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Bi-large neutrino mixing from bilinear R-parity violation with non-universality

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Abstract

We investigate how the bi-large mixing required by the recent neutrino data can be accommodated in the supersymmetric standard model allowing bilinear R-parity violation and non-universal soft terms. In this scheme, the tree-level contribution and the so-called Grossman–Haber one-loop diagrams are two major sources of the neutrino mass matrix. The relative size of these two contributions falls into the right range to generate the atmospheric and solar neutrino mass hierarchy. On the other hand, the bi-large mixing is typically obtained by a mild tuning of input parameters to arrange a partial cancellation among various contributions.

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Recently, impressive progress has been made in atmospheric and solar neutrino experiments [1,2]. They provided us convincing evidences for three active neutrino oscillations requiring two large and one small mixing angles [3]. The resulting neutrino mixing matrix [4] takes the form;

$$U \approx \begin{pmatrix} c_{12} & s_{12} & \theta_{13} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (1)$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$, and $s_{13} \approx \theta_{13} \lesssim 0.2$. Here we put $\theta_{23} = \pi/4$ for the nearly maximal atmospheric neutrino mixing angle. The solar neutrino

mixing angle θ_{12} takes the value $\tan \theta_{12} \approx 0.65$ for the so-called LMA solution which is strongly favored by the recent SNO data [2]. The mass-squared differences explaining the atmospheric and solar neutrino data are $\Delta m_{\text{atm}}^2 \approx 2.5 \times 10^{-3} \text{ eV}^2$ and $\Delta m_{\text{sol}}^2 \approx 5 \times 10^{-5} \text{ eV}^2$, respectively. Even though less favored, the so-called LOW solution with $\tan \theta_{12} \approx 0.77$ and $\Delta m_{\text{sol}}^2 \sim 10^{-7} \text{ eV}^2$ is still viable.

One of attractive ways to generate non-zero neutrino masses and mixing is to use R-parity and lepton-number violation allowed in the supersymmetric standard model [5]. The purpose of this Letter is to address the question whether the desired bi-large mixing of three active neutrinos can arise naturally from the bilinear R-parity violation. The most attractive feature of R-parity violation as the source of neutrino mass

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matrix would be that the idea can be tested in the future colliders by measuring the lifetime and the branching ratios of the lightest supersymmetric particle which decay through R-parity violation [6–8]. The scheme under consideration also predicts similar collider signatures as studied before.

The superpotential of the supersymmetric standard model may contain the following bilinear terms;

$$W = \epsilon_i \mu L_i H_2, \quad (2)$$

generalizing the usual μ -term, $\mu H_1 H_2$. Then, there are also six soft supersymmetry breaking terms in the scalar potential;

$$V_{\text{soft}} = \epsilon_i \mu B_i L_i H_2 + m_{L_i H_1}^2 L_i H_1^\dagger + \text{h.c.}, \quad (3)$$

where we used the same notations for the superfields and their scalar components. Let us note that B_i in the first term is dimension-one and the corresponding term for the Higgs bilinear is $\mu B H_1 H_2$.

If the universal boundary condition is imposed on the soft-terms, the differences between the soft-terms of the Higgs boson H_1 and slepton L_i such as

$$\Delta B_i \equiv B - B_i \quad \text{and} \quad \Delta m_i^2 \equiv m_{H_1}^2 - m_{L_i}^2,$$

vanish at the mediation scale of supersymmetry breaking and their non-zero values are generated at the weak scale through renormalization group evolution (RGE), while $m_{L_i H_1}^2$ remain vanishing. In this case, there are only three free parameters ϵ_i which makes the model very economic. However, this model cannot accommodate the bi-large mixing consistently with small U_{e3} . It is easy to understand it qualitatively as one can expect that the three parameters ϵ_i control all the mixing angles. A small θ_{13} and a large θ_{23} requires $\epsilon_1 \ll \epsilon_2 \approx \epsilon_3$ leading to $\theta_{12} \approx \theta_{13}$ [8,9]. Thus, in order to accommodate the bi-large neutrino mixing, one has to go beyond this minimal scheme. One way is to allow trilinear couplings while keeping the universality. In this case, the five couplings related to the third generation fermions may play a major role to generate the desired neutrino mass matrix [8,9]. Another way is to allow non-universal soft-terms [10–13]. Introduction of general flavor-mixing soft-masses is, of course, tightly constrained by the flavor changing neutral current processes, such as $\mu \rightarrow e\gamma$ or $\tau \rightarrow \mu\gamma$ [14]. However, the non-universality in the flavor-diagonal soft-parameters is not severely constrained. Generi-

cally, one could expect $\Delta m_i^2/m_{H_1}^2$ and $\Delta B_i/B$ to be of order one. One can also have $m_{L_i H_1}^2 \sim \epsilon_i m_{H_1}^2$.

In this Letter, we investigate how the desired neutrino mass and mixing pattern can arise under such a generic non-universality condition. We will see that the right values of the mixing angles and the mass hierarchy can be obtained in reasonable ranges of parameter space without severe fine-tuning. In the below, we will first quantify all the tree-level and one-loop contributions to the neutrino mass matrix and identify the dominant contributions. Obtaining a rather simple form of the leading neutrino mass matrix, we will make qualitative discussions to understand how the desired masses and mixing arise. This will be completed by presenting our numerical analysis.

Let us start our main discussion by describing the structure of neutrino mass matrix coming from R-parity violation. Adopting the notations of Ref. [9], the most general one-loop renormalized neutrino mass matrix can be written as

$$M_{ij}^\nu = -\frac{M_Z^2}{F_N} \xi_i \xi_j c_\beta^2 - \frac{M_Z^2}{F_N} (\xi_i \delta_j + \delta_i \xi_j) c_\beta + \Pi_{ij}^\nu, \quad (4)$$

where $F_N \equiv M_1 M_2 / M_{\tilde{\gamma}} + M_Z^2 c_{2\beta} / \mu$ with $M_{\tilde{\gamma}} \equiv c_W^2 M_1 + s_W^2 M_2$. Here, the first term is the neutrino mass matrix arising at tree-level, the second terms containing δ_i come from the one-loop correction to the neutrino–neutralino mixing masses projected on to the neutrino direction, and the last term Π_{ij}^ν is the one-loop correction to the ν_i – ν_j Majorana mass matrix. The non-zero values of $\xi_i \equiv \langle L_i^0 \rangle / \langle H_1^0 \rangle - \epsilon_i$ arise due to non-universal soft terms in the slepton–Higgs sector as follows;

$$\xi_i = \epsilon_i \frac{\Delta m_i^2 + \Delta B_i \mu t_\beta}{m_{\tilde{\nu}_i}^2} - \frac{m_{L_i H_1}^2}{m_{\tilde{\nu}_i}^2}, \quad (5)$$

where the sneutrino mass-squared is $m_{\tilde{\nu}_i}^2 = m_{L_i}^2 + M_Z^2 c_{2\beta} / 2$. As is well known, the tree-level mass matrix makes massive only one neutrino in the direction of $\vec{\xi}$, which is typically the heaviest one, ν_3 . In fact, the quantity ξ_i controls the neutrino–neutralino mixing and thus could be probed by lepton flavor violating decays of the lightest neutralino in the future colliders [6–8]. Here, let us introduce another quantity,

$$\eta_i \equiv \frac{\langle L_i^0 \rangle}{\langle H_1^0 \rangle} - \epsilon_i \frac{B_i}{B} = \xi_i + \epsilon_i \frac{\Delta B_i}{B} \quad (6)$$

which governs the mixing between the sleptons and Higgs bosons. As we will see, the flavor structure of the neutrino mass matrix depends on these two R-parity violating parameters, ξ_i and η_i , as well as non-universal slepton masses.

A simplification of the full neutrino mass matrix comes from the observation that *the second term on the right-hand side of Eq. (4) can be ignored in our case [15].* This can be seen immediately by going to the basis where the tree-level mass matrix is diagonalized by the eigenvector $\hat{\xi}$ and any two orthogonal unit vectors. In this basis, one finds that the second mass matrix has vanishing components in the 1–2 plane orthogonal to $\hat{\xi}$. Thus, leaving the heaviest ν_3 untouched, approximate see-saw diagonalization can be applied to get the contribution to the 1–2 plane of the order of $M_Z \delta^2$. This is like a two-loop contribution much smaller than the (non-vanishing) 1–2 components of the last term Π^ν . Thus, there is no need to compute the second mass term in most cases even though we included it in our analysis.

The main contribution to the last term Π^ν of Eq. (4) comes from the one-loop diagrams exchanging sneutrinos/Higgs bosons and gauginos [15,16] in the case of generic non-universality under consideration. Here we present the explicit formula of this one-loop mass matrix which is calculated by the use of approximate see-saw rotation [9];

$$\begin{aligned} \Pi_{ij}^\nu = & -\frac{g^2}{32\pi^2} \sum_a (t_W N_{1a} - N_{2a})^2 m_{\tilde{\chi}_a^0} \\ & \times \left(\sum_\phi \frac{1}{2} \theta_{i\phi} \theta_{j\phi} B_0(m_{\tilde{\chi}_a^0}^2, m_\phi^2) \right. \\ & \quad \left. + \frac{Z_{ij}}{m_{\tilde{\nu}_i}^2 - m_{\tilde{\nu}_j}^2} \right. \\ & \quad \left. \times [B_0(m_{\tilde{\chi}_a^0}^2, m_{\tilde{\nu}_i}^2) - B_0(m_{\tilde{\chi}_a^0}^2, m_{\tilde{\nu}_j}^2)] \right), \end{aligned} \tag{7}$$

where N_{ab} is the 4×4 neutralino diagonalization matrix, $\tilde{\chi}^0$ denotes the neutralino mass eigenstates, ϕ represents the neutral Higgs bosons ($\phi = h, H$ and A), and the loop-function B_0 is given by $B_0(x, y) = -\frac{x}{x-y} \ln \frac{x}{y} - \ln \frac{x}{Q^2} + 1$ with the renormalization scale Q . The effect of the bilinear R-parity violat-

ing terms are encoded in the coefficients $\theta_{i\phi}$ and Z_{ij} which are given by

$$\begin{aligned} \theta_{ih} &= +\xi_i s_\alpha + \eta_i s_\beta m_A^2 \frac{m_{\tilde{\nu}_i}^2 c_{\alpha-\beta} - M_Z^2 c_{2\beta} c_{\alpha+\beta}}{(m_{\tilde{\nu}_i}^2 - m_h^2)(m_{\tilde{\nu}_i}^2 - m_H^2)}, \\ \theta_{iH} &= -\xi_i c_\alpha + \eta_i s_\beta m_A^2 \frac{m_{\tilde{\nu}_i}^2 s_{\alpha-\beta} - M_Z^2 c_{2\beta} s_{\alpha+\beta}}{(m_{\tilde{\nu}_i}^2 - m_h^2)(m_{\tilde{\nu}_i}^2 - m_H^2)}, \\ \theta_{iA} &= -i\xi_i s_\beta + i\eta_i s_\beta \frac{m_A^2}{m_A^2 - m_{\tilde{\nu}_i}^2}, \\ Z_{ij} &= \eta_i \eta_j m_A^4 M_Z^2 c_\beta^2 s_\beta^4 \left[\frac{m_{\tilde{\nu}_i}^2}{F_S^i} + \frac{m_{\tilde{\nu}_j}^2}{F_S^j} \right], \end{aligned} \tag{8}$$

where η_i is defined in Eq. (6), α is the usual diagonalization angle of two CP even Higgs bosons, and $F_S^i \equiv (m_{\tilde{\nu}_i}^2 - m_A^2)(m_{\tilde{\nu}_i}^2 - m_h^2)(m_{\tilde{\nu}_i}^2 - m_H^2)$. Recall that the angle α is defined by $c_{2\alpha} = c_{2\beta}(m_A^2 - M_Z^2)/(m_h^2 - m_H^2)$ and $s_{2\alpha} = s_{2\beta}(m_A^2 + M_Z^2)/(m_h^2 - m_H^2)$.

A few remarks are in order: (i) The coefficients $\theta_{i\phi}$ are the linear combinations of θ_{ij}^S 's defined in Eq. (9) of Ref. [8]. They are related by the Higgs mass diagonalization. In Eq. (8), the quantity ξ_i appears to include the effect of neutrino–neutralino mixing by ϵ_i . This ξ_i dependence can be easily understood if one goes to the basis where ϵ_i vanishes [9]. (ii) The same diagrams have been considered in Ref. [15] using the mass-insertion method which must yield the equivalent results to ours. These diagrams involve two mass-insertions which can be seen here as products of two induced R-parity odd $\nu - \phi - \chi^0$ vertices, $\theta_{i\phi} \theta_{j\phi}$, and as individual sneutrino vertices, Z_{ij} , which is R-parity even. (iii) Among various contributions in $\theta_{i\phi} \theta_{j\phi}$, the term proportional to $\xi_i \xi_j$ can be absorbed into the tree-level mass term giving a negligible effect. The term proportional to $\xi_i \eta_j$ is suppressed due to the similar reason discussed before, but cannot be neglected completely. (iv) The term Z_{ii} is nothing but the contribution due to the sneutrino–anti-sneutrino mass splitting induced by R-parity violation, a la Grossman–Haber [16], and Z_{ij} with $i \neq j$ comes from the effective sneutrino mixing vertices, $\nu_i - \tilde{\nu}_j^* - \chi^0$. (v) The terms with Z_{ij} are proportional to $M_Z^2 c_\beta^2 / m_{\tilde{\nu}_i}^2$, and thus give smaller contributions than the terms with $\eta_i \eta_j$ from $\theta_{i\phi} \theta_{j\phi}$ in a reasonable range of parameters. However, they can give a sizable effect in general.

Now, let us consider the other one-loop contributions and show that (7) dominates over them in the case of the general non-universality. Among various contributions, we take the well-known diagram with squark–quark exchange to be compared with (7). Considering the trilinear couplings induced from bottom quark Yukawa couplings h_b such as $\lambda'_{i33} = \epsilon_i h_b$, one has

$$\tilde{\Pi}_{ij}^v \approx \frac{3}{8\pi^2} \frac{h_b^2 m_b^2 \mu t_\beta}{m_b^2} \epsilon_i \epsilon_j. \quad (9)$$

Taking the ratio of the above two contributions, one typically gets (9)/(7) $\approx 5 \times 10^{-6} t_\beta^3 (\epsilon/\eta)^2$ with $m_{\tilde{\chi}^0} = 100$ GeV, $\mu = m_{\tilde{b}} = 250$ GeV. Therefore, (9) can be neglected as far as $\tan\beta$ is not too large and $\epsilon_i \sim \eta_i$. In the similar way, one can find that the other diagrams are also sub-leading to (7). In Ref. [12], a slight deviation of non-universality has been assumed to yield $\epsilon/\eta \sim 10^3$ and thus (9) was considered as the main one-loop correction. In fact, this is a typical situation in the case of universality. The importance of the contribution (7) in the case of large deviation from universality has been notified in Ref. [15] and its impact on viable neutrino mass matrices has been considered in Refs. [11,13].

From the previous discussions, we can write down the leading contributions to the full mass matrix (4) as follows:

$$M_{ij}^v \approx -\frac{M_Z^2}{F_N} \xi_i \xi_j c_\beta^2 - \frac{g^2}{32\pi^2} \sum_a m_{\tilde{\chi}_a^0} \eta_i \eta_j f_{ij}^a, \quad (10)$$

where f_{ij}^a derivable from Eqs. (7) and (8) is the function of the masses of neutralinos, sneutrinos and Higgs bosons and its flavor dependence comes from the non-universal slepton masses.

We are ready to discuss how the desirable neutrino masses and mixing can be realized by the bilinear R-parity violation with generic non-universal soft masses. For this, we will take the following representative set of R-parity conserving parameters;

$$\begin{aligned} t_\beta &= 5, & m_A &= 300, \\ \mu &= -250, & M_2 &= 2M_1 = 200, \end{aligned} \quad (11)$$

throughout this Letter. This choice gives the light and heavy neutral Higgs boson masses, $m_h = 84$ GeV and $m_H = 302$ GeV, respectively. Other choices will not change the main features of our results. Concerning

the R-parity violating parameters, we allow the general flavor dependence for the supersymmetric ϵ_i and soft B_i parameters. To make our discussion simpler, we will take $m_{L_i H_1}^2 = 0$ in this Letter. This would be a plausible choice for the minimal lepton flavor violation as it may arise due to some mechanism of generating the μ and $\epsilon_i \mu$ terms. Note that our choice of the soft parameters are made at the electroweak scale. Since we introduce the non-universality, there is no need to connect them to the ultraviolet values through the RGE.

Now, let us start with the simplest case: (A) the “minimal” deviation from the universality, that is, sleptons have a universal soft-mass: $m_{H_1}^2 \neq m_{L_1}^2 = m_{L_2}^2 = m_{L_3}^2$. This was the scheme employed in the analysis of Refs. [11,15]. In this case, the lepton flavor dependence in f_{ij} disappears and thus the neutrino mass matrix (10) takes the following simple form:

$$M_{ij}^v \approx m_x \hat{x}_i \hat{x}_j + m_y \hat{y}_i \hat{y}_j \quad (12)$$

where $m_x = |\xi|^2 M_Z^2 c_\beta^2 / F_N$, $m_y \sim |\eta|^2 m_{\tilde{\chi}^0} g^2 / 64\pi^2$. Here, \hat{x} and \hat{y} are nothing but the unit vectors in the direction of $\vec{\xi}$ and $\vec{\eta}$, respectively. As analyzed in Ref. [17], the mass matrix (12) has two non-vanishing eigenvalues, $m_3 \approx m_x$ and $m_2 \approx m_y s_\varphi^2$, whose eigenvectors are in the directions of \hat{x} and $\hat{x} \times (\hat{x} \times \hat{y})$, respectively. Here the angle φ is defined by $c_\varphi = \hat{x} \cdot \hat{y}$. From these, one finds that the desired neutrino mixing matrix (1) is obtained for

$$\begin{aligned} \hat{x} &\simeq \left(\theta_{13}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right), \quad \text{and} \\ \hat{y} &\propto (s_{12}, \sqrt{2}(1+k)c_{12}, \sqrt{2}kc_{12}), \end{aligned} \quad (13)$$

with an arbitrary number k . The ratio of two mass eigenvalues is given by

$$\frac{m_2}{m_3} \sim \frac{g^2}{32\pi^2} \frac{m_{\tilde{\chi}^0} F_N}{M_Z^2} t_\beta^2 \frac{|\eta|^2}{|\xi|^2} s_\varphi^2. \quad (14)$$

Note that one can easily obtain its right value to accommodate the atmospheric and solar neutrino (LMA) mass scales; namely, $m_2/m_3 \approx \sqrt{\Delta m_{\text{sol}}^2 / \Delta m_{\text{atm}}^2} \sim 0.16$ putting $m_{\tilde{\chi}^0} = F_N = 200$ GeV, $t_\beta = 5$, $|\eta|/|\xi| = 1$ and $s_\varphi^2 = 1$. Furthermore, the relation (13) can also be arranged by an appropriate choice of two independent set of parameters ξ_i and η_j . In the similar way, the LOW solution can also be easily accommodated.

However, it remains to be seen how such an arrangement for ξ_i and η_i can be made in terms of the input parameters, ϵ_i , $\Delta B_i/B$ and $\Delta m_i^2/m_{H_1}^2$. In order to answer this question, let us choose the following set of values;

$$\xi_i = (0.1, 1, 1), \quad \eta_i \propto (\sqrt{2}t_3, 1, -1)$$

with $t_3 = 0.65$

which give rise to the desired bi-large mixing of the atmospheric and solar neutrino oscillations. Note that the above choice corresponds to $c_\varphi = 0$. The normalization of η will be chosen to reproduce a right value of $\Delta m_{\text{sol}}^2/\Delta m_{\text{atm}}^2 \sim 2 \times 10^{-2}$. Since we will calculate the ratios of neutrino mass eigenvalues and mixing angles, we put, e.g., $\xi_2 = \xi_3 = 1$. In order to obtain the mass scale of $m_3 = 0.05$ eV, one can take an overall rescaling of R-parity violating variables, ξ , η and ϵ , by factor of 5×10^{-6} . We now give three examples realizing the above choice of ξ_i and η_i as follows.

(A1) $\Delta m_i^2/m_{H_1}^2 = 0.7$: This corresponds to the neutrino mass, $m_{\tilde{\nu}_i} = 67$ GeV, and gives the neutrino mass matrix,

$$M_{ij}^{\nu} = -2.12\xi_i\xi_j + 0.18\eta_i\eta_j.$$

Therefore, the choice of $\eta_i = (1.1, 1.2, -1.2)$ leads to the desired results as follows;

$$\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2} = 0.03, \quad U_{e3} = 0.08,$$

$$\sin^2 2\theta_{\text{atm}} = 0.99, \quad \sin^2 2\theta_{\text{sol}} = 0.82. \quad (15)$$

Our choice of $\xi_i = (0.1, 1, 1)$ and the above η_i is realized by the following input parameters; $\epsilon_i = (4.5, 1.1, -9.5)$ and $\Delta B_i/B = (0.22, 0.18, 0.23)$.

(A2) $\Delta m_i^2/m_{H_1}^2 = -1$: It gives rise to $m_{\tilde{\nu}_i} = 228$ GeV and

$$M_{ij}^{\nu} = -2.12\xi_i\xi_j + 0.089\eta_i\eta_j.$$

Taking $\eta_i = (1.4, 1.5, -1.5)$, we find

$$\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2} = 0.018, \quad U_{e3} = 0.07,$$

$$\sin^2 2\theta_{\text{atm}} = 0.99, \quad \sin^2 2\theta_{\text{sol}} = 0.83. \quad (16)$$

The corresponding input parameters are $\epsilon_i = (-4.2, -3.4, 5.9)$ and $\Delta B_i/B = (-0.31, -0.15, -0.42)$.

(A3) $\Delta m_i^2/m_{H_1}^2 = 0.1$: It leads to $m_{\tilde{\nu}_i} = 146$ GeV and

$$M_{ij}^{\nu} = -2.12\xi_i\xi_j - 0.0022\eta_i\eta_j.$$

With the choice of $\eta_i = (9.2, 10, -10)$, we get

$$\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2} = 0.02, \quad U_{e3} = 0.02,$$

$$\sin^2 2\theta_{\text{atm}} = 0.98, \quad \sin^2 2\theta_{\text{sol}} = 0.84, \quad (17)$$

and the input parameters; $\epsilon_i = (283, 287, -334)$ and $\Delta B_i/B = (0.032, 0.031, 0.033)$.

For the cases (A1) and (A2), our general parameter scan showed that the realistic neutrino masses and mixing can be obtained within the range of input parameters: $1 \lesssim |\epsilon_i| \lesssim 10$ and $0.1 \lesssim |\Delta B_i/B| \lesssim 1$ leading to $|\xi|, |\eta| \sim 1$. From the above samples, one can see that there need certain arrangements in the flavor structure of the input parameters realizing the required mixing angles. This would be the case in many class of models. In our case, the smallness of $|\xi_1|$ is arranged not by the smallness of $|\epsilon_1|$ but by a partial cancellation between two terms: $\Delta m_1^2 \approx -\Delta B_1 \mu t_\beta$ leading to $\Delta B_1/B \approx 0.22$ and -0.31 for (A1) and (A2), respectively. This pattern arises also in more general cases as we will see shortly. Since $|\epsilon_1|$ is not necessarily smaller than $|\epsilon_{2,3}|$, it is favored to have $\Delta B_1/B \sim \Delta B_{2,3}/B$. Thus, a vanishingly small $|U_{e3}|$ cannot be naturally realized in our scheme. In the case (A3), the universality is maintained to a certain degree, as we can see, this requires $|\epsilon_i| \gg |\eta_i|, |\xi_i|$ and a strong correlation for the fine-tuned values of $|\Delta B_i/B| \ll 1$. In fact, this is a characteristic property of the universality case where the small deviation of $\Delta m_i^2/m_{H_1}^2$ and $\Delta B_i/B$ arises due to RGE of soft parameters. We excluded such cases in our analysis.

Let us now relax the universality condition of the slepton and Higgs boson masses, which leads to the following form of the neutrino mass matrix;

$$M_{ij}^{\nu} = c_0\xi_i\xi_j + c_{ij}\eta_i\eta_j, \quad (18)$$

where $c_0 = -2.12$ again with the choice of Eq. (11) and the flavor dependence in c_{ij} appears due to non-universal slepton masses. We first consider an interesting case where (B) *the flavor independence is assumed for ϵ_i* to see whether only non-universality in soft-parameters can be the source of the bi-large mixing. As an example, we take

(B1) $\epsilon_i = 1$, $\Delta m_i^2/m_{H_1}^2 = (-3.3, -3.1, -4.3)$ and $\Delta B_i/B = (-1.0, -2.6, -3.5)$:

This gives us $\xi_i = (-0.047, 1.25, 1.27)$, $\eta_i = (-1.05, -1.35, -2.23)$ and thus

$$c_{ij} = \begin{pmatrix} 0.41 & 0.48 & 0.15 \\ 0.48 & 0.57 & 0.18 \\ 0.15 & 0.18 & 0.046 \end{pmatrix}.$$

As a result, we get

$$\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2} = 0.017, \quad U_{e3} = 0.14, \\ \sin^2 2\theta_{\text{atm}} = 0.94, \quad \sin^2 2\theta_{\text{sol}} = 0.73. \quad (19)$$

Again, one needs a relation $\Delta m_1^2 \approx -\Delta B_1 \mu t \beta$. We find that this case (B) is not particularly fine-tuned compared to the previous case (A) and can be a viable option.

Finally, we consider (C) *the most general case* where we take arbitrary values of the 9 input parameters, ϵ_i , $\Delta m_i^2/m_{H_1}^2$ and $\Delta B_i/B$, whose sizes are however restricted within the range of (0.1–10). In Figs. 1 and 2, we present the scatter plot in term of $x_i = m_{L_i}^2/m_{H_1}^2$ and $p_i = \Delta B_i/B$ with $i = 1$ and 2, respectively, which generate the desired neutrino masses and mixing.

Fig. 1 shows that a solution set are centered around the values of x_1 and p_1 for which the cancellation in ξ_1 happens as discussed before. Another solution set is allowed around $x_1 = 3.4$ or 0.4 for which the sneutrino mass is close to the heavy or light Higgs mass, respectively. In this region, the mixing elements (8) and thus the coefficients c_{ij} in Eq. (18) become large to enhance the one-loop contribution. As a consequence, U_{e3} can be arranged to be small without making ξ_1 small. In Fig. 2, one sees that the points (x_2, p_2) close to (x_1, p_1) are favored although those points Fig. 1 allowing the cancellation in ξ_1 are excluded as can be expected. The plot in terms of (x_3, p_3) is also very similar to Fig. 2. In Figs. 1 and 2, we plotted only the points where the tree mass is three times larger than the loop mass. Here, let us remark that the one-loop mass can be even larger than the tree mass. That is, it is possible that the one-loop contribution proportional to $\eta_i \eta_j$ is the main source of the atmospheric neutrino mass and mixing angle while the tree mass generates the solar neutrino mass and mixing angle. Even though such cases of the loop dominance cannot be neglected, there is a much

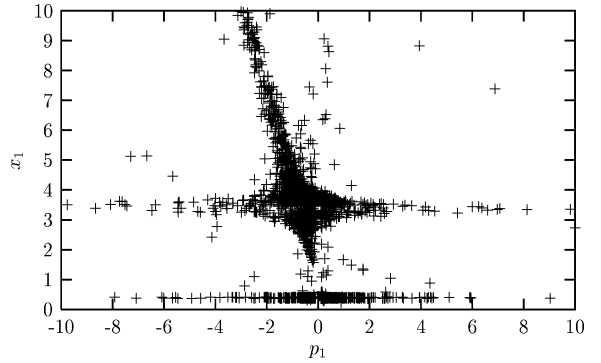


Fig. 1. The tree-dominant points allowing the atmospheric and solar neutrino masses and mixing in terms of the two input variables, $x_1 = m_{L_1}^2/m_{H_1}^2$ and $p_1 = \Delta B_1/B$.

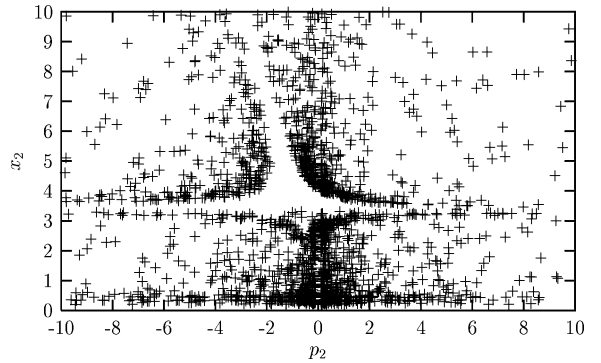


Fig. 2. Same as Fig. 1 with $x_2 = m_{L_2}^2/m_{H_1}^2$ and $p_2 = \Delta B_1/B$.

larger parameter space allowed in the case of the tree dominance as one can expect. This can be seen in Figs. 3 and 4 which plotted all the allowed points in terms of the induced variables ξ_i which determine the tree mass matrix as in Eq. (4). These two figures show that there appears the pattern, $|\xi_1| \ll |\xi_2| \approx |\xi_3|$, which gives rise to $\theta_{13} \ll 1$, $s_{23} \approx c_{23} \approx 1/\sqrt{2}$ as shown in Eq. (13) for the tree-dominance case.

To conclude, we showed how naturally the realistic neutrino mass matrix can arise from bilinear R-parity violation assuming non-universal soft-terms. When generic non-universality is allowed and $\tan \beta$ is not too large, the neutrino mass matrix is dominated by two contributions; the tree-level mass and the one-loop mass from the so-called Grossman–Haber diagrams arising due to the sneutrino–Higgs mixing. This was checked by our numerical calculation taking the full one-loop renormalized neutrino mass matrix. In this

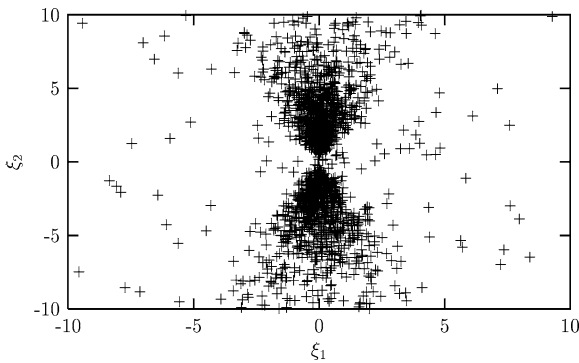


Fig. 3. All the points allowing the atmospheric and solar neutrino masses and mixing in terms of the two induced variables, ξ_1 and ξ_2 , controlling the tree-level mass matrix as in Eq. (4).

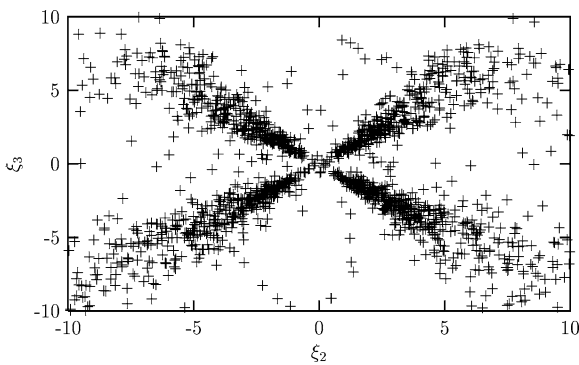


Fig. 4. Same as in Fig. 3 with ξ_2 and ξ_3 .

scheme, the loop-to-tree mass ratio falls naturally into the right range to generate the desired values for $\Delta m_{\text{sol}}^2/\Delta m_{\text{atm}}^2$. Considering nine input parameters, ϵ_i , ΔB_i and Δm_i^2 , we analyzed the parameter space accommodating two large (θ_{12} and θ_{23}) and one small (θ_{13}) mixing angles. Typically, the smallness of θ_{13} is realized by a cancellation between the terms contributing to ξ_1 . This was shown by some examples and also by the scatter plot of Fig. 1. Such an arrangement would not be a severe fine-tuning of input parameters. However, our scheme cannot provide a natural reason for vanishingly small θ_{13} if it turns out so. We presented the results accommodating only the LMA solution, but the similar conclusion can be

drawn also in the case of the LOW solution as can be inferred from our discussions.

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