A contribution of the density-of-states term to the paraconductivity in a $d$-wave superconductor

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Abstract

A theoretical study is carried out to calculate the density-of-states (DOS) contribution to the paraconductivity (PC) for a $d$-wave superconductor in the context of our previous $d$-wave PC approach. Resonant scattering by nonmagnetic impurities is considered under the unitary limit condition in a clean layered superconductor. In calculating the DOS term, we also include consequences of short-wavelength fluctuations. It is found that a negative contribution from the DOS term compensates the whole positive Maki-Thompson terms, leaving a small negative contribution depending on the scattering rate. Therefore, the $d$-wave PC in the absence of impurities is characterized only as the Aslamazov-Larkin contribution, in terms of which experiments in a cuprate superconductor with the optimal doping could be explained.

Keywords: Paraconductivity; $d$-wave pairing; density-of-states term; short-wavelength fluctuations; cuprate superconductors

1. Introduction

Theoretical expressions for the fluctuation conductivity above $T_c$ (the paraconductivity, PC) are composed of three different contributions: the Aslamazov-Larkin (AL) term, Maki-Thompson (MT) terms and density-of-states (DOS) term. In the PC analysis with the conventional $s$-wave pairing theories, these overall contributions to PC must be considered to better describe experimental results for high-$T_c$ materials [1]. A theoretical study of PC in the case of a superconductor with the $d$-wave pairing mechanism has been carried out [2] to give the following findings in the MT process: i) an additional contribution of the regular-MT form appears instead of the anomalous MT term; ii) the resultant contributions from the whole regular-MT terms become positive in contrast to the negative one in the $s$-wave case. Given the evidence of these important modifications by the $d$-wave effects, it is necessary to investigate how the DOS term plays a role in a $d$-wave PC regime for the sake of completeness.

In this study we present calculations on the DOS term in the scheme of our previous $d$-wave PC theory [2]. We also take account of short-wavelength fluctuations (SWF) in a total-energy cutoff (EC) approach [3,4] since PC properties are observable at high temperatures for cuprate superconductors to which the $d$-wave pairing mechanism may be applied. From the present calculations, we can obtain an explicit expression for the sum of the AL, MT and DOS terms.A point

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of interest in the $d$-wave PC characters is that a negative DOS term removes the MT term completely, and provides only a small contribution to PC. Numerical results for overall contributions to PC and comparison to recent experiments are presented.

2. The Aslamazov-Larkin and Maki-Thompson terms

To begin with calculations on the DOS term, we present summaries of our previous study [2] which are necessary for the present work. The system under consideration is a clean layered superconductor with nonmagnetic localized impurities where electrons in a two dimensional (2D) squared lattice (a lattice parameter $a$) are described in terms of the $t$-$J$ model [5,6] and are weakly coupled between the layers (an interlayer distance $s$). The Green’s function of a normal-state quasiparticle is written as

$$ G(\mathbf{k}, i\omega_n) = [i\omega_n + i\mathbf{f} \text{sgn}(\omega_n) - \varepsilon_{\mathbf{k}}]^{-1}, \quad \varepsilon_{\mathbf{k}} = -t \left( \cos k_x a + \cos k_y a \right) - \varepsilon_F + J_z \cos k_z s $$

(1)

where $\mathbf{k} = (k_x, k_y, k_z)$ is the wave vector, $t$ is the nearest-neighbor transfer integral, $\varepsilon_F$ is the Fermi energy, $J_z$ is the interlayer hopping energy, and $\omega_n$ is the Matsubara frequency of Fermions. The scattering rate $\Gamma$ associated with the resonant scattering by localized impurities is given in the unitary limit by $\Gamma = n_i/(\pi N_0)$ with the normal electron density of states $N_0$ (per a single spin) at $\varepsilon_F$ and the impurity density $n_i$.

The fluctuation propagator $L_d(\mathbf{q}, i\omega_n)$ with $\omega_n \to 0$ in the $d$-wave pairing regime is given as

$$ L_d^{-1}(\mathbf{q}, 0) = \tilde{N}_0 \left\{ \varepsilon_d + \frac{-1}{(4\pi k_BT)^2} \psi^{(2)} \left( \frac{1}{2} + \rho \right) \left[ \left( \frac{\alpha_B}{\pi} \right)^2 + 4J_z^2 \sin^2 \left( \frac{2\varepsilon_d}{\pi} \right) \right] \right\}, $$

(2)

where $\mathbf{q} = (\tilde{q}, q_3)$ is the wave vector of fluctuating Cooper pairs with the 2D interlayer component $\tilde{q}$, $\psi^{(2)}(z)$ is the tetra-gamma function, and we have neglected the Boson frequency $\nu_m$ since such dependence is less important in the calculations on the DOS term. Moreover, we define $\tilde{N}_0 = N_0/(1 + n_i)$ and $\rho = \Gamma/(2\pi k_BT)$. Due to the pair-breaking effects of nonmagnetic impurities, the reduced temperature $\varepsilon = \ln(T/T_c) \approx (T - T_c)/T_c$ is renormalized as

$$ \varepsilon_d = \ln \left( \frac{\Gamma}{\nu_m} \right) + \psi \left( \frac{1}{2} + \rho \right) - \psi \left( \frac{1}{2} \right), $$

(3)

where $\psi(z)$ is the digamma function and $T_{co}$ is a $T_c$ value in the absence of impurities (Note that we have neglected the dilution effect of the resonant scattering on $T_c$). The $T_c$ equation is given by $\varepsilon_d = 0$ [6].

The AL term in the $d$-wave pairing scheme is expressed as

$$ \sigma_d^{AL}(\varepsilon_d, \alpha_g) = e^2 \left[ \psi^{(1)} \left( \frac{1}{2} + \rho \right) \right]^2 \left( \frac{\varepsilon_d}{\pi} + r_a^2 \varepsilon_d \right)^{-1/2} - 2a_{\tilde{q}} \left( \frac{\varepsilon_d + 2\alpha_g}{\alpha_{\tilde{q}}^2} \right), \quad r_a^2 = \frac{-\frac{1}{4(4\pi k_BT)^2}}{\psi^{(2)} \left( \frac{1}{2} + \rho \right)} $$

(4)

with the tri-gamma function $\psi^{(1)}(z)$. In this equation, the SWF effects in the EC approach [3,4] are taken into consideration, where the range of the $\tilde{q}$-integral is restricted to the condition $\left| \tilde{N}_d L_d(\mathbf{q}, 0) \right|^{-1} \leq a_{\tilde{q}}^2$ with the cutoff parameter $\alpha_g$. In Eq. (4), $r_a$ is the renormalized Lawrence-Doniach anisotropic parameter given by $r_a = 2\xi_0^2/s$ with $\xi_0^2$ being the out-of-plane coherence length amplitude at $T=0$.

The contribution from the MT process (shown in Fig. 1(a)) in an $s$-wave superconductor is classified into the anomalous and regular terms. The regular MT term has been derived, in the case of the $d$-wave PC with EC, to be

$$ \sigma_d^{MT(\text{reg})}(\varepsilon_d, \alpha_g) = -\frac{e^2}{4m \hbar s} \left[ \psi^{(3)} \left( \frac{1}{2} + \rho \right) \right] \cdot 2 \ln \left[ \frac{2a_{\tilde{q}}}{\varepsilon_d^{1/2} + (\varepsilon_d + \alpha_g)^{1/2}} \right], $$

(5)

with the penta-gamma function $\psi^{(3)}(z)$. The above formula gives a negative contribution and recovers the corresponding equation for the $s$-wave case in the absence of impurities ($\rho=0$). On the contrary, the anomalous MT term is absent in the $d$-wave case since vertex corrections of the impurity scattering provide no diffusion pole due to the $d$-wave pairing symmetry [5]. Interestingly, we have pointed out that an additional positive term $\sigma_d^{MT(\text{add})}$ of the type of the regular MT term, which amounts to having the same $\varepsilon$ dependence as in Eq. (5), could appear instead of the anomalous MT term, that is $\sigma_d^{MT(\text{add})} = (-4/3)\sigma_d^{MT(\text{reg})}$.

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Fig.1. Diagrams contributing to the PC in the Maki processes. (a): MT diagrams and (b)–(e): DOS diagrams. The solid lines, wavy lines and black half-circles denote $G(\mathbf{k}, i\omega_n)$, $L_d(\mathbf{q}, 0)$ and $\Lambda_d$, respectively, while the broken lines with crosses indicate renormalization effects by the impurity scattering.
Therefore, all the contributions obtained from the MT diagram take the form

\[ \sigma^{\text{MT(all)}}_{\text{d}}(\epsilon_d, \alpha_d) = \frac{e^2}{3 \pi n s} \left[ \frac{\psi^{(3)}(\frac{1}{2} p)}{\psi^{(2)}(\frac{1}{2} p)} \right] \cdot 2 \ln \frac{2a_n}{\epsilon_{d_{1/2}}^2 + (\epsilon_d + r_d^2)^{1/2}}, \]  

(6)

giving a positive contribution.

3. The density-of-states term

Feynman diagrams for the electromagnetic response function \(Q_d\) in the Maki processes are displayed in Fig. 1, in which the diagram (a) contributes the MT terms while others (b)-(c) provide the DOS term [7]. The equation for \(Q_d^{\text{DOS(b)}}\) corresponding to the diagram (b) is expressed as

\[ Q_d^{\text{DOS(b)}}(\omega_n) = -2(k_B T)^2 \sum_{\omega_n} \sum_k [j_k(k)]^2 \cdot \hat{\theta}_k L_d(q, 0) \hat{\theta}_q - k \times G_z(k, i \omega_n) G(q - k, -i \omega_n) \Lambda_d^2, \]

(7)

where \(\omega_n = \omega_0 + \omega_0, j_k(k) = -e h^{-1}(\partial \epsilon_k / \partial k_x)\) is the bare (external) current vertex, \(\hat{\theta}_k = \cos k_x a - \cos k_y a\) denotes the angular dependent part of the fluctuation propagator, \(\Lambda_d\) is the impurity vertex correction simply given by \((1 + n_i)^{-1}\). In the evaluation of Eq. (7), we neglect the \(q\)-dependence of the Green’s function since the small \(q\) term is important in the fluctuations. Replacing the summation \(\sum_k \rightarrow N_q \int d\theta / (2\pi)\), the angular average \((j_k(k))^2 \hat{\theta}_k \hat{\theta}_q \approx \frac{1}{2} (\epsilon a / h)^2\) could be obtained [8]. On the summation over \(\omega_n\), we define the function

\[ f^{(b)-(1)}(\omega_0) = \frac{(k_B T)^2}{N_q} \int d\theta G^2(\epsilon, i \omega_n) G(\epsilon, -i \omega_n) G(\epsilon, i \omega_n). \]

(8)

for the evaluation of which the range of the \(\omega_n\)-summation must be divided into three domains: (1) \([-\omega_0 < \omega_n < 0]\), (2) \([\omega_n > 0]\) and (3) \([\omega_n < -\omega_0]\). In the domain (1), we obtain an analytical expression for Eq. (8) as shown in column (b)-(1) of Table 1, together with formulae in other diagrams as well as other domains. In Table 1, we have defined

\[ D_{\Omega_0} = \psi \left( \frac{1}{2} + \rho + \Omega_0 \right), \quad D_{\Omega_0/2} = \psi \left( \frac{1}{2} + \rho + \frac{\Omega_0}{2} \right), \quad D_0 = \psi \left( \frac{3}{2} + \rho \right), \quad T_{\Omega_0} = \psi (\frac{1}{2} + \rho + \Omega_0), \quad T_0 = \psi (\frac{1}{2} + \rho), \]

(9)

where \(\Omega_0 = \omega_0 / (2\pi k_B T)\). For comparison, formulae for the diagram (a) (MT terms) are also shown in Table 1.

The dc conductivity is derived as the coefficient of the first order of the external frequency \(\omega_0\), so that the formulae listed in Table 1 must be expanded as series of \(\omega_0\) under the clean-limit condition. Furthermore, the \(q\)-summation of the fluctuation propagator is calculated, in the EC approach, as

\[ \sum_{\text{q}} L_d(q, 0) = \frac{(\pi k_B T)^2}{N_q} \cdot \frac{(a_n / 2)^2}{N} \cdot 2 \ln \frac{2a_n}{\epsilon_{d_{1/2}}^2 + (\epsilon_d + r_d^2)^{1/2}}. \]

(10)

With the above-mentioned results, we obtain the DOS term contributed from the diagrams (b)-(c) as

\[ \sigma^{\text{DOS}}_{\text{d}}(\epsilon_d, \alpha_d) = \frac{e^2}{3 \pi n s} \left[ \frac{\psi^{(3)}(\frac{1}{2} p)}{\psi^{(2)}(\frac{1}{2} p)} \right] \cdot 2 \ln \frac{2a_n}{\epsilon_{d_{1/2}}^2 + (\epsilon_d + r_d^2)^{1/2}}. \]

(11)

Finally, overall contributions to PC from the AL, MT and DOS (AMD) terms are expressed as

\[ \sigma^{\text{AMD}}_{\text{d}}(\epsilon_d, \alpha_d) = \frac{e^2}{16 n s} \left[ \frac{\psi^{(4)}(\frac{1}{2} p)}{\psi^{(3)}(\frac{1}{2} p)} \right] \cdot \left[ (\epsilon_d^2 + r_d^2)^{1/2} \right] \cdot 2 \ln \frac{2a_n}{\epsilon_{d_{1/2}}^2 + (\epsilon_d + r_d^2)^{1/2}}. \]

(12)

**Table 1. Formulae of \(F^{(i)}(\omega_0)\) with \(i=b-e\) in the domains (1)-(3).**

<table>
<thead>
<tr>
<th>Diagram</th>
<th>Domain (1)</th>
<th>Domain (2)</th>
<th>Domain (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(N_0 k_B T ) (a_0^2 ) (D_{\Omega_0} - D_0) (D_{\Omega_0} + D_0 - 2D_{\Omega_0/2})</td>
<td>(N_0 k_B T ) (a_0^2 ) (D_{\Omega_0} + D_0 - 2D_{\Omega_0/2})</td>
<td>The same as in (a)-(2)</td>
</tr>
<tr>
<td>(b)</td>
<td>(N_0 k_B T ) (a_0^2 ) (D_{\Omega_0} - D_0) (D_{\Omega_0/2} - D_0)</td>
<td>(N_0 k_B T ) (a_0^2 ) (D_{\Omega_0/2} - D_0)</td>
<td>(N_0 k_B T ) (a_0^2 ) (D_{\Omega_0} - D_0)</td>
</tr>
<tr>
<td>(c)</td>
<td>The same as in (b)-(1)</td>
<td>The same as in (b)-(3)</td>
<td>The same as in (b)-(2)</td>
</tr>
<tr>
<td>(d)</td>
<td>(N_0 k_B T ) (a_0^2 ) (D_{\Omega_0} - D_0)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(e)</td>
<td>The same as in (d)-(1)</td>
<td>0</td>
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term provides significant contributions to the PC properties even for a small calculations to the DOS term, showing that the DOS term completely cancels out the positive whole MT term reported indicated in the caption, which are roughly available to cuprate systems [9]. It can be seen from Fig. 2(a) that the NT t
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re
the corrective term could give a significant contribution to the PC characteristics in the SWF regime. Finally, the AL term provides a large contribution to the PC in a Bi
2
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2
Ca
Cu
2
O
8
+ optimals and theory (Eq. (12) with
\right) = \alpha
\text{g} = 100 K, \gamma = 1 or 10. The lowest line is nothing but the s-wave AL term. (b) Main panel: Comparison between experiment (Bi2212-optimal) and theory (Eq. (12) with \gamma = 0) in \ln(\sigma'/\sigma_0) vs. ln \tau plots, where \sigma_0 is the conductivity at T=280 K and parameter values are
\tau_c = 93.2 K, \gamma = -0.1, \alpha = 0.2 and \alpha_E = 0.33. Inset: Resistivity data \rho(T) with a linear normal-state resistivity \rho_0(T) = AT + B (with constants A and B).

4. Discussion

The final expression for the overall contributions to PC, Eq. (12), indicates that a negative contribution from the DOS term completely removes the whole positive MT terms which we have found in the previous study. Consequently, the PC in a d-wave superconductor is given by the sum of the AL term and a negative term (MT+DOS terms: hereafter referred to as “the NT term”) proportional to the scattering rate \tau. Under the unitary limit condition, \pi N_0 \tau^\alpha in Eq. (12) is equal to the impurity density \n, and therefore the contribution from the NT term may be small. Interestingly, the PC of a d-wave superconductor is given by that of an s-wave case in the absence of impurities. Examples of numerical results for bi-logarithmic plots of Eq. (12) are displayed in Fig. 2(a), where calculations are carried out with parameter values indicated in the caption, which are roughly available to cuprate systems [9]. It can be seen from Fig. 2(a) that the NT term provides significant contributions to the PC properties even for a small \n, the effect of which is more pronounced in the larger value of \alpha, in which case the SWF effect is less important. On the basis of the present results, we analyze our recent experimental data on the PC in a Bi
2
Sr
2
Ca
Cu
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O
8
+ (Bi2212) single crystal with the optimal doping [10]. Fig. 2(b) shows results for a theoretical fit of experiments with Eq. (12) using similar procedures as in previous reports [1]. Good agreement between theory and experiment is attained in terms of Eq. (12) with \gamma = 0, which amounts to the s-wave AL term.

5. Conclusion

In the scheme of our previous study of d-wave pairing effects on AL and MT terms, we have extended the calculations to the DOS term, showing that the DOS term completely cancels out the whole positive MT term reported previously, giving a negative corrective term proportional to the impurity density. Therefore, it is anticipated that only the AL term provides a large contribution to the d-wave PC. Nevertheless, we have shown that, from numerical results, the corrective term could give a significant contribution to the PC characteristics in the SWF regime. Finally, experimental data in a Bi2212 crystal with the optimal doping are shown to be describable by overall contributions of AL, MT and DOS terms in the absence of impurities.

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References