# Doppler Color Flow "Proximal Isovelocity Surface Area" Method for Estimating Volume Flow Rate: Effects of Orifice Shape and Machine Factors 

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#### Abstract

Previously described Doppler color flow mapping methods for estimating the severity of valvular regurgitation have focused on the distal jet. In this study, a newer Doppler color flow technique, focusing on the flow proximal to an orifice, was used. This method identifies a proximal isovelocity surface area (PISA) by displaying an aliasing interface. Volume flow rate ( $\mathrm{cm}^{3} / \mathrm{s}$ ) can be calculated as PISA $\left(\mathrm{cm}^{2}\right) \times$ aliasing velocity $(\mathrm{cm} / \mathrm{s})$. For planar circular orifices, a hemi-elliptic model accurately approximated the shape of PISA.

Clinically, however, orifice shapes may be noncircular. In vitro flow experiments ( $n=226$ ) using orifices of various shapes (ellipse, square, triangle, star, rectangle) were performed. Volume flow rate calculated using a hemi-elliptic model for PISA was accurate, with average percent differences from actual flow rate $=$ $+4.3 \%$ for a square, $-\mathbf{4 . 2 \%}$ for a triangle, $\mathbf{- 4 . 7 \%}$ for a star, $-4.5 \%$ for an ellipse and $-2.8 \%$ for a rectangle. However, average percent differences for calculated volume flow rates using a hemispheric model for PISA shape ranged from $\mathbf{- 1 1 . 6 \%}$ (square) to $\mathbf{- 3 4 . 8 \%}$ (rectangle).

In addition, to evaluate whether PISA is influenced by machine


Previous attempts (1-7) to use Doppler color flow mapping to estimate the severity of shunts and valvular regurgitation or stenosis by quantitating the area, width or momentum in the jet distal to an orifice have been subject to various technical limitations. For example, methods for measuring color jet area are known to be influenced by machine factors (5-7). More recently, we and others ( $8-11$ ) have shown that for circular orifices, in vitro Doppler color flow mapping can identify a red-blue aliasing interface, corresponding to a proximal isovelocity surface area (PISA). Volume flow rate $\left(\mathrm{cm}^{3} / \mathrm{s}\right)$ across the orifice can be calculated as PISA $\left(\mathrm{cm}^{2}\right) \times$ isovelocity ( $\mathrm{cm} / \mathrm{s}$ ).

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factors, in vitro studies $(\mathrm{n}=83)$ were performed. For a volume flow rate of 13 liters $/ \mathrm{min}$, the color aliasing radius was not affected by: 1) system gain (radius $=9.9 \mathrm{~mm}$ at $\mathbf{- 2 0} \mathrm{dB}$ versus 10.4 mm at $+20 \mathrm{~dB})$; 2) wall filter ( $\mathbf{1 0 . 8} \mathrm{mm}$ at high versus 11.2 mm at low); 3) frame rate ( 11.2 mm at $6 / \mathrm{s}$ versus 10.6 mm at $22 / \mathrm{s}$ ); 4) transmit power ( $\mathbf{1 0 . 3} \mathbf{~ m m}$ at high versus 10.1 mm at low); and 5) packet size ( 10.6 mm at 4 samples/line versus 10.5 mm at 8 samples/line). The aliasing radius was lower at higher aliasing velocities. However, the calculated volume flow rate was not affected by changes in aliasing velocity.

It is concluded that differences in planar orifice shape do not affect volume flow rate calculated using a hemi-elliptic model. Furthermore, changes in machine factors do not alter volume flow rate calculated using the PISA method. The PISA method may have advantages over previous Doppler color flow methods in calculating volume flow rate and may be useful clinically in estimating valvular regurgitant or shunt volume.
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According to the flow net theory in fluid dynamics, radial streamlines converge as lines of increasing velocity proximal to a narrowed orifice. Figure 1 (left panel) shows the flow net at a narrowed orifice, with streamlines (solid lines) converging at increasing velocity. A series of isopotential lines (dotted lines), nearly equivalent to isovelocity lines, can be drawn tangential to the radial streamlines. Using Doppler color flow mapping, a series of PISAs (S1 to S3) can be identified as red-blue aliasing interfaces corresponding to the isovelocities V1 to V3 (Fig. 1). For planar circular orifices, a hemi-elliptic approximation for the shape of PISA, using color aliasing radii measured from orthogonal views, has been shown by us (11) in an in vitro Doppler color flow system to be accurate for calculating PISA and volume flow rate.

However, in clinical shunts and valvular stenosis and regurgitation, orifice shape may be noncircular (12-14). To evaluate the effect of orifice shape and size, 226 in vitro experiments were performed in constant and pulsatile flow systems using orifices of various shapes and sizes. Furthermore, to evaluate whether the volume flow rate calculated


Figure 1. Principle of the proximal isovelocity surface area (PISA) method. This drawing (left panel) depicts the flow convergence region proximal to a narrowed orifice. Solid lines indicate the streamlines of increasing velocity proximal to the orifice; dotted lines indicate the isopotential (or isovelocity) lines. If PISA can be quantified, volume flow rate ( $\mathrm{cm}^{3} / \mathrm{s}$ ) can be calculated as PISA ( $\mathrm{cm}^{2}$ ) $\times$ isovelocity ( $\mathrm{cm} / \mathrm{s}$ ). Using Doppler color flow mapping, a series of PISA values ( S 1 to S 3 ) can be identified as red-blue aliasing interfaces with central aliasing radii corresponding to the isovelocities (nearly equal to the isopotential lines) V1 to V3 (right panel).

Figure 2. Orthogonal long-axis (left), short-axis (middle) and $90^{\circ}$ short-axis (right) views of flow were recorded proximal to the orifice of the submerged box.
by the PISA method is influenced by alterations in machine factors, 83 in vitro studies were performed in a constant flow system.

## Methods

Flow system. Constant and pulsatile flow were produced using the flow systems. In each flow system, a box with a narrowed planar orifice was used to study the proximal isovelocity surface area (PISA) phenomenon by Doppler color flow mapping. The constant flow system had three tanks. The height of the water column between the middle and lower tank was kept constant, so that constant flow was produced at the orifice of a box submerged in the lower tank. The pulsatile flow system consisted of a motorized piston

pump connected to a tank that contained a submerged box. The constant flow system was used for evaluating the effects of machine factors on PISA. Both constant and pulsatile flow systems were used to evaluate the effects of orifice shape and size.

Doppler recording. Doppler color flow mapping was performed using a Hitachi-Biosound CVC-151 ultrasound machine. It should be noted that the proximal isovelocity surface area (PISA) could not be directly planimetered from long- or short-axis views, which would have resulted in inaccuracies due to the necessity to angle-correct the velocities representing the Doppler color signal along much of the PISA interface. Rather, PISA was calculated from geometric models (a hemi-ellipse or hemisphere) using aliasing radii measured in orthogonal views from pixels along the colorPISA interface that were parallel to the central axis of the interrogating ultrasound beam. Orthogonal long-, short- and $90^{\circ}$ short-axis views of flow proximal to the orifice were recorded using a 5 MHz transducer with the aliasing velocity set at $27 \mathrm{~cm} / \mathrm{s}$.

Orifice shape. Circular orifices of various sizes, ranging from 3 to 16 mm in diameter, were tested. In addition, square, triangular, star, elliptic and rectangular orifice shapes were evaluated.

Machine factor settings. Doppler color flow mapping in the region of the orifice was performed using the following machine factor settings:

1) System gain (dB): $-20,-10,0,+10,+20$
2) Wall filter: low, medium, high
3) Transmit power: low, medium, high
4) Frame rate (frames/s): 7, 11, 15, 22
5) Packet size (pulses/line): $4,6,8$
6) Aliasing velocity ( $\mathrm{cm} / \mathrm{s}$ ): $27,30,40,60$

Measurements. Long-axis aliasing radius (a), short-axis aliasing radius (b) and $90^{\circ}$ short-axis radius (c) were measured from two orthogonal scanning planes (X-Y plane and $\mathrm{Y}-\mathrm{Z}$ plane) (Fig. 2). These radii were in the center ray of the ultrasound beam and were thus not affected by a non-zero Doppler angle. For circular orifices, aliasing radii in the short- and $90^{\circ}$ short-axis views were equal ( $\mathrm{b}=\mathrm{c}$ ).

Proximal isovelocity surface area (PISA) (Fig. 3). This was calculated using hemispheric and hemi-elliptic models to describe the area of the isovelocity surface.

1) Hemispheric model: PISA $=2 \times \pi \times \mathrm{a}^{2}$, where a is the aliasing radius in the long-axis view.
2) Hemi-elliptic model: The PISA value was calculated using three radii ( $\mathrm{a}, \mathrm{b}$ and c ) and integration, where a is the aliasing radius in the long-axis view, b is the aliasing radius in the short-axis view and c is the aliasing radius in the $90^{\circ}$ short-axis view (see Appendix for additional mathematic details).

For circular orifices (where $b=c$ ), this equation is simplified as follows: PISA $=\pi \times b \times\left[\left(b+a^{2} / \sqrt{b^{2}-a^{2}}\right) \times\right.$ $\left.\operatorname{Ln}\left(b+\sqrt{b^{2}-a^{2}} / a\right)\right]$, where $a$ is the aliasing radius in the


Figure 3. Calculation of the proximal isovelocity surface area (PISA) from geometric models. Two models were used for calculating PISA: hemispheric (left) and hemi-elliptic (right). The radii of the geometric shape are denoted by $A$ and $B$; Axes are indicated by $X$, Y and Z .
long-axis view, b is the aliasing radius in the short-axis view and Ln is the natural logarithm.

Volume flow rate. Volume flow rate (FR) at the orifice was calculated using the following equation for the constant flow experiments: $\mathrm{FR}($ liters $/ \mathrm{min})=$ PISA $\left(\mathrm{cm}^{2}\right) \times \mathrm{V}(\mathrm{cm} / \mathrm{s})$ $\times 60 / 1,000\left(\mathrm{~s}-\right.$ liters $/ \mathrm{min}-\mathrm{cm}^{3}$ ), where PISA is the proximal isovelocity surface area, V is the aliasing velocity and $60 / 1,000$ is a correction factor (to convert $\mathrm{cm}^{3} / \mathrm{s}$ to liters/min) (11).

Volume flow rate ( $F R$ ) was calculated using the following equation for the pulsatile flow experiments: $\mathrm{PR}=\mathrm{PISA}_{\text {max }}$ $\times \mathrm{V} \times(\mathrm{TVI} / \mathrm{PFV}) \times \mathrm{HR}$, where $\mathrm{PISA}_{\text {max }}$ is the maximal PISA in one cycle ( $\mathrm{cm}^{2}$ ), V is the aliasing velocity $(\mathrm{cm} / \mathrm{s})$, TVI is the time-velocity integral recorded by continuous wave Doppler ultrasound at the orifice ( cm ), PFV is the peak flow velocity recorded by continuous wave Doppler ultrasound ( $\mathrm{cm} / \mathrm{s}$ ) and HR is the heart (pump) rate (beats $/ \mathrm{min}$ ).

## Results

Effect of orifice shape. Figure 2 displays examples of Doppler color flow mapping of the proximal isovelocity surface area (PISA) for a rectangular orifice. Radius a was measured in the long-axis view, radius $b$ in the short-axis view and radius c in the $90^{\circ}$ short-axis view. Figure 4 shows the relation between actual and calculated flow rates in circular, square and elliptic shapes in the constant and pulsatile flow states. Figure 5 shows the relation in triangular, star and rectangular orifice shapes in the constant and pulsatile flow states. Calculated volume flow rate using the hemi-elliptic model to describe PISA was accurate for all six orifice shapes. However, calculated volume flow rate using the hemispheric model to describe PISA underestimated the actual volume flow rate, especially in the rectangular orifice shapes.

In the pulsatile flow experiments, calculated volume flow rate using the hemi-elliptic model to describe PISA was also accurate for the various orifice shapes. Correlation coefficients for the comparison between actual and calculated flow rates are shown in Table 1.

Actual and percent differences between calculated and actual flow rates are shown in Table 2. Actual difference was determined as calculated flow rate minus actual flow rate.



Figure 4. Relation between actual and calculated flow rates for circular, square and elliptic orifice shape constant flow experiments. Closed circles indicate the calculated flow rate using the hemi-elliptic model; Xs indicate the calculated flow rate using the hemispheric model.

The percent difference was calculated as (actual difference/ actual flow rate) $\times 100 \%$. For the constant flow system, actual difference using the hemi-elliptic model to describe PISA ranged from -0.42 to +0.89 liters $/ \mathrm{min}$. However, the actual differences using the hemispheric model to describe PISA were larger (range -1.17 to -3.05 liters $/ m i n$ ). Percent differences for the constant flow system using the hemielliptic model ranged from $-9.2 \%$ to $+4.3 \%$. However, percent differences using the hemispheric model ranged from $-11.6 \%$ to $-34.8 \%$. The percent differences between calculated and actual flow rates using the hemi-elliptic model were significantly less than using the hemispheric model ( $\mathrm{p}<$ 0.05 ). For pulsatile flow, actual differences using the hemielliptic model ranged from -0.38 to +0.30 liters $/ \mathrm{min}$. Percent differences using the hemi-elliptic model ranged from $-13 \%$ to $+10.1 \%$.

Effect of machine variable settings. The effects of changes in system gain, wall filter, frame rate, transmit power, packet size and color aliasing velocity on the proximal isovelocity surface area (PISA) were calculated. System gain was varied from -20 to 0 to +20 dB (Fig. 6). Aliasing radius (distance from leading edge of orifice to leading edge of red-blue aliasing interface) and calculated volume flow rates remained constant. Figure 7 (left panel) shows the relation of system
gain to aliasing radius and blue color area proximal to PISA. Although the blue color area increased significantly with increasing system gain, aliasing radius was constant. Figure 7 (right panel) shows the relation of wall filter to aliasing radius and the blue color area proximal to PISA. Although the blue color area decreased with an increased wall filter setting, aliasing radius remained constant.

Table 3 shows the effects of frame rate, transmit power and packet size on aliasing radius. Aliasing radius was constant under the four different frame rate conditions, three transmit power conditions and three packet size conditions. For pulsatile flow, at a frame rate of 6 to 7 frames $/ \mathrm{s}$, aliasing radius demonstrated wide fluctuations and underestimated actual flow rate; however, at frame rates of 11,15 and 22 frames/s, flow rate was accurately calculated using an average of three cycles.

Figure 8 shows the effect of changes in color aliasing velocity from 27 to 40 to $60 \mathrm{~cm} / \mathrm{s}$. Aliasing radius varied from 1.06 to 0.84 to 0.68 cm , respectively ( $\mathrm{p}<0.01$ ), but calculated volume flow rates were constant. Figure 9 shows the relation of color aliasing velocity to aliasing radius and calculated volume flow rate. Although aliasing radius decreased with increasing aliasing velocity, calculated volume

TRIANGLE,CONSTANT FLOW


Figure 5. Relation between actual and calculated flow rates for triangular (constant flow), star (constant flow) and rectangular orifice shapes. Data are expressed as in Figure 4 (constant and pulsatile flow).

RECTANGLE,CONSTANT FLOW


RECTANGLE,PULSATILE FLOW

flow rate was constant for the four aliasing velocity conditions.

Therefore, changes in system gain, wall filter, color aliasing velocity, frame rate, transmit power and packet size had no significant effect on volume flow rate calculated using the PISA method.

## Discussion

Previously described (1-7) Doppler color flow mapping methods for estimating volume flow rate or regurgitation or shunt severity (for example, jet area distal to an orifice) have been subject to various technical limitations, such as measurement variability related to variability in machine factors. A new method for quantitation of a proximal isovelocity

Table 1. Correlation Coefficients (r) for the Comparisons Between Actual and Calculated Flow Rate for Pulsatile Flow Across Various Orifice Shapes

|  | Circle | Ellipse | Square | Triangle | Star | Rectangle |
| :--- | ---: | ---: | ---: | :---: | ---: | ---: |
| r Value | 0.99 | 0.96 | 0.98 | 0.97 | 0.96 | 0.99 |
| p Value | $<0.01$ | $<0.01$ | $<0.01$ | $<0.01$ | $<0.01$ | $<0.01$ |

surface area (PISA), defined by a red-blue aliasing interface, can be used to estimate volume flow rate across a narrowed orifice ( $8-10$ ). In vitro constant and pulsatile flow experiments have shown this method to be accurate for calculating volume flow rate for circular orifices. In this study, a total of

Table 2. Actual and Percent Differences Between Calculated Volume Flow Rate (by PISA method) and Actual Volume Flow Rate

| Model | Circle | Ellipse | Square | Triangle | Star | Rectangle |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Actual difference for constant flow (liters/min) |  |  |  |  |  |  |
| Elliptic | -0.26 | -0.42 | +0.89 | -1.08 | -1.09 | -0.08 |
| Hemispheric | -1.17 | -1.59 | -1.82 | -2.81 | -1.94 | -3.05 |
| Percent difference for constant flow |  |  |  |  |  |  |
| Elliptic | -5.5 | -4.5 | +4.3 | -9.2 | -4.7 | -2.8 |
| Hemispheric | -17.5 | -14.5 | -11.6 | -22.9 | -21.5 | -34.8 |
| Actual difference for pulsatile flow (liters/min) |  |  |  |  |  |  |
| $\quad$ Elliptic | -0.26 | -0.06 | -0.38 | +0.30 | +0.30 | +0.20 |
| Percent difference for pusatile flow |  |  |  |  |  |  |
| Elliptic | -1.8 | -0.9 | -13.0 | +5.8 | +8.5 | +10.1 |

Results are shown for both constant and pulsatile flow experiments. Hemielliptic and hemispheric models were used to calculate the proximal isovelocity surface area (PISA). See text for details.

+20 dB
1.04
13.6


## SYSTEM GAIN <br> $\begin{array}{ll}\text { ALIASING RADIUS (CM) }=0.99 \\ \text { FLOW RATE (L/MIN) }= & 12.5\end{array}$

Figure 6. Upper left. Effect of system gain on aliasing radius. Examples of proximal isovelocity surface area (PISA) recorded at
 (right panel). The aliasing radius and calculated flow rate were not significantly different over the range of system gains.

Figure 7. Upper right. Effects of system gain and wall filter on aliasing radius. Left panel, Effect of variations in system gain. The blue color area proximal to the proximal isovelocity surface area
 affected by system gain. Right panel, Effect of wall filter. The blue color area proximal to PISA decreases at the high (compared with
lower) wall filter setting. However, aliasing radius is not affected by wall filter.

Figure 8. Lower left. Effect of color aliasing velocity. Examples of proximal isovelocity surface area recorded at aliasing velocities of 27 (left), 40 (middle) and 60 (right) $\mathrm{cm} / \mathrm{s}$. The aliasing radius varies inversely with the aliasing velocity; however, calculated flow rate

$\begin{array}{ll}\text { ALIASING RADIUS (CM) } & 1.06 \\ \text { FLOW RATE (L/MIN)= } & 14.4\end{array}$

Table 3. Effects of Frame Rate, Transmit Power and Packet Size on Doppler Color Aliasing Radius

|  | Aliasing Radius |
| :--- | :---: |
| Frame rate (frames/s) |  |
| 6 | $10.6 \pm 0.3$ |
| 11 | $10.9 \pm 0.6$ |
| 15 | $11.1 \pm 0.7$ |
| 22 | $10.3 \pm 0.7$ |
| Transmit power (dB) | $10.1 \pm 0.9$ |
| Low | $10.3 \pm 0.7$ |
| Medium | $10.0 \pm 0.7$ |
| High | $10.6 \pm 0.6$ |
| Packet size (pulses/line) | $10.2 \pm 0.3$ |
| 4 | $10.5 \pm 0.2$ |
| 8 |  |

For each machine factor, $p=$ nonsignificant for comparisons among settings.

309 in vitro experiments were performed in constant and pulsatile flow systems to evaluate whether the PISA method is influenced by orifice shape or size or machine factors.

Effect of orifice shape. In our experiments, the accuracy of the proximal isovelocity surface area (PISA) method using a hemi-elliptic model to calculate volume flow rate was not affected by orifice shape. The reason why the PISA method using a hemi-elliptic model is accurate for calculating actual volume flow rate is outlined diagrammatically in Figure 10. According to fluid dynamics theory, three-dimensional flow nets (isopotential lines, nearly equal to isovelocity lines, tangential to the accelerating flow streamlines) are characterized by very smooth curves at orifices of various shapes. The PISA may have an intermediate shape between the shape of the orifice and a circular shape, as shown for the example of a rectangular orifice in Figure 10 (top panel). The actual rectangular orifice and the measured PISA diameters (a and b) are drawn in the Y-Z axis plane in Figure 10 (bottom panel). The actual orifice shape is rectangular, with

Figure 9. Effect of color aliasing velocity on aliasing radius and calculated flow rate. Aliasing radius varied inversely with aliasing velocity (left panel); however, there was no significant difference in calculated flow rate for the four aliasing velocity conditions (right panel).



Figure 10. Schematic drawing of the shape of the proximal isovelocity surface area (PISA) for a rectangular orifice. Upper panel shows the three-dimensional scheme of PISA for the rectangular orifice. Lower panel depicts the shape of the actual orifice and of corresponding PISA values in the X-Y plane. Note that the PISA diameter increases with volume flow rate (FR). Actual orifice shape is rectangular, with a b/a ratio $=5$; however, the PISA shape is an ellipse, with a b/a ratio $=1.6$.
a b/a (long length/short length) ratio $=5$; however, in our experiments, the shape of PISA appeared more rounded, with a $\mathrm{b} / \mathrm{a}$ (long-axis/short-axis) ratio $=1.6$. Because the shape of PISA approximates that of a hemi-ellipse, the volume flow rate can be accurately calculated using a hemi-elliptic model.

In contrast, the PISA method using a hemispheric model underestimated the actual volume flow rate in our experiments, especially for rectangular shapes. For a rectangularshaped orifice, PISA calculated using the hemispheric model and a one-dimensional radius (c in Fig. 10, top panel) underestimates actual flow rate because PISA is actually a "football" shape (b is longer than a or c in Fig. 10, top panel).

However, in the clinical setting, the PISA method using a hemi-elliptic model to calculate PISA is limited by the requirement that orthogonal long-, short- and $90^{\circ}$ short-axis views must be recorded.

Effect of machine variable settings. Calculated volume flow rate using the proximal isovelocity surface area (PISA) method was not affected by changes in system gain, wall filter, frame rate, transmit power, packet size or Doppler color aliasing velocity. This relative insensitivity to variability in machine settings represents a distinct advantage over previously described Doppler color flow methods for estimating volume flow rate or regurgitation severity. For example, in the distal color jet, increasing system gain increases the number of pixels in which apparent velocity exceeds the minimal color-encoded velocity and, therefore, the measured jet area. Furthermore, a higher wall filter setting increases the threshold for the frequency (velocity) display; therefore, the total number of pixels that exceed the minimal color-encoded velocity required for display of the
distal color jet is smaller, resulting in a smaller measured jet area. In contrast, the pixel velocity at the aliasing red-blue interface is determined by the aliasing velocity and volume flow rate, but is not affected by the other machine settings evaluated in this study.

Potential clinical applications of the PISA method. In preliminary studies, we and others have shown that the proximal isovelocity surface area (PISA) approach may be useful in patients with ventricular septal defect (14), moderate and severe mitral regurgitation (15) and mitral stenosis (16). Important considerations clinically include: 1) correction of the PISA calculation for the nonplanar (funnel-shaped) orifice in mitral stenosis; 2) attempts to use lower aliasing velocities (in the range of 10 to 15 frames $/ \mathrm{s}$ to record aliasing radii in cases of smaller volume flow (for example, mild regurgitation) (17); and 3) adaptation of the PISA method for use with a single aliasing radius in the long axis of flow. The Doppler color flow PISA method may provide an important alternative method for estimating regurgitant and shunt volumes, as well as stenotic valve areas.

## Appendix

## Derivation of Equation for Hemi-Elliptic Shape Using Computer BASIC Language

The ellipsoid shape is described by the following equation: $(\mathrm{X} / \mathrm{A})^{2}+$ $(\mathrm{Y} / \mathrm{B})^{2}+(\mathrm{Z} / \mathrm{C})^{2}=1$, where A is the radius in the X axis, B is the radius in the Y axis and C is the radius in the Z axis.

The surface area defined in this equation is calculated using elliptic integrals (18). The final equation for the hemi-ellipse is:
$\mathrm{PISA}=\pi \times \mathrm{A} \times \mathrm{B} \times\left(\frac{\mathrm{C}^{2}}{\mathrm{AB}}+\mathrm{d} \int_{0}^{\phi 0} \sqrt{1-\mathrm{k}^{2} \sin ^{2} \phi} \mathrm{~d} \phi\right.$

$$
\left.+\frac{1-\alpha^{2}}{\alpha} \int_{0}^{\phi_{0}} \frac{\mathrm{~d} \phi}{\sqrt{1-\mathrm{k}^{2} \sin ^{2} \phi}}\right)
$$

where

$$
\mathrm{A}>\mathrm{B}>\mathrm{C}>0, \alpha=\sqrt{1-\frac{\mathrm{C}^{2}}{\mathrm{~A}^{2}}}, \mathrm{k}=\sqrt{\frac{1-\frac{\mathrm{C}^{2}}{\mathrm{~B}^{2}}}{1-\frac{\mathrm{C}^{2}}{\mathrm{~A}^{2}}}}
$$

and

$$
\phi_{0}=\operatorname{Arcsin} \sqrt{1-\frac{\mathrm{C}^{2}}{\mathrm{~A}^{2}}} .
$$

The derivation below in computer BASIC language outlines the program for calculating the surface area of the ellipsoid shape (approximate value). The three radii must be $\mathrm{A}>\mathrm{B}>\mathrm{C} ; \mathrm{F}$ (in line 70) is expressed in radians. SQR is the square root and $\operatorname{ArcSin}(A)$ is the $\sin ^{-1}(\mathrm{~A})$.
10: Input " $\mathrm{A}=$ "'; $\mathrm{A}:$ INPUT " $\mathrm{B}=$ "; $\mathrm{B}:$ INPUT " $\mathrm{C}=$ "; C
20: $P=\left(S Q R\left(A^{2}-C^{2}\right)\right) / A$
30: $\mathrm{Q}=\left(\mathrm{SQR}\left(\mathrm{B}^{2}-\mathrm{C}^{2}\right)\right) / \mathrm{B}$

40: $\mathrm{K}=\mathrm{Q} / \mathrm{P}$
50: $\mathrm{F}=\operatorname{ArcSin}(\mathrm{P})$
60: $\mathrm{G}=2 \times \mathrm{P} \times \mathrm{SQR}\left(1-\mathrm{p}^{2}\right)$
70: $\mathrm{X}=-1 / 4$ * $\mathrm{G}+\mathrm{F} / 2$
80: $\mathrm{Y}=-1 / 16^{*} \mathrm{G} *\left(2 * \mathrm{p}^{2}+3\right)+3 / 8 * \mathrm{~F}$
90: $\mathrm{Z}=-1 / 96^{*} \mathrm{G} *\left(8^{*} \mathrm{p}^{4}+10^{*} \mathrm{p}^{2}+15\right)+5 / 16 * \mathrm{~F}$
100: $\mathrm{S}=\mathrm{F}-1 / 2 * \mathrm{~K}^{2} * \mathrm{X}-1 / 8^{*} \mathrm{~K}^{4} * \mathrm{Y}-3 / 48 * \mathrm{~K}^{6}{ }^{*} \mathrm{Z}$
110: $\mathrm{T}=\mathrm{F}+1 / 2 * \mathrm{~K}^{2} * \mathrm{X}+3 / 8 * \mathrm{~K}^{4} * \mathrm{Y}+15 / 48 * \mathrm{~K}^{6} * \mathrm{Z}$
120: $\mathrm{W}=\pi^{*} \mathrm{~A}^{*} \mathrm{~B}^{*}\left(\mathrm{c}^{2} / \mathrm{A} / \mathrm{B}+\mathrm{P}^{*} \mathrm{~S}+\left(1-\mathrm{p}^{2}\right) / \mathrm{P}^{*} \mathrm{~T}\right)$
130: PRINT "Surface Area="; W

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