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CPT groups for spinor field in de Sitter space

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Abstract

A group structure of the discrete transformations (parity, time reversal and charge conjugation) for spinor field in de Sitter space are studied in terms of extraspecial finite groups. Two *CPT* groups are introduced, the first group from an analysis of the de Sitter–Dirac wave equation for spinor field, and the second group from a purely algebraic approach based on the automorphism set of Clifford algebras. It is shown that both groups are isomorphic to each other.

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1. Introduction

Quantum field theory in de Sitter spacetime has been extensively studied during the past two decades with the purpose of understanding the generation of cosmic structure from inflation and the problems surrounding the cosmological constant. It is well known that discrete symmetries play an important role in the standard quantum field theory in Minkowski spacetime. In recent paper [1] discrete symmetries for spinor field in de Sitter space with the signature $(+,-,-,-,-)$ have been derived from the analysis of the de Sitter–Dirac wave equation. Discrete symmetries in de Sitter space with the signature $(+,+,+,+,-)$ have been considered in the work [2] within an algebraic approach based on the automorphism set of Clifford algebras.

In the present Letter we study a group structure of the discrete transformations in the framework of extraspecial finite groups. In Section 3 we introduce a *CPT* group for discrete symmetries in the representation of the work [1] (for more details about *CPT* groups see [3–6]). It is shown that the discrete transformations form a non-Abelian finite group of order 16. Group isomorphisms and order structure are elucidated for this group. Other realization of the *CPT* group is given in Section 4. In this section we consider an automorphism set of the Clifford algebra associated with the de Sitter space. It is proven that a *CPT* group, formed within the automorphism set, is isomorphic the analogous group considered in Section 3.

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2. Preliminaries

Usually, the de Sitter space is understood as a hyperboloid embedded in a five-dimensional Minkowski space $\mathbb{R}^{1,4}$

$$X_H = \{x \in \mathbb{R}^{1,4}\colon\ x^2 = \eta_{\alpha\beta}x^\alpha x^\beta = -H^{-2}\},\quad \alpha,\beta = 0,1,2,3,4, \tag{1}$$

where $\eta_{\alpha\beta} = \mathrm{diag}(1,-1,-1,-1,-1)$.

The spinor wave equation in the de Sitter spacetime[1] (1) has been given in the works [1,8], and can be derived via the eigenvalue equation for the second order Casimir operator,

$$(-i\not{x}\gamma\cdot\bar{\partial} + 2i + \nu)\psi(x) = 0, \tag{2}$$

where $\not{x} = \eta^{\alpha\beta}\gamma_\alpha x_\beta$ and $\bar{\partial}_\alpha = \partial_\alpha + H^2 x_\alpha x\cdot\partial$. In this case the 4×4 matrices γ_α are spinor representations of the units of the Clifford algebra $\mathcal{C}\ell_{1,4}$ and satisfy the relations

$$\gamma_\alpha\gamma_\beta + \gamma_\beta\gamma_\alpha = 2\eta_{\alpha\beta}\mathbf{1}.$$

An explicit representation[2] for γ_α chosen in [1] is

$$\gamma_0 = \begin{pmatrix}\mathbf{1}_2 & 0\\ 0 & -\mathbf{1}_2\end{pmatrix},\qquad \gamma_1 = \begin{pmatrix}0 & i\sigma_1\\ i\sigma_1 & 0\end{pmatrix},$$
$$\gamma_2 = \begin{pmatrix}0 & -i\sigma_2\\ -i\sigma_2 & 0\end{pmatrix},\qquad \gamma_3 = \begin{pmatrix}0 & i\sigma_3\\ i\sigma_3 & 0\end{pmatrix},$$
$$\gamma_4 = \begin{pmatrix}0 & \mathbf{1}_2\\ -\mathbf{1}_2 & 0\end{pmatrix}, \tag{3}$$

where $\mathbf{1}_2$ is a 2×2 unit and σ_i are Pauli matrices.

Discrete symmetries (parity transformation P, time reversal T and charge conjugation C), obtained from analysis of Eq. (2), in spinor notation have the form [1]

$$P = \eta_p\gamma_0\gamma_4,\qquad T = \eta_t\gamma_0,\qquad C = \eta_c\gamma_2, \tag{4}$$

where η_p, η_t, η_c are arbitrary unobservable phase quantities.

3. The *CPT* group

In this section we will show that the transformations (4) form a finite group of order 16, a so-called *CPT* group. Moreover, this group is a subgroup of the more large finite group associated with the algebra $\mathcal{C}\ell_{1,4}$.

As is known [10–13], a structure of the Clifford algebras admits a very elegant description in terms of finite groups. In accordance with a multiplication rule

$$\gamma_i^2 = \sigma(p-i)\mathbf{1},\qquad \gamma_i\gamma_j = -\gamma_j\gamma_i, \tag{5}$$

$$\sigma(n) = \begin{cases}-1 & \text{if } n\leqslant 0,\\ +1 & \text{if } n>0,\end{cases} \tag{6}$$

basis elements of the Clifford algebra $\mathcal{C}\ell_{p,q}$ (the algebra over the field of real numbers, $\mathbb{F}=\mathbb{R}$) form a finite group of order 2^{n+1},

$$G(p,q) = \{\pm 1, \pm\gamma_i, \pm\gamma_i\gamma_j, \pm\gamma_i\gamma_j\gamma_k, \ldots, \pm\gamma_1\gamma_2\cdots\gamma_n\}\quad (i<j<k<\cdots). \tag{7}$$

The finite group $G(1,4)$, associated with the algebra $\mathcal{C}\ell_{1,4}$, is a particular case of (7),

$$G(1,4) = \{\pm 1, \pm\gamma_0, \pm\gamma_1, \ldots, \pm\gamma_0\gamma_1\gamma_2\gamma_3\gamma_4\},$$

where γ_i have the form (3). It is a finite group of order 64 with an order structure (23, 40). Moreover, $G(1,4)$ is an *extraspecial two-group* [12,13]. In Salingaros notation the following isomorphism holds:

$$G(1,4) = \Omega_4 \simeq N_4\circ(\mathbb{Z}_2\otimes\mathbb{Z}_2) \simeq Q_4\circ D_4\circ(\mathbb{Z}_2\otimes\mathbb{Z}_2),$$

where Q_4 is a quaternion group, D_4 is a dihedral group, $\mathbb{Z}_2\otimes\mathbb{Z}_2$ is a Gauss–Klein viergroup, $\circ$ is a *central product* (Q_4 and D_4 are finite groups of order 8).

As is known, the orthogonal group $O(p,q)$ of the real space $\mathbb{R}^{p,q}$ is represented by a semidirect product $O_0(p,q)\odot\{1, P, T, PT\}$, where $O_0(p,q)$ is a connected component, $\{1, P, T, PT\}$ is a discrete subgroup (reflection group). If we take into account the charge conjugation C, then we come to the product

[1] Originally, relativistic wave equations in a five-dimensional pseudo-Euclidean space (de Sitter space) were introduced by Dirac in 1935 [7]. They have the form $(i\gamma_\mu\partial_\mu + m)\psi = 0$, where five 4×4 Dirac matrices form the Clifford algebra $\mathcal{C}\ell_{4,1}$.

[2] In general, the Clifford algebra $\mathcal{C}\ell_{1,4}$, associated with the de Sitter space $\mathbb{R}^{1,4}$, has a double quaternionic ring $\mathbb{K} = \mathbb{H}\oplus\mathbb{H}$ [9], the type $p-q\equiv(\mathrm{mod}\,8)$. For this reason, the algebra $\mathcal{C}\ell_{1,4}$ admits the following decomposition into a direct sum: $\mathcal{C}\ell_{1,4}\simeq\mathcal{C}\ell_{1,3}\oplus\mathcal{C}\ell_{1,3}$, where $\mathcal{C}\ell_{1,3}$ is a *spacetime algebra*. There is a homomorphic mapping $\epsilon: \mathcal{C}\ell_{1,4}\to{}^\epsilon\mathcal{C}\ell_{1,3}$, where ${}^\epsilon\mathcal{C}\ell_{1,3}\simeq\mathcal{C}\ell_{1,3}/\mathrm{Ker}\,\epsilon$ is a quotient algebra, $\mathrm{Ker}\,\epsilon$ is a kernel of ϵ. The basis (3) is one from the set of isomorphic spin bases obtained via the homomorphism ϵ.

Table 1

	1	γ_{04}	γ_0	γ_4	γ_2	γ_{024}	γ_{02}	γ_{24}
1	1	γ_{04}	γ_0	γ_4	γ_2	γ_{024}	γ_{02}	γ_{24}
γ_{04}	γ_{04}	1	$-\gamma_4$	$-\gamma_0$	$-\gamma_{024}$	$-\gamma_2$	γ_{24}	γ_{02}
γ_0	γ_0	γ_4	1	γ_{04}	γ_{02}	γ_{24}	γ_2	γ_{024}
γ_4	γ_4	γ_0	$-\gamma_{04}$	-1	$-\gamma_{24}$	$-\gamma_{02}$	γ_{024}	γ_2
γ_2	γ_2	$-\gamma_{024}$	$-\gamma_{02}$	γ_{24}	-1	γ_{04}	γ_0	$-\gamma_4$
γ_{024}	γ_{024}	$-\gamma_2$	γ_{24}	$-\gamma_{02}$	γ_{04}	-1	γ_4	$-\gamma_0$
γ_{02}	γ_{02}	$-\gamma_{24}$	$-\gamma_2$	γ_{024}	$-\gamma_0$	γ_4	1	$-\gamma_{04}$
γ_{24}	γ_{24}	$-\gamma_{02}$	γ_{024}	$-\gamma_2$	γ_4	$-\gamma_0$	γ_{04}	-1

$O_0(p,q)\odot\{1, P, T, PT, C, CP, CT, CPT\}$. Universal coverings of the groups $O(p,q)$ are Clifford–Lipschitz groups $\mathbf{Pin}(p,q)$ which are completely constructed within the Clifford algebras $C\!\ell_{p,q}$ [9]. It has been recently shown [4,5] that there exist 64 universal coverings of the orthogonal group $O(p,q)$:

$$\mathbf{Pin}^{a,b,c,d,e,f,g}(p,q)\simeq\frac{\mathbf{Spin}_+(p,q)\odot C^{a,b,c,d,e,f,g}}{\mathbb{Z}_2},$$

where

$$C^{a,b,c,d,e,f,g} = \{\pm 1, \pm P, \pm T, \pm PT, \pm C, \pm CP, \pm CT, \pm CPT\}$$

is *a full CPT-group*. $C^{a,b,c,d,e,f,g}$ is a finite group of order 16 (a complete classification of these groups is given in [5]). At this point, the group

$$\mathrm{Ext}(C\!\ell_{p,q})=\frac{C^{a,b,c,d,e,f,g}}{\mathbb{Z}_2}$$

is called *a generating group*.

Let us define a *CPT* group for the spinor field in de Sitter space. The invariance of the dS–Dirac equation (2) with respect to P-, T-, and C- transformations leads to the representation (4). For simplicity, we suppose that all the phase quantities are equal to the unit, $\eta_p=\eta_t=\eta_c=1$. Thus, we can form a finite group of order 8

$$\{1, P, T, PT, C, CP, CT, CPT\} \sim \{1, \gamma_0\gamma_4, \gamma_0, \gamma_4, \gamma_2, \gamma_0\gamma_2\gamma_4, \gamma_0\gamma_2, \gamma_2\gamma_4\}. \quad (8)$$

It is easy to verify that a multiplication table of this group has the form of Table 1.

Here $\gamma_{04}\equiv\gamma_0\gamma_4$, $\gamma_{024}\equiv\gamma_0\gamma_2\gamma_4$ and so on. Hence it follows that the group (8) is a non-Abelian finite group of the order structure $(3,4)$. In more details, it is the group $\overset{*}{\mathbb{Z}}_4\otimes\mathbb{Z}_2$ with the signature $(+,+,-,-,-,+,-)$. Therefore, the *CPT* group in de Sitter spacetime is

$$C^{+,+,-,-,-,+,-}\simeq\overset{*}{\mathbb{Z}}_4\otimes\mathbb{Z}_2\otimes\mathbb{Z}_2.$$

It is easy to see that $C^{+,+,-,-,-,+,-}$ is a subgroup of $G(1,4)$. In this case, the universal covering of the de Sitter group is defined as

$$\mathbf{Pin}^{+,+,-,-,-,+,-}(1,4) \simeq \frac{\mathbf{Spin}_+(1,4)\odot\overset{*}{\mathbb{Z}}_4\otimes\mathbb{Z}_2\otimes\mathbb{Z}_2}{\mathbb{Z}_2}.$$

4. Discrete symmetries and automorphisms of the Clifford algebras

Within the Clifford algebra $\mathbb{C}_n$ (the algebra over the field of complex numbers, $\mathbb{F}=\mathbb{C}$) there exist eight automorphisms [4,14] (including an identical automorphism Id). We list these transformations and their spinor representations:

$$\begin{aligned}
&\mathcal{A}\to\mathcal{A}^\star, && \mathsf{A}^\star=\mathsf{W}\mathsf{A}\mathsf{W}^{-1},\\
&\mathcal{A}\to\widetilde{\mathcal{A}}, && \widetilde{\mathcal{A}}=\mathsf{E}\mathsf{A}^{\mathsf{T}}\mathsf{E}^{-1},\\
&\mathcal{A}\to\widetilde{\mathcal{A}}^\star, && \widetilde{\mathsf{A}}^\star=\mathsf{C}\mathsf{A}^{\mathsf{T}}\mathsf{C}^{-1}, \quad \mathsf{C}=\mathsf{E}\mathsf{W},\\
&\mathcal{A}\to\bar{\mathcal{A}}, && \bar{\mathsf{A}}=\Pi\mathsf{A}^*\Pi^{-1},\\
&\mathcal{A}\to\bar{\mathcal{A}}^\star, && \bar{\mathsf{A}}^\star=\mathsf{K}\mathsf{A}^*\mathsf{K}^{-1}, \quad \mathsf{K}=\Pi\mathsf{W},\\
&\mathcal{A}\to\bar{\widetilde{\mathcal{A}}}, && \bar{\widetilde{\mathsf{A}}}=\mathsf{S}(\mathsf{A}^{\mathsf{T}})^*\mathsf{S}^{-1}, \quad \mathsf{S}=\Pi\mathsf{E},\\
&\mathcal{A}\to\bar{\widetilde{\mathcal{A}}}^\star, && \bar{\widetilde{\mathsf{A}}}^\star=\mathsf{F}(\mathsf{A}^*)^{\mathsf{T}}\mathsf{F}^{-1}, \quad \mathsf{F}=\Pi\mathsf{C},
\end{aligned}$$

where the symbol T means a transposition, and $*$ is a complex conjugation. In general, the real algebras $C\!\ell_{p,q}$ also admit all the eight automorphisms, excluding the case $p-q\equiv 0,1,2\ (\mathrm{mod}\ 8)$ when a pseudoautomorphism $\mathcal{A}\to\bar{\mathcal{A}}$ is reduced to the identical automorphism Id. It is easy to verify that an

Table 2

	1	γ_{01234}	γ_{24}	γ_{013}	γ_{13}	γ_{024}	γ_{1234}	γ_0
1	1	γ_{01234}	γ_{24}	γ_{013}	γ_{13}	γ_{024}	γ_{1234}	γ_0
γ_{01234}	γ_{01234}	1	γ_{013}	γ_{24}	γ_{024}	γ_{13}	γ_0	γ_{1234}
γ_{24}	γ_{24}	$-\gamma_{013}$	-1	$-\gamma_{01234}$	$-\gamma_{1234}$	$-\gamma_0$	γ_{13}	γ_{024}
γ_{013}	γ_{013}	γ_{24}	$-\gamma_{01234}$	-1	$-\gamma_0$	$-\gamma_{1234}$	γ_{024}	γ_{13}
γ_{13}	γ_{13}	γ_{024}	$-\gamma_{1234}$	$-\gamma_0$	-1	$-\gamma_{01234}$	γ_{24}	γ_{013}
γ_{024}	γ_{024}	γ_{13}	$-\gamma_0$	$-\gamma_{1234}$	$-\gamma_{01234}$	-1	$-\gamma_{013}$	γ_{24}
γ_{1234}	γ_{1234}	γ_0	γ_{13}	γ_{024}	γ_{24}	γ_{013}	1	γ_{01234}
γ_0	γ_0	γ_{1234}	γ_{024}	γ_{13}	γ_{013}	γ_{24}	γ_{01234}	1

automorphism set $\mathrm{Ext}(\mathbb{C}_n) = \{\mathrm{Id}, \star, \widetilde{\ }, \widetilde{\star}, \bar{\ }, \bar{\star}, \widetilde{\bar{\ }}, \widetilde{\bar{\star}}\}$ of $\mathbb{C}_n$ forms a finite group of order 8. Moreover, there is an isomorphism between $\mathrm{Ext}(\mathbb{C}_n)$ and a *CPT*-group of the discrete transformations, $\mathrm{Ext}(\mathbb{C}_n) \simeq \{1, P, T, PT, C, CP, CT, CPT\}$. In this case, space inversion P, time reversal T, full reflection PT, charge conjugation C, transformations CP, CT and the full CPT-transformation correspond to the automorphism $\mathcal{A} \to \mathcal{A}^\star$, antiautomorphisms $\mathcal{A} \to \widetilde{\mathcal{A}}$, $\mathcal{A} \to \widetilde{\mathcal{A}}^\star$, pseudoautomorphisms $\mathcal{A} \to \bar{\mathcal{A}}$, $\mathcal{A} \to \bar{\mathcal{A}}^\star$, pseudoantiautomorphisms $\mathcal{A} \to \widetilde{\bar{\mathcal{A}}}$ and $\mathcal{A} \to \widetilde{\bar{\mathcal{A}}}^\star$, respectively (for more details, see [4]).

Let us study an automorphism group of the algebra $C\ell_{1,4}$. First of all, $C\ell_{1,4}$ has the type $p-q \equiv 5 \pmod 8$, therefore, all the eight automorphisms exist. Using the γ-matrices of the basis (3), we will define elements of the group $\mathrm{Ext}(C\ell_{1,4})$. At first, the matrix of the automorphism $\mathcal{A} \to \mathcal{A}^\star$ has the form $\mathsf{W} = \gamma_0\gamma_1\gamma_2\gamma_3\gamma_4 \equiv \gamma_{01234}$. Further, since

$$\gamma_0^{\mathsf{T}} = \gamma_0, \qquad \gamma_1^{\mathsf{T}} = \gamma_1, \qquad \gamma_2^{\mathsf{T}} = -\gamma_2,$$
$$\gamma_3^{\mathsf{T}} = \gamma_3, \qquad \gamma_4^{\mathsf{T}} = -\gamma_4,$$

then in accordance with $\widetilde{\mathsf{A}} = \mathsf{E}\mathsf{A}^{\mathsf{T}}\mathsf{E}^{-1}$ we have

$$\gamma_0 = \mathsf{E}\gamma_0\mathsf{E}^{-1}, \qquad \gamma_1 = \mathsf{E}\gamma_1\mathsf{E}^{-1},$$
$$\gamma_2 = -\mathsf{E}\gamma_2\mathsf{E}^{-1}, \qquad \gamma_3 = \mathsf{E}\gamma_3\mathsf{E}^{-1},$$
$$\gamma_4 = -\mathsf{E}\gamma_4\mathsf{E}^{-1}.$$

Hence it follows that E commutes with γ_0, γ_1, γ_3 and anticommutes with γ_2 and γ_4, that is, $\mathsf{E} = \gamma_2\gamma_4$. From the definition $\mathsf{C} = \mathsf{E}\mathsf{W}$ we find that a matrix of the antiautomorphism $\mathcal{A} \to \widetilde{\mathcal{A}}^\star$ has the form $\mathsf{C} = \gamma_0\gamma_1\gamma_3$. The basis (3) contains both complex and real matrices:

$$\gamma_0^* = \gamma_0, \qquad \gamma_1^* = -\gamma_1, \qquad \gamma_2^* = \gamma_2,$$
$$\gamma_3^* = -\gamma_3, \qquad \gamma_4^* = \gamma_4.$$

Therefore, from $\bar{\mathsf{A}} = \Pi\mathsf{A}^*\Pi^{-1}$ we have

$$\gamma_0 = \Pi\gamma_0\Pi^{-1}, \qquad \gamma_1 = -\Pi\gamma_1\Pi^{-1},$$
$$\gamma_2 = \Pi\gamma_2\Pi^{-1}, \qquad \gamma_3 = -\Pi\gamma_3\Pi^{-1},$$
$$\gamma_4 = \Pi\gamma_4\Pi^{-1}.$$

From the latter relations we obtain $\Pi = \gamma_1\gamma_3$. Further, in accordance with $\mathsf{K} = \Pi\mathsf{W}$ for the matrix of the pseudoautomorphism $\mathcal{A} \to \bar{\mathcal{A}}^\star$ we have $\mathsf{K} = \gamma_0\gamma_2\gamma_4$. Finally, for the pseudoantiautomorphisms $\mathcal{A} \to \widetilde{\bar{\mathcal{A}}}$, $\mathcal{A} \to \widetilde{\bar{\mathcal{A}}}^\star$ from the definitions $\mathsf{S} = \Pi\mathsf{E}$, $\mathsf{F} = \Pi\mathsf{C}$ it follows that $\mathsf{S} = \gamma_1\gamma_2\gamma_3\gamma_4$, $\mathsf{F} = \gamma_0$. Thus, we come to the following automorphism group:

$$\begin{aligned}\mathrm{Ext}(C\ell_{1,4}) &\simeq \{\mathsf{I}, \mathsf{W}, \mathsf{E}, \mathsf{C}, \Pi, \mathsf{K}, \mathsf{S}, \mathsf{F}\} \\ &\simeq \{1, \gamma_0\gamma_1\gamma_2\gamma_3\gamma_4, \gamma_2\gamma_4, \gamma_0\gamma_1\gamma_3, \gamma_1\gamma_3, \\ &\qquad \gamma_0\gamma_2\gamma_4, \gamma_1\gamma_2\gamma_3\gamma_4, \gamma_0\}. \end{aligned} \tag{9}$$

The multiplication table of this group has the form of Table 2.

As follows from this table, the group $\mathrm{Ext}(C\ell_{1,4})$ is non-Abelian. More precisely, the group (9) is a finite group $\overset{*}{\mathbb{Z}}_4 \otimes \mathbb{Z}_2$ with the signature $(+,-,-,-,-,+,+)$. In this case we have the following universal covering:

$$\mathbf{Pin}^{+,-,-,-,-,+,+}(1,4) \simeq \frac{\mathbf{Spin}_+(1,4) \odot C^{+,-,-,-,-,+,+}}{\mathbb{Z}_2},$$

where

$$C^{+,-,-,-,-,+,+} \simeq \overset{*}{\mathbb{Z}}_4 \otimes \mathbb{Z}_2 \otimes \mathbb{Z}_2$$

is a full *CPT* group of the spinor field in de Sitter space $\mathbb{R}^{1,4}$. In turn, $C^{+,-,-,-,-,+,+}$ is a subgroup of $G(1,4)$.

Moreover, we see that the generating group (8) and (9) are isomorphic,

$$\{1,\ P,\ T,\ PT,\ C,\ CP,\ CT,\ CPT\} \simeq \{\mathsf{I},\ \mathsf{W},\ \mathsf{E},\ \mathsf{C},\ \Pi,\ \mathsf{K},\ \mathsf{S},\ \mathsf{F}\} \simeq \overset{*}{\mathbb{Z}}_4 \otimes \mathbb{Z}_2.$$

Thus, we come to the following result: the finite group (8), derived from the analysis of invariance properties of the dS–Dirac equation with respect to discrete transformations C, P and T, is isomorphic to the automorphism group of the algebra $\mathcal{C}\ell_{1,4}$. This result allows us to study discrete symmetries and their group structure for physical fields without handling to analysis of relativistic wave equations.

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