Abstract

In this study supersonic mixing of two-parallel and non-parallel gaseous streams has been simulated numerically. The streams are of air and hydrogen, which come into contact after passing over a finite thickness base. Two gas streams are considered from a high-pressure reservoir and entering into the domain with atmospheric pressure. Two-dimensional unsteady state Navier-Stokes equations, energy, mass diffusion and species continuity equations are numerically simulated to analyze the two-dimensional mixing layer in supersonic flow field. An explicit Harten-Yee Non-MUSCL Modified flux-type TVD (total variation diminishing) scheme has been used to solve the system of equations. An algebraic turbulence model is used to calculate the eddy viscosity coefficient. Keeping constant the inlet pressure and velocity of the streams, the merging angle is varied to observe the physics of flow fields, mixing fields of two-streams and mixing efficiency. The result shows that when merging angle increases interaction between two streams, high momentum exchange occurs and eventually enhances the mixing of two streams.

Keywords: Supersonic combustor, mixing, shear layers, merging angle

1. Introduction

Turbulent mixing layers occur in flow fields of many engineering applications e.g., combustion chambers, pre-mixers for gas turbine combustors, chemical lasers, propulsion systems and flow reactors. Particularly, the mixing of reactants and their complete combustion in supersonic combustion ramjet (scramjet) engines has drawn special attention of present scientists. In supersonic combustion systems, the flow speeds are so high that the fuel and oxidizer have little time to mix. The shear layers are naturally unstable and usually lead to a large scale mixing. The higher the Mach number, the longer length it takes for the shear layers to become unstable. This reduces mixing accomplished in a given length. Several configurations of combustor have been studied to seek the enhancement of mixing. Generally parallel, normal or oblique type mixing is used and most of the researchers carried out their study on two parallel supersonic streams.

Guirguis et al. [1] performed two-dimensional time-dependent numerical simulation of the convective mixing of two supersonic parallel streams of air. They simulated a supersonic shear layer in a two dimensional channel of 20 cm long and 2.4 cm high and used flux corrected transport algorithm neglecting all diffusion transport processes. Comparisons were made for the vorticity, density and pressure contour of confined and unconfined shear layer. Farouk et al. [2] performed numerical simulation of the mixing of two supersonic streams of air in a 25cm x 3cm flow field considering laminar velocity profile of the streams at inlet. They solved Euler equation and studied the effects of density, velocity and pressure variation on mixing. Brown et al. [3] experimentally investigated the effects of density ratio on plane turbulent mixing between two...
streams of different gases. It was observed that, for all ratios of densities in the two streams, the mixing layer was dominated by large coherent structures. These structures made convection at nearly constant speed, and increased their sizes and spaces discontinuously by the process of amalgamation with neighboring ones. Papamoschuo et al. [4] observed that the spreading rate was dependent on Mach number but independent on transverse density gradients.

Ali and Islam [5] studied the mixing mechanisms, investigated the mixing characteristics for several flow configurations and observed that recirculation in flow field can play an important role in the mixing enhancement. Ali et al. [6] found that the inlet configuration of air stream can play an important role on the enhancement of mixing. Ali et al. [7] studied the physics of mixing in two-dimensional supersonic stream and showed that mixing is only possible by incorporating the molecular diffusion terms in the Navier-Stokes equations. Gerlinger et al. [8] found that increase in injector lip thickness results in increased shear layer thickness and also in larger total pressure losses because of the stronger recompression shocks. They also found that increase in mixing layer thickness did not have significant effect on the mixing efficiency. In another investigation Guirguis et al. [9] studied the effect of bluff center bodies on mixing enhancement in supersonic shear layers. They observed that the shear layer became unstable faster than with the streamlined body. As a result, a large amount of convective mixing occurs within the length of the domain. Azim et al. [10] investigated plane mixing layers from parallel and non-parallel merging of two streams. The authors reported that both types of mixing layers were found to decrease in growth with increasing velocity ratio, though they spread more at the high speed side.

In present research, numerical investigation on supersonic mixing layers has been performed by solving two-dimensional unsteady Navier-Stokes equations, energy equation, mass diffusion equation and continuity equation. The problem was defined by allowing two streams past over a finite-thickness base (2.3 mm) confined between two parallel plates. The air stream is at the upper side of the base plate and the hydrogen stream is underneath the base plate as shown in Fig. 1. After separating from the base, the streams form shear layers and mix with one another that usually occurs in combustor. This study has been made for the following reasons: (i) to increase the mixing efficiencies of a supersonic combustor and (ii) to study the physics of fluid dynamics including shocks and turbulence. The inlet pressure ratio and velocity ratio of the two streams have been kept constant which is equal to unity and the merging angle is varied from 0° to 20° with the increment of 5° for this study. The calculations of flow field with different merging angles are denoted as case 1 (merging angle 0°), case 2 (merging angle 5°), case 3 (merging angle 10°), case 4 (merging angle 15°), and case 5 (merging angle 20°).

![Fig. 1. Schematic diagram of the calculation domain](image)

2. Governing equations and boundary conditions

The following continuity equation, Navier-Stokes equation, energy equation and mass diffusion equation are used to solve this flow field where body forces are neglected.

Continuity equation:

\[
\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0
\]

Navier-Stokes equation: X component:

\[
\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(p + \rho u^2) + \frac{\partial}{\partial y}(\rho uv) = \frac{\partial}{\partial x}(\sigma_x) + \frac{\partial}{\partial y}(\tau_{yx})
\]

Y Component:

\[
\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho uv) + \frac{\partial}{\partial y}(p + \rho v^2) = \frac{\partial}{\partial x}(\tau_{yx}) + \frac{\partial}{\partial y}(\sigma_y)
\]
Energy equation:
\[
\frac{\partial}{\partial t}(E) + \frac{\partial}{\partial x}[(E + p)u] + \frac{\partial}{\partial y}[(E + p)v] = \frac{\partial}{\partial x}[u\sigma_x - v\tau_{xy} - \dot{q}_x] + \frac{\partial}{\partial y}[u\tau_{yx} - v\sigma_y - \dot{q}_y]
\]

Species continuity equation:
\[
\frac{\partial}{\partial t}(\rho Y_i) + \frac{\partial}{\partial x}(\rho Y_i u) + \frac{\partial}{\partial y}(\rho Y_i v) + \frac{\partial}{\partial x}(\dot{m}_{i_x}) + \frac{\partial}{\partial y}(\dot{m}_{i_y}) = 0
\]

- The top and bottom boundaries of the computational region are considered as solid walls. A Navier-Stokes analysis imposes that the normal and tangential velocity components are zero on the walls. The walls are assumed to be thermally adiabatic, so that \(\frac{\partial T}{\partial n}\bigg|_w = 0\).
- Inflow boundary conditions are used on the left boundary of the computational domain. The inflow condition is supersonic with fully developed turbulent boundary layers which is kept constant throughout the computations.
- The outflow boundary conditions \((X = L)\) are considered to be zero-gradient for all variables.

![Fig 2. Velocity vector and streamline representation of near flow circulating region for all cases.](image-url)
3. Results and discussion

3.1 Physics of fluid dynamics

Figure 2 shows the velocity vectors with streamlines just behind the finite base for case 1–5. In Fig 2(a) the upper recirculation rotates clockwise while the lower recirculation rotates counterclockwise. The flows expand and high interaction occurs after recirculation. The recirculation zones spread downstream, increasing the length in longitudinal direction. The stream lines indicate that both of the recirculations are created by the hydrogen flow. For other cases in Figs. 2(b) to (e), after entering into the first recirculation, portion of the hydrogen flow cannot complete the recirculation. This portion of hydrogen makes intimate contact with air and deflects 180° due to the high momentum of air stream. The velocity in recirculation is low and therefore hydrogen has much time to contact with air resulting in high diffusion. Throughout the study, the momentum of air is higher than that of hydrogen, due to which the expansion of hydrogen is high behind the base while the expansion of the air stream is low. Due to expansion and interaction between two streams hydrogen enters in the recirculation region and mixes with air by diffusion and convection. So recirculation plays a vital role to enhance mixing.

Figure 2(b) shows the shear layer mixing regions spread with longitudinal distance until impingement occurs at approximately X/H = 0.27, which is shorter than case 1. Therefore, the area of recirculation zones in case-2 are smaller than case-1. Figures 2(a–e) shows that with the increment of merging angle the size of both recirculations decreases but more hydrogen molecules enter in the recirculation region due to strong interactions and eventually more molecular and convection diffusion occurs. Another observation is that in Fig. 2(a) two recirculations are very clear but in Fig. 2(b) the upper recirculation vanishes and the size of lower recirculation decreases. Moreover, the streamlines generated from the same location of Fig. 2(a–b) indicate that more hydrogen molecules enter into the upper side of the recirculation region and make intimate contact with the air stream for case 2 than that of case 1. Velocity distribution curve in Fig. 3 shows that velocities of the upper section are similar for all merging angles at constant pressure ratio. But the velocity of the lower part (hydrogen) increases at the downstream. The maximum velocity occurs at merging angle of 20. The maximum velocity 3200 m/s is found at X/H= 5.98 for pressure ratio 1.0.

![Image](image-url)

**Fig. 3. Velocity distribution at section X/H=5.98 for Case 1 to Case 5**

3.2 Structure of shear layers

The mole fraction contours give a structure of free shear layers created by the mixing of the two streams. Figure 4 shows the mole fraction contours of hydrogen for pressure ratio 1.0. The mole fraction of hydrogen close to bottom wall is 0.95 and the contour line varies from 0.95 to 0.05 towards the upper wall. The increment of mole fraction between two adjacent contour lines is 0.05. As stated earlier, a thin base is located from Y/H = 0.45 to 0.55 in the middle of the two streams. Throughout the study hydrogen has less momentum than that of the air stream and eventually hydrogen will occupy more space after the thin base. For case 1, there is no initial deflection of shear layer due to identical pressure. But for cases 2-5, the shear layer deflects towards the bottom wall due to non-parallel mixing and higher density of air. The deflection angles for case 2, 3, 4 and 5 are 5°, 8°, 10° and 18°, respectively. The spreading rate of free shear layers increases with the
increment of merging angle. Further deflections of shear layer at downstream are fairly understandable for higher merging angles.

3.3 Mixing efficiency

Mixing efficiency has been calculated on the basis of flammability limits of hydrogen and air. So, in the calculation of mixing efficiency the region having the mole fraction range of hydrogen from 0.05 to 0.75 has been taken into consideration. The mixing of hydrogen in air can be occurred by means of (i) interaction between two streams, (ii) turbulence and convection due to recirculation and velocity of the flow, or (iii) molecular diffusion due to density gradient. The performance of different cases is evaluated by calculating the mixing efficiency. Figure 5 shows the mixing efficiency along the physical model for different cases. For all the cases the mixing efficiency increases sharply just behind the base due to expansion at the thin base corner and recirculation. The rate of increment in efficiency further increases due to the interaction of two streams. In downstream the mixing is very slow in mixing shear-layer because weak molecular diffusion due to supersonic nature of the flow.

Figure 5 shows the mixing efficiency along the physical model for cases 1–5. Generally, in the upstream region, the increasing of mixing is high and in downstream it is very slow. In Fig.5 at X/H= 2 the mixing efficiencies of the cases 1, 2, 3, 4 and 5 are approximately 5.0, 6.0, 8.5, 14.0 and 20.5%, respectively, i.e., in upstream mixing efficiency increases with the increase of mixing angle. Also increment of mixing can be found along the length of physical model between X/H = 4.0 to 5.0 for all the cases due to the impingement of the streams. In downstream, cases 3, 4 and 5 have negligible increment of mixing in the shear layer. Case 1 has the maximum increment of mixing at downstream. The difference between the value of mixing efficiency of cases 1 and 2 starts to decrease from X/H = 4.0 and the efficiency curve of case 1 coincides with that of case 2 at X/H = 7.0 and both curves coincide the curve of case 3.

Fig. 4. Mole fraction contour of hydrogen φ(0.05,0.95,0.05) for different cases.

4. Conclusions

For good combustion in a supersonic combustor the efficient mixing is mandatory. Many experiments, theoretical and numerical studies have been performed on mixing, ignition and combustion in supersonic flow. In supersonic combustion, high penetration and mixing of fuel with oxidizer is difficult due to their short residence time in combustor. In the present study the effects of merging angle (ranges from 0 ~ 20º) on supersonic mixing have been studied. Due to finite base,
hydrogen and air expand behind the base creating a separation region and a recirculation region. Both hydrogen and air streams move to each other and strike behind base. The velocity in recirculation is low and therefore hydrogen has much time to contact with air resulting in high diffusion. By varying merging angle it has been found that, interaction between the two streams increases with increase of merging angle but the area of recirculation decreases. By the detail investigation of the recirculation region, it has been found that although recirculation area decreases with the increase of merging angle, high amount of hydrogen enters into the recirculation region and eventually mixing efficiency increases. Due to high interaction of the streams high momentum exchange occurs and eventually high mixing occurs at upstream for high merging angle. At high merging angle shocks created in the flow-field are strong. Due to these strong shocks, pressure loss increases as the merging angle increases. Strong interactions and shocks in high merging angles reduce the pressure in outflow boundary.

Fig. 5. Mixing efficiency based on flammability limit for different cases.

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5. References