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Introductory Remarks to a Lecture of Erich H. Rothe*

BY

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Erich Rothe's distinguished career began in Berlin where he studied under Erhard Schmidt. After a position of Privatdozent in Breslau, he came to this country in 1937 and to the University of Michigan in 1943. Erich Rothe has been working mainly on functional analysis and topology in function spaces. By saying functional analysis we must understand both linear and nonlinear functional analysis, in particular, linear and nonlinear partial or integral equations, boundary value problems, and calculus of variations: calculus of variations as a theory of maxima and minima of generally nonlinear functionals.

Erich Rothe has investigated deeply the concept of gradient mapping of a functional in a Banach space, or, in a different terminology, Fréchet differentials. The result obtained by Erich Rothe in 1937 has been widely applied: A functional in a Hilbert space is weakly continuous if and only if its Fréchet differential is completely continuous. The condition is still sufficient in a Banach space. The weak continuity of the functional is understood here as the continuity in the bounded weak topology.

This theorem is important in the calculus of variations. Erich Rothe has proved that, under conditions, certain functions are continuous in the bounded weak topology, and therefore, have maximum and minimum in any given compact set. Erich Rothe has used the same theorem, or variants of it, to prove the existence of solutions of large classes of functional equations, in particular, the nonlinear equations of the Hammerstein type.

Erich Rothe has given a satisfactory theory of critical points and critical levels for functions in a Hilbert space, making use of the concepts of type number and homology group for mappings in Hilbert spaces. The concept of mapping degree in Banach spaces, as introduced by Leray and Schauder, allow them to define the index of a singular point, and Erich Rothe has proved, in this general situation, that the expected numerical relations hold which express the index of a singular points as the algebraic sum of the type numbers

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of various dimensions, and the mapping degree as the sum of the indices of the singular points. The theory of critical points and critical levels for functions in Hilbert spaces can thus be based only on topological considerations.

In 1955 Erich Rothe found ways to define the Leray-Schauder mapping degree directly in the given Banach space, without making recourse to the usual process of limit from finitely dimensional Euclidean spaces. He then proved the homotopy properties of the mapping degree, for instance, the product theorem.

Existence theorems for generalized solutions of partial differential equations, or for generalized solutions of problems of the calculus of variations, follow from the use of the methods of functional analysis. For instance, in the calculus of variations, the spaces of Calkin and Morrey, also called Sobolev spaces in a more general setting, are weakly compact, as was proved by Morrey by direct analysis. Erich Rothe has proved that they are weakly compact simply because they are uniformly convex by a theorem of Clarkson, therefore reflexive by a theorem of Kakutani, and therefore weakly compact by a theorem of Alaoglu.