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SciVerse ScienceDirect
Procedia - Social and Behavioral Sciences 57 (2012) 231 – 235
Procedia
Social and Behavioral Sciences

International Conference on Asia Pacific Business Innovation and Technology Management

The Relationships between Implied Volatility Indexes and Spot Indexes

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Abstract

This article adopt bivariate GARCH model with TAR to investigate the extent of volatility as well as return transmission between S&P 500 (NASDAQ 100) and VIX (VXN) since their introduction of VIX and VXN. Results show that the performance of VIX index is the best among the four indices during the whole sample period. But the volatility of VIX is also higher than other index. Further, only lagged negative return (change) has a bidirectional casual effect in the low-fear regime for the SP500/VIX series. The results also indicate that VIX index market has a stronger pricing effect on SP500. However, there is no obvious lead-lag relationship between NASDAQ100 and VXN index. Moreover, the return and volatility responses to high-fear and low-fear gauge are asymmetrical.

Key words: VIX; VXN; Bivariate-GARCH model

1. Introduction

As the implied volatility index exhibits the viewpoint on the expected future realized stock index volatility, it is therefore referred to as the investors fear gauge. Besides, when a negative or positive shock to the market, which will induce adjustments in the hedging and trading strategies and consequently launch changes in the prices of put or call option. The implied volatility index then moves in the direction of the market demand of option and underlying asset. Accordingly, the implied volatility index is a superior device to investigate the relationship between market risk and returns.

Although many studies have showed that the relationship between returns and changes in the implied volatility index is strongly negative and asymmetric (Whaley, 2000[1]; Simon, 2003[2]; Skiadopoulos, 2004[3]; Giot, 2005[4]; Bollerslev and Zhou, 2006[5]; Hibbert et al., 2008[6]; Badshah, 2009[7]), most of their analysis are confined to the consideration of one-way direction. However, if implied volatility index represents the market volatility prediction, then changes in the volatility index may be related to future spot returns (Whaley, 2000[1]). Oppositely, spot returns also can determine the

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doi:10.1016/j.sbspro.2012.09.1179

changes in current implied volatility (Hibbert et al., 2008[6]). Nevertheless, the direction of the causality between the return and implied volatility index is important if investors aim to profit from the two markets.

S&P 500 index is a free-float capitalization-weighted index compiled based on the stock prices of 500 large-capitalization common stocks actively traded in the United States. Comparatively, NASDAQ-100 index includes 100 of the largest domestic and international non-financial securities listed on the NASDAQ stock market based on market capitalization. In view of the description, the market characteristics between the two indexes might be different. We therefore further compare the difference of relationship between S&P 500 index versus VIX along with NASDAQ-100 index versus VXN to provide the investors with thorough comprehension in the dynamic variation of the two markets.

On the basis of previous statement, the objective of this paper is to employ a bivariate GARCH model with TAR to examine the extent of volatility as well as return transmission between S&P 500 (NASDAQ 100) and VIX (VXN) since their introduction of VIX and VXN. In this way, we can not only examine the asymmetric and dynamic phenomenon between return and implied volatility indexes, but also investigate the investors' behaviors when confronting different fear regime.

2. Data and Methodology

2.1. Data

We obtain the daily time series price data for the S&P 500 index, NASDAQ-100 index, VIX and VXN from the Yahoo! Finance website. The daily data for S&P 500 stock and VIX covers an eleven-year period, from January 1, 2001 to October 5, 2011, a total of 2,707 trading days; that for the NASDAQ-100 index and VXN is form January 23, 2001 to October 5, 2011, a total of 2,689 trading days. All the analysis is conducted on return data.[1]

2.2. Methodology

As the possibility to allow for the simultaneous analysis of return and volatility transfer in the model, we employ a bivariate GARCH model with VIX Threshold effect to investigate the dynamic relationships between S&P 500 (NASDAQ-100) index and VIX (VXN). In the mean equation, we include positive and negative returns or percentage changes of other markets in addition to threshold effect to examine if transmission effects across markets exist. Additionally, we check the spillover effect of price shocks through lagged squared residuals from other markets in the variance equation. Hence, we construct the conditional mean equation of the bivariate GARCH model with Threshold effect as follows.

$$\begin{aligned} r_{spot,t} &= \mu_{10} + \sum_{i=1}^{m} a_{10,i} r_{spot,t-i} + \sum_{j=1}^{k} \left(b_{10} r_{iv,t-j}^{+} + b_{12} r_{iv,t-j}^{-} \right) \\ &\quad + I_{\{VIX_t > \tau\}} \left(\mu_{11} + \sum_{i=1}^{m} a_{11,i} r_{spot,t-i} + \sum_{j=1}^{k} (b_{11} r_{iv,t-j}^{+} + b_{13} r_{iv,t-j}^{-}) \right) + \varepsilon_{1,t} \end{aligned} \tag{1}$$

$$\begin{aligned} r_{iv,t} &= \mu_{20} + \sum_{i=1}^{m} a_{20,i} r_{iv,t-i} + \sum_{j=1}^{k} (b_{20} r_{spot,t-j}^{+} + b_{22} r_{spot,t-j}^{-}) \\ &\quad + I_{\{VIX_t > \tau\}} \left(\mu_{21} + \sum_{i=1}^{m} a_{21,i} r_{iv,t-i} + \sum_{j=1}^{k} (b_{21} r_{spot,t-i}^{+} + b_{23} r_{spot,t-i}^{-}) \right) + \varepsilon_{2,t} \end{aligned} \tag{2}$$

where $r_{spot,t}$ and $r_{iv,t}$ represent the return of S&P 500 (NASDAQ-100) index and the change in the VIX (VXN) at time t. $r_{spot,t}^{+}$, $r_{spot,t}^{-}$ $r_{iv,t}^{+}$ and $r_{iv,t}^{-}$ denote the positive and negative changes of S&P 500 (NASDAQ-100) returns and VIX (VXN) index at time t, respectively. $I_{\{\}}$ is the indicator function,

[1] Unit root tests (ADF and PP tests) are conducted for the four series. As the results show that both series have unit roots, we use first-order difference (return) to implement our empirical study. To save space, the related results are not shown here. All results are available on request.

VIX_t is a threshold variable and τ is the threshold value. ε_t is a mean-zero innovation with a normal stochastic process and is assumed to be $\varepsilon_t = h_t z_t\,,\, z_t \sim NID(0,1)$.

If the estimation results of b_{10}, b_{12}, b_{20} and b_{22} are significant, we can infer that the changes in return can influence the other. Besides, the significance or insignificance of threshold-related variables could help us judge the participants' behaviors when they run into different fear regime.

In the following, we consider the transmission of price shocks through lagged squared residuals from other markets in the variance equation. The model considers the asymmetric volatility transmission from other markets in the variance equation to investigate if the reaction of volatility to bullish news and bearish news is the same across markets. The specification for the variance equations are as follows.

$$H_{11,t} = \omega_{10} + \sum_{i=1}^{p} \alpha_{10,i}\varepsilon^2_{11,t-i} + \sum_{j=1}^{q} \beta_{10,j} H_{11,t-j} + I_{\{VIX_t > \tau\}}\left(\omega_{11} + \sum_{i=1}^{p} \alpha_{11,i}\varepsilon^2_{11,t-i} + \sum_{j=1}^{q} \beta_{11,j} H_{11,t-j}\right) \tag{3}$$

$$H_{12,t} = \omega_{12} + \sum_{i=1}^{p} \alpha_{12,i}\varepsilon_{11,t-i}\varepsilon_{22,t-1} + \sum_{j=1}^{q} \beta_{12,j} H_{12,t-j} + I_{\{\mathrm{VIX}_t > \tau\}}\left(\omega_{13} + \sum_{i=1}^{p} \alpha_{13,i}\varepsilon_{11,t-i}\varepsilon_{22,t-1} + \sum_{j=1}^{q} \beta_{13,j} H_{12,t-j}\right) \tag{4}$$

$$H_{22,t} = \omega_{20} + \sum_{i=1}^{p} \alpha_{20,i}\varepsilon^2_{22,t-i} + \sum_{j=1}^{q} \beta_{20,j} H_{22,t-j} + I_{\{VIX_t > \tau\}}\left(\omega_{21} + \sum_{i=1}^{p} \alpha_{21,i}\varepsilon^2_{22,t-i} + \sum_{j=1}^{q} \beta_{21,j} H_{22,t-j}\right) \tag{5}$$

where H_t denotes the conditional volatility of S&P 500 (nasdaq-100) returns at time t, ε_t is a mean-zero innovation with a normal stochastic process and is assumed to be $\varepsilon_t = h_t z_t\,,\, z_t \sim NID(0,1)$. In addition, we utilize the maximum likelihood estimation (MLE) to estimate the parameters of the model. The concise log likelihood function is stated as follows.

$$L(\theta) = -\frac{1}{2}\sum_{t=1}^{T}(\ln(|H_t|) + \varepsilon_t' H_t^{-1}\varepsilon_t) \tag{6}$$

where θ is the vector of all parameters; T is the number of observations. [8]

3. Estimation Results

Table 1 presents the estimated results of bivariate GARCH model with threshold effect. For the mean equation of SP500 series, we find that the relationship between SP500 and VIX is negative in the low-fear periods. However, in the high-fear period, the impact of VIX index change on SP500 reverses.

Relatively, for the mean equation of VIX index, the change of VIX series is significantly negatively affected by its own lagged change and positively influenced by the lagged negative return of SP500 index in the low-fear regimes. But in the high-fear period, the impact of SP500 index on the change of VIX index is indifferent from the low-fear period. These results imply that VIX index market has a stronger pricing impact on that of SP500.

For the mean equation of NASDAQ100 series, we find that the return of NASDAQ100 series is significantly negatively affected by its own lagged return and by the lagged positive change of VXN series in the low-fear periods. But in the high-fear period, the impact of VXN index on NASDAQ100 is indifferent from the low-fear period. Oppositely, for the mean equation of VXN index, the influence of VXN lagged change and the return of NASDAQ100 index on the VXN index is insignificant in the low-fear regime.

For the variance equation of SP500 series, the result of α_{10} indicates that the previous unexpected impulse of SP500 to its current volatility is significant in the low-fear regime. In the high-fear period, the impact is even stronger. Besides, though the previous volatility of SP500 has significant impact on its current volatility in the low-fear regime, the effect decreases in the in the high-fear regime.

For the variance equation of VIX series, the result of α_{20} also indicates that the previous unexpected impulse of VIX to its current volatility is significant in the low-fear regime. In the high-fear period, the impact is stronger. In addition, although the previous volatility of VIX has significant impact on its current volatility in the low-fear regime, the effect decreases in the in the high-fear regime. These con-

Table 1 Parameter estimates of bivariate GARCH model with threshold effect for S&P 500/VIX and NASDAQ100/VXN model

Variables	SP500/VIX		NASDAQ100/VXN	
	Coeff.	S.E.	Coeff.	S.E.
Conditional mean equation				
μ_{10}	0.0684***	0.0177	0.0936***	0.0230
μ_{11}	-0.3345***	0.1022	-0.2479*	0.1413
a_{10}	-0.0850***	0.0245	-0.0646***	0.0236
a_{11}	0.0736	0.0588	0.0712	0.0644
b_{10}	-0.0076**	0.0038	-0.0132**	0.0058
b_{11}	0.0411**	0.0171	0.0064	0.0425
b_{12}	-0.0078*	0.0045	-0.0087	0.0068
b_{13}	0.0350*	0.0190	0.0599	0.0456
μ_{20}	-0.1933	0.1191	-0.2166*	0.0884
μ_{21}	1.3731***	0.3112	-0.0956	0.2621
a_{20}	-0.0470**	0.0239	0.0103	0.0243
a_{21}	-0.0857	0.0602	-0.0695	0.0659
b_{20}	-0.0093	0.1569	0.1300	0.0891
b_{21}	-0.3005	0.2605	-0.0706	0.1425
b_{22}	0.3130*	0.1717	0.1586	0.1111
b_{23}	0.0631	0.2186	-0.5079***	0.1755
ω_{10}	0.0112***	0.0016	0.0128***	0.0030
α_{10}	0.0853**	0.0368	0.4133***	0.1333
β_{10}	0.0612***	0.0057	0.0538***	0.0051
Condition variance equation				
ω_{11}	0.0509***	0.0182	0.0390	0.0254
α_{11}	0.9241***	0.0059	0.9373***	0.0058
β_{11}	-0.0482***	0.0135	-0.0873**	0.0361
ω_{12}	-0.0798***	0.0133	-0.0579***	0.0119
α_{12}	-0.2473**	0.1045	-0.3865**	0.1509
β_{12}	0.0546***	0.0055	0.0463***	0.0046
ω_{13}	0.0224	0.0139	0.0150	0.0162
α_{13}	0.9257***	0.0073	0.9395***	0.0062
β_{13}	-0.0309**	0.0156	-0.0566**	0.0283
ω_{20}	0.9788***	0.1628	0.5044***	0.1036
α_{20}	0.9566*	0.5785	0.6190*	0.3555
β_{20}	0.0576***	0.0060	0.0477***	0.0055
ω_{21}	0.0189	0.0146	0.0162	0.0125
α_{21}	0.9146***	0.0092	0.9326***	0.0082
β_{21}	-0.0405*	0.0223	-0.0553**	0.0228
Diagnostics on standard residuals				
INDEX	SP500	VIX	NASDAQ100	VXN
Q(1)	0.419	0.0192	0.04663	0.236
$Q^2(1)$	9.770***	0.211	8.084***	1.934
LogL	-11070.1363		-11627.8372	

Note:

1.* indicates significance at the 10% level; ** indicates significance at the 5% level; and *** indicates significance at the 1% level.

2. Q(1) represents Ljung-Box's Q statistics of the standard residuals.

3. $Q^2(1)$ is Ljung-Box's Q statistics of the squared standard residuals.

ditions are the same as those in the SP500 market. Similarly, we can reach the same result through the outcome of NASDA100 and VXN variance equation.

4. Conclusion

This article adopt bivariate GARCH model with TAR to investigate the extent of volatility between S&P 500 (NASDAQ 100) and VIX (VXN) since their introduction. Results show that the performance of VIX index is the best among the four indices. But the volatility of VIX is also higher than other index. Further, for the mean equation of SP500/VIX series, we find that only lagged negative return (change) has a bidirectional casual effect in the low-fear regime. However, for the mean equation of NASDAQ100 series, only the return of NASDAQ100 series is significantly negatively affected by its own lagged return and by the lagged positive change of VXN series in the low-fear periods. For the mean equation of VXN index, only lagged negative return of NASDAQ100 index has significantly negatively impact in the high-fear regime. There is no obvious lead-lag relationship between NASDAQ100 and VXN index.

As to the variance equation, the significant result of coefficients in the high-fear and low-fear regimes along with feedback effect may demonstrate that the bivariate GARCH adopted in this paper is appropriate.

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