# Heavy baryonic $\Lambda_{b} \rightarrow \Lambda \eta^{(\prime)}$ decays 

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#### Abstract

We calculate the branching ratios of the hadronic $\Lambda_{b}$ decays to $\eta$ and $\eta^{\prime}$ in the factorization approximation where the form factors are estimated via QCD sum rules and the pole model. Our results indicate that, contrary to $B \rightarrow K \eta^{(\prime)}$ decays, the branching ratios for $\Lambda_{b} \rightarrow \Lambda \eta$ and $\Lambda_{b} \rightarrow \Lambda \eta^{\prime}$ are more or less the same in the hadronic $\Lambda_{b}$ transitions. We find that the anomaly contribution is crucial in $\Lambda_{b} \rightarrow \Lambda \eta^{(1)}$ decays. We obtain the branching ratio of $\Lambda_{b} \rightarrow \Lambda \eta^{(1)}$ to be $11.47(11.33) \times 10^{-6}$ in QCD sum rules, and $2.95(3.24) \times 10^{-6}$ in the pole model. We also consider the contribution of the charm content in the $\eta^{\prime}$ production in $\Lambda_{b} \rightarrow \Lambda$ transition.


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For the last few years, different experimental groups have been accumulating plenty of data for the charmless hadronic $B$ decay modes. CLEO, Belle and BaBar Collaborations are providing us with the information on the branching ratio ( BR ) and the CP asymmetry for different decay modes. A clear picture is about to emerge from these information. Among the $B \rightarrow P P$ ( $P$ denotes a pseudoscalar meson) decay modes, the BR for the decay $B^{+} \rightarrow K^{+} \eta^{\prime}$ is found to be larger

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than that expected within the standard model (SM). The observed BR for this mode in three different experiments are [1-3]

$$
\begin{array}{rll}
\mathcal{B} & \left(B^{ \pm} \rightarrow K^{ \pm} \eta^{\prime}\right) & \\
& =\left(80_{-9}^{+10} \pm 7\right) \times 10^{-6} & {[\text { CLEO }]} \\
& =\left(77.9_{-5.9-8.7}^{+6.2+9.3}\right) \times 10^{-6} & {[\text { Belle }]} \\
& =(67 \pm 5 \pm 5) \times 10^{-6} & {[\text { BaBar }] .} \tag{1}
\end{array}
$$

In order to explain the unexpectedly large branching ratio for $B \rightarrow K \eta^{\prime}$, different assumptions have been proposed, e.g., large form factors [4], the QCD anomaly effect [5,6], high charm content in $\eta^{\prime}$ [7-9], a new mechanism in the Standard Model [10,11], the
perturbative QCD approach [12], the QCD improved factorization approach [13,14], or new physics like supersymmetry without R-parity [15-17]. Even though some of these approaches turn out to be unsatisfactory, the other approaches are still waiting for being tested by experiment. Therefore, it would be much better if besides using $B$ meson system, one can have an alternative way to test the proposed approaches in experiment.

Weak decays of the bottom baryon $\Lambda_{b}$ can provide a fertile testing ground for the SM. $\Lambda_{b}$ decays can also be used as an alternative and complementary source of data to $B$ decays, because the underlying quark level processes are similar in both $\Lambda_{b}$ and $B$ decays. For example, $\Lambda_{b} \rightarrow \Lambda \eta^{(1)}$ decay involves similar quark level processes as $B \rightarrow K \eta^{(1)}$, i.e., $b \rightarrow q \bar{q} s(q=u, d, s)$. In the coming years, large number of $\Lambda_{b}$ baryons are expected to be produced in hadron machines, like Tevatron and LHC, and a highluminosity linear collider running at the $Z$ resonance. For instance, the BTeV experiment, with a luminosity $2 \times 10^{32} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$, is expected to produce $2 \times 10^{11}$ $b \bar{b}$ hadrons per $10^{7}$ seconds [18], which would result in the production of $2 \times 10^{10} \Lambda_{b}$ baryons per year of running [19]. One of peculiar properties of $\Lambda_{b}$ decays is that, unlike $B$ decays, these decays can provide valuable information about the polarization of the $b$ quark. Experimentally the polarization of $\Lambda_{b}$ has been measured [20].

In this Letter, we study $\Lambda_{b} \rightarrow \Lambda \eta^{(\prime)}$ decay. The calculation of the BR for $\Lambda_{b} \rightarrow \Lambda \eta^{(\prime)}$ involves hadronic form factors which are highly model-dependent. Using different models for the form factors, we calculate the BR for $\Lambda_{b} \rightarrow \Lambda \eta^{(1)}$ and investigate the modeldependence of the theoretical predictions. In particular, we focus on the anomaly contribution in the production of $\eta^{\prime}$. In fact, our results indicate that this effect could be very important in $\Lambda_{b} \rightarrow \Lambda \eta^{(\prime)}$.

We also investigate the effect of the charm content in the $\eta^{\prime}$ production in the $\Lambda_{b} \rightarrow \Lambda$ transition, proposed originally for understanding the large $\mathcal{B}\left(B^{+} \rightarrow\right.$ $K^{+} \eta^{\prime}$ ) [7-9]. In this mechanism, the CKM allowed transition $b \rightarrow s c \bar{c}$ in conjunction with the $c \bar{c}$ component of the $\eta^{\prime}$ is suggested to be partly responsible for the enhancement of the $\mathcal{B}\left(B \rightarrow K \eta^{\prime}\right)$. Even though the effect on $\mathcal{B}\left(B^{+} \rightarrow K^{+} \eta^{\prime}\right)$ is generally agreed to be small, it will be interesting to investigate the contribution to the $\mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta^{\prime}\right)$ due to the charm content
of $\eta^{\prime}$. Indeed, the enhancement arising from the charm content turns out to be tiny in the main parameter region. However, this contribution can be as high as $15 \%$ in a specific range of parameter.

This Letter is organized as follows. First, we present the effective Hamiltonian for the usual $\Delta B=1$ transition and calculate the BR for $\Lambda_{b} \rightarrow \Lambda \eta^{(1)}$ decay within the factorization assumption using two different form factor models- QCD sum rules and the pole model. Then, the anomaly contribution to the $\Lambda_{b} \rightarrow \Lambda \eta^{(1)}$ is investigated and its effects on the $\eta$ and $\eta^{\prime}$ production in the $\Lambda_{b}$ decay is pointed out. Finally, we discuss the possible additional contribution to $\eta^{\prime}$ production in $\Lambda_{b} \rightarrow \Lambda$ transition due to the charm content of $\eta^{\prime}$.

The effective Hamiltonian $H_{\text {eff }}$ for the $\Delta B=1$ transition is

$$
\begin{align*}
H_{\mathrm{eff}}=\frac{4 G_{F}}{\sqrt{2}}[ & V_{u b} V_{u q}^{*}\left(c_{1} O_{1 u}^{q}+c_{2} O_{2 u}^{q}\right) \\
& +V_{c b} V_{c q}^{*}\left(c_{1} O_{1 c}^{q}+c_{2} O_{2 c}^{q}\right) \\
& \left.-V_{t b} V_{t q}^{*} \sum_{i=3}^{12} c_{i} O_{i}^{q}\right]+ \text { h.c. } \tag{2}
\end{align*}
$$

where the definition for each operators $O$ and the numerical values for the Wilson coefficients can be found in the literature [21-23]. In this Letter, we shall take into account the chromomagnetic operator $O_{11}$, but neglect the extremely small contribution from the electromagnetic operator $O_{12}$. Considering the gluon splitting into two quarks, the chromomagnetic operator can be rewritten in the Fierz transformed form as described in Ref. [24,25].

In general, the vector and axial-vector matrix elements for the $\Lambda_{b} \rightarrow \Lambda$ transition can be parameterized as

$$
\begin{align*}
& \langle\Lambda| \bar{s} \gamma_{\mu} b\left|\Lambda_{b}\right\rangle \\
& \quad=\bar{u}_{\Lambda}\left[f_{1} \gamma_{\mu}+i \frac{f_{2}}{m_{\Lambda_{b}}} \sigma_{\mu \nu} q^{\nu}+\frac{f_{3}}{m_{\Lambda_{b}}} q_{\mu}\right] u_{\Lambda_{b}}, \\
& \langle\Lambda| \bar{s} \gamma_{\mu} \gamma_{5} b\left|\Lambda_{b}\right\rangle \\
& \quad=\bar{u}_{\Lambda}\left[g_{1} \gamma_{\mu} \gamma_{5}+i \frac{g_{2}}{m_{\Lambda_{b}}} \sigma_{\mu \nu} q^{v} \gamma_{5}+\frac{g_{3}}{m_{\Lambda_{b}}} q_{\mu} \gamma_{5}\right] u_{\Lambda_{b}}, \tag{3}
\end{align*}
$$

where the momentum transfer $q^{\mu}=p_{\Lambda_{b}}^{\mu}-p_{\Lambda}^{\mu}$ and $f_{i}$ and $g_{i}(i=1,2,3)$ are Lorentz invariant form factors.

Alternatively, using the HQET , the hadronic matrix elements for the $\Lambda_{b} \rightarrow \Lambda$ transition can be parameterized [26] as
$\langle\Lambda| \bar{s} \Gamma b\left|\Lambda_{b}\right\rangle=\bar{u}_{\Lambda}\left[F_{1}\left(q^{2}\right)+\psi F_{2}\left(q^{2}\right)\right] \Gamma u_{\Lambda_{b}}$,
where $v=p_{\Lambda_{b}} / m_{\Lambda_{b}}$ is the four-velocity of $\Lambda_{b}$ and $\Gamma$ denotes the possible Dirac matrix. The relations between $f_{i}, g_{i}$ and $F_{i}$ can be easily given by
$f_{1}=g_{1}=F_{1}+r F_{2}$,
$f_{2}=f_{3}=g_{2}=g_{3}=F_{2}$,
where $r=m_{\Lambda} / m_{\Lambda_{b}}$.
The decay constants of the $\eta$ and $\eta^{\prime}$ mesons, $f_{\eta^{(\prime)}}^{q}$, are defined by
$\langle 0| \bar{q} \gamma^{\mu} \gamma_{5} q\left|\eta^{(\prime)}\right\rangle=i f_{\eta^{\prime \prime}}^{q} p_{\eta^{(\prime)}}^{\mu} \quad(q=u, s, c)$.
Due to the $\eta-\eta^{\prime}$ mixing, the decay constants of the physical $\eta$ and $\eta^{\prime}$ are related to those of the flavor $\mathrm{SU}(3)$ singlet state $\eta_{0}$ and octet state $\eta_{8}$ through the relations [27,28]
$f_{\eta}^{u}=\frac{f_{8}}{\sqrt{6}} \cos \theta_{8}-\frac{f_{0}}{\sqrt{3}} \sin \theta_{0}$,
$f_{\eta}^{s}=-2 \frac{f_{8}}{\sqrt{6}} \cos \theta_{8}-\frac{f_{0}}{\sqrt{3}} \sin \theta_{0}$,
$f_{\eta^{\prime}}^{u}=\frac{f_{8}}{\sqrt{6}} \sin \theta_{8}+\frac{f_{0}}{\sqrt{3}} \cos \theta_{0}$,
$f_{\eta^{\prime}}^{s}=-2 \frac{f_{8}}{\sqrt{6}} \sin \theta_{8}+\frac{f_{0}}{\sqrt{3}} \cos \theta_{0}$,
where $\theta_{8}$ and $\theta_{0}$ are the mixing angles and phenomenologically $\theta_{8}=-21.2^{\circ}$ and $\theta_{0}=-9.2^{\circ}$ [28]. We use $f_{8}=166 \mathrm{MeV}$ and $f_{0}=154 \mathrm{MeV}$ [21].

The decay amplitude of $\Lambda_{b} \rightarrow \Lambda \eta^{\prime}$ can be written as [25]
$\mathcal{M} \equiv\left\langle\Lambda \eta^{\prime}\right| H_{\text {eff }}\left|\Lambda_{b}\right\rangle=i \bar{u}_{\Lambda}\left(a+b \gamma_{5}\right) u_{\Lambda_{b}}$,
where
$a=(X+Y)\left[\left(m_{\Lambda_{b}}-m_{\Lambda}\right) f_{1}+\frac{m_{\eta^{\prime}}^{2}}{m_{\Lambda_{b}}} f_{3}\right]$,
$b=(X-Y)\left[\left(m_{\Lambda_{b}}+m_{\Lambda}\right) g_{1}-\frac{m_{\eta^{\prime}}^{2}}{m_{\Lambda_{b}}} g_{3}\right]$,

$$
\begin{align*}
X= & \frac{G_{F}}{\sqrt{2}}\left[\left\{V_{u b} V_{u s}^{*} a_{2}\right.\right. \\
& \left.-V_{t b} V_{t s}^{*}\left(2 a_{3}-2 a_{5}-\frac{1}{2} a_{7}+\frac{1}{2} a_{9}\right)\right\} f_{\eta^{\prime}}^{u} \\
& -V_{t b} V_{t s}^{*}\left\{a_{3}+a_{4}-a_{5}+\frac{1}{2} a_{7}-\frac{1}{2} a_{9}\right. \\
& \left.\left.-\frac{1}{2} a_{10}+\left(1+\frac{2 p_{b} \cdot q}{m_{b}^{2}}\right) a_{f}\right\} f_{\eta^{\prime}}^{s}\right], \\
Y= & -\frac{G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*} \chi_{\eta^{\prime}}\left(a_{6}-\frac{1}{2} a_{8}+\frac{5}{4} a_{f}\right)\left(f_{\eta^{\prime}}^{s}-f_{\eta^{\prime}}^{u}\right), \\
a_{i} \equiv & c_{i}^{\text {eff }}+\frac{1}{N_{c}} c_{i+1}^{\text {eff }} \quad(\text { for } i=\text { odd }), \\
a_{i} \equiv & c_{i}^{\text {eff }}+\frac{1}{N_{c}} c_{i-1}^{\text {eff }} \quad(\text { for } i=\text { even }), \\
\chi_{\eta^{\prime}}= & \frac{m_{\eta^{\prime}}^{2}}{m_{b} m_{s}}, \quad a_{f}=\frac{\alpha_{s}}{16 \pi k^{2}} m_{b}^{2} \frac{N_{c}^{2}-1}{N_{c}^{2}} c_{11 .} . \tag{9}
\end{align*}
$$

In the above amplitude, we have taken into account the anomaly contribution ${ }^{1}$ to the matrix element $\left\langle\eta^{\prime}\right| \bar{s} \gamma_{5} s|0\rangle[8,23,29,30]$, which leads to

$$
\begin{equation*}
\left\langle\eta^{\prime}\right| \bar{s} \gamma_{5} s|0\rangle=i \frac{\left(f_{\eta^{\prime}}^{s}-f_{\eta^{\prime}}^{u}\right) m_{\eta^{\prime}}^{2}}{2 m_{s}} \tag{10}
\end{equation*}
$$

The similar expression for the decay amplitude of $\Lambda_{b} \rightarrow \Lambda \eta$ can be obtained by replacing $\eta^{\prime}$ by $\eta$ in the above Eqs. (8) and (9).

We will see that this anomaly contribution plays an important role in $\Lambda_{b} \rightarrow \Lambda \eta^{\prime}$ and $\Lambda_{b} \rightarrow \Lambda \eta$ decays. In the case of neglecting the anomaly effect, we find that the BR of $\Lambda_{b} \rightarrow \Lambda \eta^{\prime}$ is much larger (e.g., about 5 times) than that of $\Lambda_{b} \rightarrow \Lambda \eta$ for most of the relevant parameter space. However, taking into account the anomaly effect, we find that the result drastically changes: both BRs become comparable.

For numerical calculations, we need specific values for the form factors in the $\Lambda_{b} \rightarrow \Lambda$ transition which are model-dependent. We use the values of the form factors from both the QCD sum rule approach [31] and the pole model [26,32]. In the QCD sum rule ap-

[^1]proach, the form factors $F_{1}$ and $F_{2}$ are given by
\[

$$
\begin{align*}
F_{1}= & -\frac{e^{2 \bar{\Lambda} / M+m_{\Lambda}^{2} / T}}{2 f_{\Lambda_{b}} f_{\Lambda}}\left[\int_{0}^{v_{c}} d v \int_{0}^{2 v z} d s \rho_{\mathrm{pert}}^{1}\right. \\
& \times e^{-s / T-\nu / M}-\frac{1}{3}\langle\bar{q} q\rangle^{2} \\
& -\frac{1}{32 \pi^{4}}\left\langle\alpha_{s} G G\right\rangle \int_{0}^{T / 4}\left(1-\frac{4 \beta}{T}\right) \\
& \left.\times e^{-4 \beta(1-4 \beta / T) / M^{2}-8 \beta z /(T M)} d \beta\right], \\
F_{2}= & -\frac{e^{2 \bar{\Lambda} / M+m_{\Lambda}^{2} / T}}{2 f_{\Lambda_{b}} f_{\Lambda}}\left[\int_{0}^{v_{c}} d \nu \int_{0}^{2 v z} d s \rho_{\mathrm{pert}}^{2} e^{-s / T-\nu / M}\right. \\
& +\frac{1}{8 \pi^{4}}\left\langle\alpha_{s} G G\right\rangle \int_{0}^{T / 4}\left(1-\frac{4 \beta}{T}\right) \frac{\beta}{M} \\
& \left.\times e^{-4 \beta(1-4 \beta / T) / M^{2}-8 \beta z /(T M)} d \beta\right], \tag{11}
\end{align*}
$$
\]

where

$$
\begin{align*}
& \rho_{\text {pert }}^{1}= \frac{1}{32 \pi^{4} \sigma^{3}}\left\{-2 z^{3} \sigma^{3}-[-s+z(v+2 z)]^{3}\right. \\
&\left.+3 z^{2}[-s+z(v+2 z)] \sigma^{2}\right\}, \\
& \rho_{\text {pert }}^{2}=-\frac{1}{64 \pi^{4} \sigma^{3}}\left[s-2 z^{2}+z(-v+\sigma)\right]^{2} \\
& \times\left[v s+8 z^{3}-4 z^{2}(-2 v+\sigma)\right. \\
&\left.-2 z\left(-v^{2}+5 s+v \sigma\right)\right], \\
& \sigma=\sqrt{-4 s+(v+2 z)^{2}} . \tag{12}
\end{align*}
$$

Here $z=\frac{p_{\Lambda} \cdot p_{\Lambda_{b}}}{m_{\Lambda_{b}}}=\frac{m_{\Lambda_{b}}^{2}+m_{\Lambda}^{2}-q^{2}}{2 m_{\Lambda_{b}}}\left(q^{\mu}=p_{\Lambda_{b}}^{\mu}-p_{\Lambda}^{\mu}\right)$ and the Borel parameter $M=\frac{4 T}{m_{b}}$. For the other relevant conventions and notation, we refer to Ref. [31]. In Figs. 1 and 2 we show the form factors $F_{1}$ and $F_{2}$ as a function of the Borel parameter $M=\frac{4 T}{m_{b}}$ for $\Lambda_{b} \rightarrow \Lambda \eta^{(1)}$, respectively. In $\Lambda_{b} \rightarrow \Lambda \eta^{(1)}, F_{1}=$ $0.510(0.514)$ and $F_{2}=-0.058(-0.060)$ for $M=$ $1.5 \mathrm{GeV}, \quad F_{1}=0.476(0.481)$ and $F_{2}=-0.084$ ( -0.088 ) for $M=1.7 \mathrm{GeV}$, and $F_{1}=0.473(0.479)$ and $F_{2}=-0.117(-0.122)$ for $M=1.9 \mathrm{GeV}$. The BRs of $\Lambda_{b} \rightarrow \Lambda \eta^{\prime}$ and $\Lambda_{b} \rightarrow \Lambda \eta$ versus $\xi \equiv \frac{1}{N_{c}}$ for


Fig. 1. The form factor $F_{1}$ for the transition $\Lambda_{b} \rightarrow \Lambda$ versus the Borel parameter $M\left(=\frac{4 T}{m_{b}}\right)$. The dotted (solid) line corresponds to the case of $\Lambda_{b} \rightarrow \Lambda \eta^{(\prime)}$.


Fig. 2. The form factor $F_{2}$ for the transition $\Lambda_{b} \rightarrow \Lambda$ versus the Borel parameter $M\left(=\frac{4 T}{m_{b}}\right)$. The dotted (solid) line corresponds to the case of $\Lambda_{b} \rightarrow \Lambda \eta^{(\prime)}$.
different values of the Borel parameter $M=\frac{4 T}{m_{b}}$ are shown in Figs. 3 and 4, respectively. Our result shows
$\mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta^{\prime}\right)=(6.0-19.0) \times 10^{-6}$,
and
$\mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta\right)=(6.5-17.9) \times 10^{-6}$.
For $\xi=1 / 3$ (i.e., $N_{c}=3$ ) and $M=1.7 \mathrm{GeV}, \mathcal{B}\left(\Lambda_{b} \rightarrow\right.$ $\left.\Lambda \eta^{\prime}\right)=11.33 \times 10^{-6}$ and $\mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta\right)=11.47 \times$


Fig. 3. The BR for the decay $\Lambda_{b} \rightarrow \Lambda \eta^{\prime}$ versus $\xi=\frac{1}{N_{c}}$ for different values of the Borel parameter $M=\frac{4 T}{m_{b}}$. The shaded region denotes the case of $\xi \leqslant 0.1$, which is favored from the analysis of $B \rightarrow K \eta^{\prime}$ decays.


Fig. 4. The BR for the decay $\Lambda_{b} \rightarrow \Lambda \eta$ versus $\xi=\frac{1}{N_{c}}$ for different values of the Borel parameter $M=\frac{4 T}{m_{b}}$. The shaded region denotes the case of $\xi \leqslant 0.1$, which is favored from the analysis of $B \rightarrow K \eta^{\prime}$ decays.
$10^{-6}$. We recall that in the case of $B \rightarrow K \eta^{\prime}$ a small value of $\xi(\xi \leqslant 0.1)$ is favored to fit the experimental data on the BR in the framework of the generalized factorization $[8,21,23,30]$. In the figures, the shaded region denotes the case of $\xi \leqslant 0.1$, favored from the analysis of $B \rightarrow K \eta^{\prime}$. For $\xi=0.1, \mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta^{\prime}\right)=$ $14.53 \times 10^{-6}$ and $\mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta\right)$ is $13.91 \times 10^{-6}$.

In the pole model [26,32], the form factors are given by
$F_{i}\left(q^{2}\right)=N_{i}\left(\frac{\Lambda_{\mathrm{QCD}}}{\Lambda_{\mathrm{QCD}}+z}\right)^{2}, \quad i=1,2$,
where $\Lambda_{\mathrm{QCD}} \sim 200 \mathrm{MeV}$ and $z=\frac{p_{\Lambda} \cdot p_{\Lambda_{b}}}{m_{\Lambda_{b}}}$. Using $N_{1}=52.32$ and $N_{2}=-13.08$, we obtain the values of the form factors: $F_{1}\left(q^{2}\right)=0.225(0.217)$ and $F_{2}\left(q^{2}\right)=-0.056(-0.054)$ for $q^{2}=m_{\eta^{\prime}}^{2}\left(m_{\eta}^{2}\right)$. We note that the magnitudes of these form factors are less than a half of those obtained in the QCD sum rule method. This would result in the fact that the BRs for $\Lambda_{b} \rightarrow \Lambda \eta^{(1)}$ predicted in the case of the pole model are quite smaller than those predicted in the case of the QCD sum rule approach. Indeed, the BRs for $\Lambda_{b} \rightarrow \Lambda \eta^{\prime}$ and $\Lambda_{b} \rightarrow \Lambda \eta$ are estimated to be
$\mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta^{\prime}\right)=(1.8-4.5) \times 10^{-6}$,
and
$\mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta\right)=(1.8-3.8) \times 10^{-6}$,
which are about a quarter of those estimated in the QCD sum rule case. For $\xi=1 / 3, \mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta^{\prime}\right)=$ $3.24 \times 10^{-6}$ and $\mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta\right)=2.95 \times 10^{-6}$. For $\xi=0.1, \mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta^{\prime}\right)=4.08 \times 10^{-6}$ and $\mathcal{B}\left(\Lambda_{b} \rightarrow\right.$ $\Lambda \eta)=3.55 \times 10^{-6}$. Since the main uncertainty in our analysis arises from the uncertainty in the relevant form factors, for a more accurate analysis, the experimental test for determining the value of the form factors is called for. The form factors can be determined in the semileptonic decays, $\Lambda_{b} \rightarrow \Lambda \ell \ell$ or $\Lambda_{b} \rightarrow p \ell \bar{\nu}$.

We note that the BR of $\Lambda_{b} \rightarrow \Lambda \eta$ is similar to that of $\Lambda_{b} \rightarrow \Lambda \eta^{\prime}$, in contrast to the case of $B \rightarrow$ $K \eta^{(1)}$ where the BR of $B \rightarrow K \eta$ is about an order of magnitude smaller than that of $B \rightarrow K \eta^{\prime}$. This difference can be understood by noticing the following two points. First, the difference arises from the fact that in the factorization scheme, the decay amplitude for $\Lambda_{b} \rightarrow \Lambda \eta^{(1)}$ consists of terms proportional to $\left\langle\eta^{(1)}\right| O|0\rangle\langle\Lambda| O^{\prime}\left|\Lambda_{b}\right\rangle$ only (see Eq. (9)), while the decay amplitude for $B \rightarrow K \eta^{(1)}$ consists of terms proportional to $\langle K| \tilde{O}|0\rangle\left\langle\eta^{(\prime)}\right| \tilde{O}^{\prime}|B\rangle$ as well as terms proportional to $\left\langle\eta^{\left({ }^{\prime}\right)}\right| O|0\rangle\langle K| O^{\prime}|B\rangle[8,21,23]$. (Here $O^{(1)}$ and $\tilde{O}^{(1)}$ denote the relevant quark currents arising from the effective Hamiltonian (2).) In the case of $B \rightarrow K \eta^{(1)}$, the destructive (constructive) interference appears between the penguin amplitude proportional to $\langle K| \tilde{O}|0\rangle\left\langle\eta^{(\prime)}\right| \tilde{O}^{\prime}|B\rangle$ and that proportional to
$\left\langle\eta^{(\prime)}\right| O|0\rangle\langle K| O^{\prime}|B\rangle$, due to the opposite (same) sign between $\langle K| \tilde{O}|0\rangle \propto f_{K}$ and $\left\langle\eta^{(\prime)}\right| O|0\rangle \propto f_{\eta^{(\prime)}}^{q}$ : in particular, $f_{\eta}^{s}=-112 \mathrm{MeV}$, while $f_{\eta^{\prime}}^{s}=+137 \mathrm{MeV}$ (see Eq. (7)). In contrast, in the case of $\Lambda_{b} \rightarrow \Lambda \eta^{(\prime)}$, there is no such interference between terms in the amplitude because the amplitude contains terms proportional to $\left\langle\eta^{(\prime)}\right| O|0\rangle \propto f_{\eta^{(\prime)}}^{q}$ only. Second, the difference also arises from the fact that there is an additional interference between the anomaly term proportional to $f_{\eta^{(1)}}^{u}$ and the other dominant terms proportional to $f_{\eta^{(\prime)}}^{s}$ both in $\Lambda_{b} \rightarrow \Lambda \eta^{(\prime)}$ and $B \rightarrow K \eta^{(\prime)}$. It turns out that in $\Lambda_{b} \rightarrow \Lambda \eta^{(\prime)}$ and $B \rightarrow K \eta^{(\prime)}$ the anomaly term interferes constructively (destructively) with the other dominant terms. In the case of $\Lambda_{b} \rightarrow \Lambda \eta^{(\prime)}$, this interference between the anomaly and the other dominant terms plays a crucial role to make the BRs of $\Lambda_{b} \rightarrow \Lambda \eta^{\prime}$ and $\Lambda_{b} \rightarrow \Lambda \eta$ comparable in magnitude (see the discussion in the following paragraphs). However, in the case of $B \rightarrow K \eta^{(\prime)}$, this interference between the anomaly and the other terms becomes less important than that between the terms proportional to $\langle K| \tilde{O}|0\rangle$ and $\left\langle\eta^{(\prime)}\right| O|0\rangle$, as mentioned above.

In order to examine how large the anomaly contribution to $\mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta^{(\prime)}\right)$ is, we calculate those BRs, neglecting the anomaly contribution. We find that in the QCD sum rule approach,

$$
\begin{align*}
& \mathcal{B}\left(\Lambda_{b}\right.\left.\rightarrow \Lambda \eta^{\prime}\right) \\
& \text { for } \xi=33.28(40.45) \times 10^{-6} \\
& \mathcal{B}(\xi=0.1) \\
& \text { for } \xi=\frac{1}{3}(\xi=0.1) \tag{18}
\end{align*}
$$

and in the pole model,

$$
\begin{align*}
\mathcal{B}\left(\Lambda_{b}\right. & \left.\rightarrow \Lambda \eta^{\prime}\right)=9.84(1.54) \times 10^{-6} \\
\text { for } \xi & =\frac{1}{3}(\xi=0.1) \\
\mathcal{B}\left(\Lambda_{b}\right. & \rightarrow \Lambda \eta)=11.83(1.90) \times 10^{-6} \\
\text { for } \xi & =\frac{1}{3}(\xi=0.1) \tag{19}
\end{align*}
$$

Compared to the corresponding results including the anomaly contribution (shown just below Eqs. (14) and (17)), the above values of $\mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta^{\prime}\right)$ are roughly 3 times larger, while those of $\mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta\right)$ are only
about a half. It shows that the anomaly contribution is indeed very crucial in $\Lambda_{b} \rightarrow \Lambda \eta^{(\prime)}$ decays: due to the anomaly contribution, $\mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta^{\prime}\right)$ is reduced by about a factor of 0.3 , but $\mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta\right)$ is increased by about a factor of 2 .

This feature can be understood by the following observation. In Eq. (9), the term proportional to the decay constant $f_{\eta^{\prime}}^{u}$ in $Y$ is due to the anomaly. For $\Lambda_{b} \rightarrow \Lambda \eta$, the similar expression appears. $\Lambda_{b} \rightarrow \Lambda \eta^{(\prime)}$ decay modes are penguin-dominated processes and the main contributions come from the QCD coefficients $a_{4}$ and $a_{6}$. There are three dominant terms in the decay amplitudes, entering through $X$ and $Y$ in Eq. (9): these are proportional to $-a_{4} f_{\eta^{(1)}}^{S}$, $-\chi_{\eta^{(\prime)}} a_{6} f_{\eta^{(\prime)}}^{s}$, and $+\chi_{\eta^{(1)}} a_{6} f_{\eta^{(\prime)}}^{u}$, respectively. Please note the following points: (i) The contribution due to anomaly appears with a negative sign compared to the other dominant terms. Also, the chiral enhancement factors are $\chi_{\eta^{\prime}} \approx 2$ and $\chi_{\eta} \approx 0.5$. (ii) The relevant decay constants are $f_{\eta^{(\prime)}}^{u}=0.078(0.063) \mathrm{GeV}$ and $f_{\eta^{(1)}}^{s}=-0.112(+0.137) \mathrm{GeV}$. (iii) The QCD coefficients $a_{4}$ and $a_{6}$ have the same sign and they are comparable in magnitude. Consequently, it is straightforward to see that $|X+Y| \gg|X-Y|$. Due to the fact $f_{1}=g_{1} \gg f_{3}=g_{3}$, we see that the dominant contribution to the decay rates arises from the term $a$ in Eq. (9) which is proportional to $(X+Y)$. Thus, now focusing only on $(X+Y)$, we observe that for the process $\Lambda_{b} \rightarrow \Lambda \eta^{\prime}$, the anomaly term that is proportional to $+\chi_{\eta^{(1)}} a_{6} f_{\eta^{(\prime)}}^{u}$, interferes destructively with the other dominant terms. In the $\Lambda_{b} \rightarrow \Lambda \eta$ case however, the anomaly term interferes constructively with the others, due to the negative value of $f_{\eta}^{s}$. It is straightforward to check that due to the anomaly contribution, the magnitude of the amplitude of $\Lambda_{b} \rightarrow \Lambda \eta^{\prime}$ is reduced by about a factor of 0.6 , while that of $\Lambda_{b} \rightarrow \Lambda \eta$ is increased by about a factor of 1.3.

Before concluding we investigate what a specific mechanism, that is proposed as a possible explanation of the large branching ratio of $B \rightarrow K \eta^{\prime}$, has to say for $\eta^{\prime}$ production in hadronic two-body $\Lambda_{b} \rightarrow \Lambda$ transition. The experimental observation of the latter decay channel may serve to distinguish the acceptable mechanism. For example, Ref. [33] predicts that the enhancement of the $\eta^{\prime}$ production in mesonic $B \rightarrow K$ decay does not apply to the baryonic $\Lambda_{b} \rightarrow \Lambda$ transition. Here we look at the enhancement of the baryonic
$\Lambda_{b} \rightarrow \Lambda \eta^{\prime}$ decay due to the possible $c \bar{c}$ component of $\eta^{\prime}$.

One can estimate the contribution from the charm content of $\eta^{\prime}$ to the decay $\Lambda_{b} \rightarrow \Lambda \eta^{\prime}$ by including $b \rightarrow s c \bar{c}$ transition in the matrix element in Eq. (8) which is given by

$$
\begin{align*}
&\left\langle\Lambda \eta_{c c}^{\prime}\right| H_{\mathrm{eff}}\left|\Lambda_{b}\right\rangle \\
&= \frac{4 G_{F}}{\sqrt{2}}\left[V_{c b} V_{c s}^{*} a_{2}\langle\Lambda| \bar{s} \gamma_{\mu} L b\left|\Lambda_{b}\right\rangle\left\langle\eta^{\prime}\right| \bar{c} \gamma^{\mu} L c|0\rangle\right. \\
&-V_{t b} V_{t s}^{*}\left(2 a_{3}-2 a_{5}-\frac{1}{2} a_{7}+\frac{1}{2} a_{9}\right) \\
& \times\langle\Lambda| \bar{s} \gamma_{\mu} L b\left|\Lambda_{b}\right\rangle\left\langle\eta^{\prime}\right| \bar{c} \gamma^{\mu} L c|0\rangle \\
& \quad-V_{t b} V_{t s}^{*} \chi_{\eta^{\prime}}\left(-a_{6}+\frac{1}{2} a_{8}-\frac{5}{4} a_{f}\right) \\
&\left.\times\langle\Lambda| \bar{s} \gamma_{\mu} R b\left|\Lambda_{b}\right\rangle\left\langle\eta^{\prime}\right| \bar{c} \gamma^{\mu} L c|0\rangle\right] \tag{20}
\end{align*}
$$

The $c \bar{c}$ vacuum annihilation of $\eta^{\prime}$, which is parameterized by the decay constant $f_{\eta^{\prime}}^{c}$ defined in Eq. (6), can be extracted from the experimental data on $\psi \rightarrow$ $\eta^{\prime} \gamma, \psi \rightarrow \eta_{c} \gamma$ and $\eta_{c} \rightarrow \gamma \gamma$ [8,34]. In fact, more careful estimate of the charm content of $\eta^{\prime}$ leads to $\left|f_{\eta^{\prime}}^{c}\right| \approx 2.4 \mathrm{MeV}$, which is substantially smaller than $f_{\eta^{\prime}}^{u}=78.0 \mathrm{MeV}$.

Inserting the parameter values in the above equation results in $\mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta^{\prime}\right)=10.60 \times 10^{-6}$ for $\xi=$ $1 / 3$ and $\mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta^{\prime}\right)=16.78 \times 10^{-6}$ for $\xi=0.1$, where the QCD sum rule is used to estimate the form factors. We observe that the shift in the branching ratio due to the charm content of $\eta^{\prime}$ is sensitive to the effective number of colors $\xi$, resulting in a reduction of around $6 \%$ for $N_{c}=3$ and an enhancement of more than $15 \%$ for $N_{c}=10$.

In this Letter, we calculated the BRs for the twobody hadronic decays of $\Lambda_{b}$ to $\Lambda$ and $\eta$ or $\eta^{\prime}$ mesons. The form factors of the relevant hadronic matrix elements are evaluated by two methods: QCD sum rules and the pole model. In QCD sum rules, the sensitivity of the form factors to the Borel parameter is roughly the same for $\eta$ and $\eta^{\prime}$. The variation of $F_{1}$ is around $7 \%$ for the Borel parameter in the range between 1.5 and 1.9. $F_{2}$ on the other hand, is quite sensitive to this parameter, changing by a factor 2 approximately, in the above range. However, due to the relative size of $F_{1}$ and $F_{2}$, one can see from Eq. (16) that the uncertainty in the amplitude for $\Lambda_{b} \rightarrow \Lambda$ transition due to
hadronic form factors is dominated by the variation in the former, leading to about $14 \%$ error in the branching ratio. Also, we have checked the variation of the BRs for $\Lambda_{b} \rightarrow \Lambda \eta^{(\prime)}$ with the effective number of colors $N_{c}$ in order to extend our results to $\xi=\frac{1}{N_{c}} \leqslant 0.1$ range, which is favored in fitting the experimental data on the $\mathcal{B}\left(B \rightarrow K \eta^{\prime}\right)$ in the framework of generalized factorization. Our results indicate that the BRs for $\Lambda_{b} \rightarrow \Lambda \eta$ and $\Lambda_{b} \rightarrow \Lambda \eta^{\prime}$ are more or less the same in QCD sum rules, $11.47 \times 10^{-6}$ and $11.33 \times 10^{-6}$, respectively, for $M=1.7 \mathrm{GeV}$ and $N_{c}=3$.

In the pole model, on the other hand, the form factor $F_{1}$ turns out to be smaller by a factor 2 , approximately. However, $F_{2}$ is roughly the same as in the sum rule case for the smaller values of the Borel parameter. As a result, the predicted branching ratios in this model, $\mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta\right)=2.95 \times 10^{-6}$ and $\mathcal{B}\left(\Lambda_{b} \rightarrow \Lambda \eta^{\prime}\right)=3.24 \times 10^{-6}$ for $N_{c}=3$, are significantly smaller than those obtained via QCD sum rules.

We showed that the anomaly contribution is very important in $\Lambda_{b} \rightarrow \Lambda \eta^{(\prime)}$ decays. Due to the anomaly contribution, the BR for $\Lambda_{b} \rightarrow \Lambda \eta^{\prime}$ is reduced by about a factor of 0.3 , while the BR for $\Lambda_{b} \rightarrow \Lambda \eta$ is increased by about a factor of 2 .

We also calculated the contribution due to the charm content of $\eta^{\prime}$ to the hadronic $\Lambda_{b} \rightarrow \Lambda \eta^{\prime}$ decay. Our results show that the shift in the branching ratio due to the charm content mechanism is about $6 \%$ downward for $\xi=1 / 3$ and more than $15 \%$ upward for $\xi=1 / 10$ which might be noticeable.

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[^1]:    ${ }^{1}$ This anomaly contribution was not taken into account in Ref. [25].

