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Response of Damaged Structure to High Speed Mass

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Abstract

The present research article analyzes the response of a single cracked cantilever beam subjected to moving load. The equation of motion of the damaged structure under moving load has been developed using continuum mechanics and Duhamel integral has been incorporated to get the solution. The deflection produced during the traversing of the load across the structure has been determined at the free end as well as each position of the mass on the beam. A computational work with various examples has been carried out for the damaged structure and the influence of numerous parameters such as speed, mass, crack depth and location on the response of the damaged structure have been analyzed.

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Keywords: single crack; Duhamel integral; moving load.

1. Introduction

Structures under moving mass have been an active research work in structural dynamics problem. The presence of cracks on the structure, crack irregularities, magnified speed and flexural rigidity affect the structure's response. So it is required to analyze the characteristics of these parameters and how it is influencing the moving mass-beam structure and impacts the structure's response. Many researchers have been working on different types of structures carrying moving mass on various end conditions and discussed their significance.

Michaltsos [1] has discussed the significance of centripetal force, Coriolis force and inertial effects on the response of a simply supported beam under moving mass. Kim et al. [2] have studied the response of different types of bridge structure to estimate the effects of vehicle mass on the measured natural frequencies using traffic induced vibration.

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Nomenclature

L	Length of the beam
b	Width of the beam
h	Thickness of the beam
l	Location of crack
a	Crack depth
α	Relative crack depth
m	Beam mass per unit length
t	Total time taken by the mass to cover the beam
δ	Dirac delta function
EI	Flexural rigidity of the undamaged beam
ω_n	Natural frequency of the beam
y	Transverse deflection of the beam
$Z_n(x)$	Function of position
$q_n(t)$	Function of time
N	Number of modes of vibration

Lin and Chang [3] have presented a theoretical expression to study the behavior of a damaged cantilever structure to intense moving mass. Al-Said [4] has developed a crack detection method in structure subjected to moving load using single natural frequency of the system. Majka and Hartnett [5] have discussed the important factors such as speed, train-to-bridge frequency ratio and damping, affecting the response of railway bridge structure.

Dehestani et al. [6] have developed a theoretical method followed by computational analysis to calculate the response of different types of structures under moving load. Bulut and Kelesoglu [7] have studied the dynamic response of beam under traversing mass using Adomian's decomposition and homotopy perturbation techniques and compare the results obtained from these two techniques. Ouyang [8] has explored a tutorial for moving mass problem for the understanding and gaining of knowledge of researchers working in the field of structural dynamics. Eftekhari and Jafari [9] have applied the combined method of differential Quadrature (DQ) and the Integral Quadrature (IQ) method to study the vibration characteristics of rectangular plates subjected to accelerated traveling masses. Nguyen [10] has carried out an analysis to identify the open and breathing cracks in damaged structures carrying moving mass. Pala and Reis [11] have investigated the significance of inertial, centripetal, and Coriolis forces on the dynamic response of cracked structure subjected to moving load. Mu and Choi [12] have examined the response of a continuous beam bridge with various kinds of track irregularities subjected to moving high-speed train. Fu [13] has discussed the importance of switching cracks on the dynamic response of continuous bridge structure along with vehicular loads. However the current analysis is dealt with the response of damaged beam with single crack carrying moving mass.

2. Problem Formulation

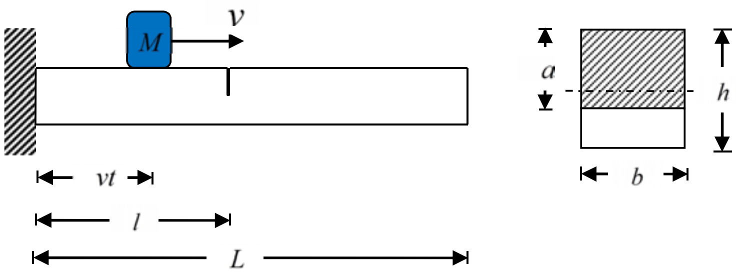

Fig. 1 Cracked Cantilever Beam with Moving

A cracked cantilever beam subjected to a moving mass is analyzed in this study. The mass ‘ M ’ is moving from the fixed end to the free end with a velocity ‘ v ’. The mechanism of the moving mass interaction structure has been shown in schematic view in Fig.1. The equation of motion of a beam subjected to a moving is given as-

$$EI\frac{\partial^4 y}{\partial x^4}+m\frac{\partial^2 y}{\partial t^2}=Mg\delta(x-vt)-M\left[\ddot{y}+v^2y''+2v\dot{y}'\right]\delta(x-vt) \tag{1}$$

Where, $\ddot{y}$ = Normal acceleration, $2v\dot{y}'$ = Coriolis's acceleration, v^2y'' = Centrifugal acceleration.

Assuming the solution of equation in series form i.e. $y(x,t)=\sum_{n=1}^{N}Z_n(x)q_n(t)$ (2)

For obtaining $Z_n(x)$ the Eq. (2) will be $Z_n^{iv}(x)-\lambda_n{}^4Z_n(x)=0,$ where $\lambda_n{}^4(x)=\dfrac{m\omega_n{}^2}{EI}$ (3)

Due to the existence of the crack and its localized effects, the beam can be stimulated into two parts. The solution of Eq. (3) can be written as

$$Z_{n1}(vt)=A_{n1}sin\ \lambda_n(vt)+B_{n1}cos\ \lambda_n(vt)+C_{n1}cinh\ \lambda_n(vt)+D_{n1}cosh\ \lambda_n(vt),\ 0\le x<l \tag{4-a}$$

$$Z_{n2}(vt)=A_{n2}sin\ \lambda_n(vt)+B_{n2}cos\ \lambda_n(vt)+C_{n2}sinh\ \lambda_n(vt)+D_{n2}cosh\ \lambda_n(vt),\ \ l\le vt<L \tag{4-b}$$

Where A_n,B_n,C_n and D_n are the constants evaluated from the different boundary conditions. Applying Eq. (2) in (1), multiplying $Z_m(x)$ and integrating it over the beam length, the equation formed can be expressed as:

$$\sum_{n=1}^{N}\left[\ddot{q}_n+\omega_n{}^2q_n\right]\int_0^L Z_m(x)Z_n(x)dx=\frac{Mg}{m}\int_0^L\delta(x-vt)Z_m(x)dx-\frac{M}{m}\left[\sum_{m=1}^{N}\ddot{q}_m\int_0^L Z_n(x)Z_m(x)\delta(x-vt)dx\right]-$$
$$\frac{M}{m}\left[\sum_{m=1}^{N}v^2q_m\int_0^L Z_n''(x)Z_m(x)\delta(x-vt)dx\right]-\frac{M}{m}\left[\sum_{m=1}^{N}2v\dot{q}_m\int_0^L Z_n'(x)Z_m(x)\delta(x-vt)dx\right] \tag{5}$$

According to the principle of orthogonality and properties of Dirac delta function, Eq. (5) can be expressed as

$$\ddot{q}_n+\omega_n^2q_n=\frac{M}{m}Z_n(vt)\left\{g-\left[\sum_{m=1}^{N}\ddot{q}_mZ_m(vt)\right]-v^2\sum_{m=1}^{N}q_mZ_m''(vt)]-\left[2v\sum_{m=1}^{N}\dot{q}_mZ_m'(vt)\right]\right\} \tag{6}$$

As the Eigen functions are orthogonal, these result in:

$$\int_0^L Z_m{}^2 dx = \int_0^L Z_n{}^2 dx = \int_0^l Z_{n1}{}^2 dx + \int_l^L Z_{n2}{}^2 dx = k \tag{7}$$

Employing Picard's method to the differential Eq. (6), the solution of the Eq. (6) is obtained considering the first term on the right side of the expression only-

$$\ddot{q}_n + \omega_n{}^2 q_n = \begin{cases} \dfrac{Mg}{m} Z_{n1}(vt) \ , \ 0<t \le \dfrac{l}{v} \\ \dfrac{Mg}{m} Z_{n2}(vt) \ , \ 0<t \le \dfrac{L}{v} \end{cases} \tag{8}$$

The solution for the homogenous part of eqn. (6)

$$\text{For } t \le \frac{l}{v} \text{ is } \left(q_{n1}\right)_h = \beta_1 \sin \omega_n t + \beta_2 \cos \omega_n t \tag{9}$$

Where β_1 and β_2 , the constant terms to be evaluated. Considering the exact solution of Eq. (6) in the form of:

$$\left(q_{n1}\right)_p = \bar{A}_{n1} \sin \Omega_n t + \bar{B}_{n1} \cos \Omega_n t + \bar{C}_{n1} \sinh \Omega_n t + \bar{D}_{n1} \cosh \Omega_n t \text{ ,where } \Omega_\mathrm{n} = \lambda_n v \tag{10}$$

Employing the values from Eq. (10) in (8),

$$\bar{A}_{n1} = \frac{Mg}{m} \frac{A_{n1}}{\left(\omega_n{}^2 - \Omega_n{}^2\right)} \ , \bar{\mathrm{B}}_{n1} = \frac{Mg}{m} \frac{\mathrm{B}_{n1}}{\left(\omega_n{}^2 - \Omega_n{}^2\right)} \ , \ \bar{\mathrm{C}}_{n1} = \frac{Mg}{m} \frac{C_{n1}}{\left(\omega_n{}^2 + \Omega_n{}^2\right)} \ , \bar{\mathrm{D}}_{n1} = \frac{Mg}{m} \frac{D_{n1}}{\left(\omega_n{}^2 + \Omega_n{}^2\right)} \tag{11}$$

The general solution of Eq. (6) for $t \le \frac{l}{v}$ may be considered in the form of:

$$q_{n1}(t) = \beta_1 \sin \omega_n t + \beta_2 \cos \omega_n t + \left(q_{n1}\right)_p \tag{12}$$

By using the initial condition $q_{n1}(0) = \dot{q}_{n1}(0) = 0$ and after simplification, it is found

$$\beta_1 = -\frac{\Omega_n}{\omega_n}(\bar{A}_{n1} + \bar{C}_{n1}) \ , \ \beta_2 = (-\bar{B}_{n1} - \bar{D}_{n1}) \tag{13}$$

The last form of $q_{n1}(t)$ can be expressed as

$$\begin{aligned} &q_{n1}(t) = q_{m1}(t) = \left(q_{n1}\right)_h + \left(q_{n1}\right)_p = \\ &\beta_1 \sin \omega_n t + \beta_2 \cos \omega_n t + \bar{A}_{n1} \sin \Omega_n t + \bar{B}_{n1} \cos \Omega_n t + \bar{C}_{n1} \sinh \Omega_n t + \bar{D}_{n1} \cosh \Omega_n t \end{aligned} \tag{14}$$

Employing Eq. (14) in (6) for the part of $t \le \frac{l}{v}$, Eq.(8)

$$\ddot{q}_{n1} + \omega_n^{\ 2} q_{n1} = \frac{2M}{mL} Z_{n1}(vt) \left\{ g - \left[\sum_{m=1}^{N} \ddot{q}_{m1} Z_{m1}(vt) \right] - v^2 \left[\sum_{m=1}^{N} q_{m1} Z''_{m1}(vt) \right] - 2v \left[\sum_{m=1}^{N} \dot{q}_{m1} Z'_{m1}(vt) \right] \right\} = Q_{n1} \tag{15}$$

The values of q_{m1} , $\dot{q}_{m1}$ and $\ddot{q}_{m1}$ are determined from eqn. (14). Employing Eq. (14) in the right part of Eq. (15), the Eq. now

$$\ddot{q}_n + \omega_n^{\ 2} q_n = Q_{n1} \quad , t \le \frac{l}{v} \tag{16}$$

From the initial conditions, the soln. of Eq. (16) can be written as

$$q_n(t) = \frac{1}{\omega_n} \int_0^t Q_{n1}(\tau) \sin \omega_n (t - \tau) d\tau \quad , t \le \frac{l}{v} \tag{17}$$

Like this way for $t \le \dfrac{L}{v}$, we have found out

$$q_n(t) = \frac{1}{\omega_n} \int_0^{l/v} Q_{n1}(\tau) \sin \omega_n (t-\tau) d\tau + \frac{1}{\omega_n} \int_{l/v}^{L/v} Q_{n2}(\tau) \sin \omega_n (t-\tau) d\tau \ , \ t \le \frac{L}{v} \tag{18}$$

The different values of $q_n(t)$ for the cracked structure are calculated.

3. Numerical Formulation and Discussion

The numerical analysis of the cracked cantilever beam with traversing mass has been carried out and explained in details from Figures 2. - 7. A mild steel beam of size (400cm×10cm×2cm) has been considered for the analysis. The moving speed ($v = 70km/h, 90km/h$), moving mass (M=3kg, 4.5kg), crack locations ($l = 150cm, 300cm$) and crack depth ratio (=0.35, 0.6) have been taken in this analysis. The beam deflections at each location of the moving mass ($x = vt$) and at the free end ($x = L$) are calculated at various crack locations, moving mass, moving speeds and crack depth ratios. From the analysis, it has been observed that with the increase of the crack depth ratio, the beam deflection also increases.

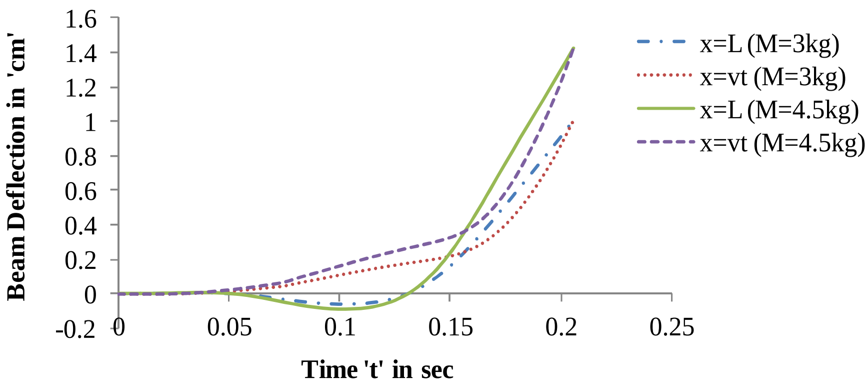

Fig. 2. For $\alpha = 0.35,\ l = 150cm,\ v = 70km/h$

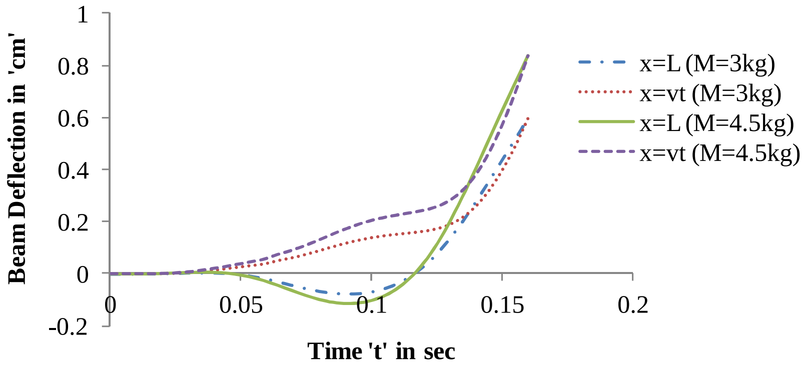

Fig. 3. For $\alpha = 0.35,\ l = 150cm,\ v = 90km/h$

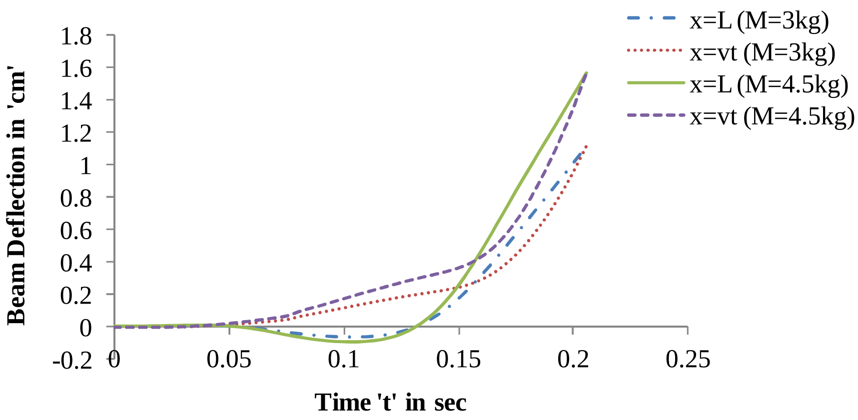

Fig. 4. For $\alpha = 0.6,\ l = 150cm,\ v = 70km/h$

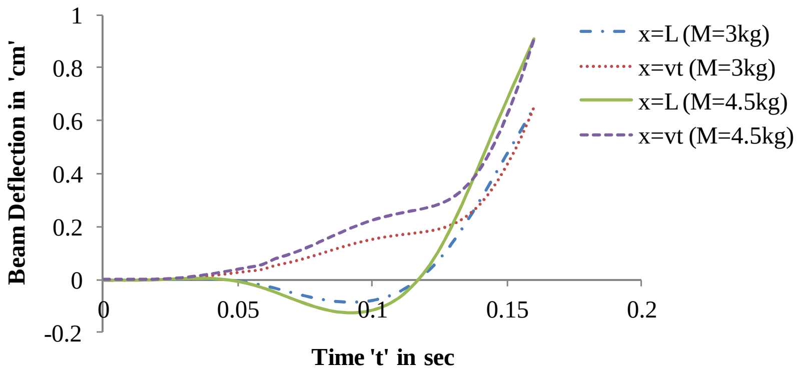

Fig. 5. For $\alpha = 0.6,\ l = 150cm,\ v = 90km/h$

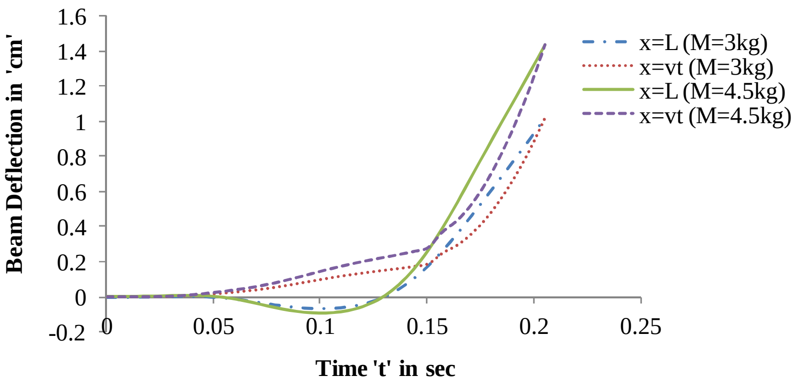

Fig. 6. For $\alpha = 0.6,\ l = 300cm,\ v = 70km/h$

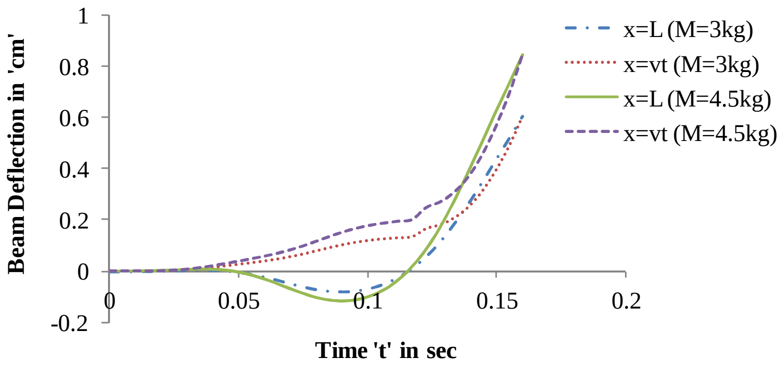

Fig. 7 For $\alpha = 0.6,\ l = 300cm,\ v = 90km/h$

As the crack location moves away from the fixed end, the deflection starts decreasing. Again with the increase of weight of the moving mass, the dynamic deflection also increases. It has been observed that (Figure 2. -7) with the increase of moving speed, the corresponding beam deflection decreases. It's because at higher speed, the lower modes of the beam are not excited. Again at higher speed, the deflection at the free end suddenly decreases and again increases in magnitude and direction. It's because the influence of higher modes of vibration are more dominant than that of lower modes. So from the above analysis, it yields that the deflection at the free end ($x = L$) is dependent upon the modal vibration of the structure whereas the deflection ($x = vt$) is dependent upon the weight of the structure.

4. Conclusions

The dynamic response of single cracked cantilever beam with moving mass has been analysed using Duhamel integral. The beam deflection produced during the traversing of the mass across the structure is calculated at various locations. Computational analysis has been carried out with numerous examples and the influence of numerous parameters such as speed, mass, crack depth and location on the response of the damaged structure have been analyzed.

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