stake decision problems, gaming and conflict dissolution, and career management. All these problems are seldom addressed in optimization theories. The concept of HD_t can play an important role in their analysis. These certainly are important areas for further research and exploration. In [1], Chan has applied the concept to conflict resolution and has suggested some methods to identify HD_t, but more needs to be done. From a theoretical point of view, how to relax the conditions relative to the existence of stable HD is worthy of exploration.

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