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Explaining why the u and d quark masses are similar

S.M. Barr, I. Dorsner

Bartol Research Institute, University of Delaware, Newark, DE 19716, USA

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Abstract

An approach is suggested for modeling quark and lepton masses and mixing in the context of grand unified theories that explains the curious fact that $m_u \sim m_d$ even though $m_t \gg m_b$. The structure of the quark mass matrices is such as to allow a non-Peccei–Quinn solution of the Strong CP Problem.

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It is well known that the grand unification group $SU(5)$ relates the mass matrices of the down quarks and charged leptons. There is some empirical support for the existence of such a relationship in the fact that when the fermion masses are extrapolated to the GUT scale in the MSSM one finds $m_b \cong m_\tau$ [1,2], $m_s \cong m_\mu/3$, and $m_d \cong 3m_e$ [3]. However, the pattern of masses of the up quarks is very different. One difference is that the t mass is much greater than the b and τ masses, which is usually explained by saying that the ratio of VEVs $v_u/v_d \equiv \tan\beta$ is large compared to one. Another difference is that the interfamily mass hierarchies are much stronger for the up quarks than for the down quarks and charged leptons (e.g., $m_c/m_t \ll m_s/m_b$ and $m_u/m_c \ll m_d/m_s$). It is tempting to say that the up quark mass matrix (M_U) is more distantly related to the down quark and charged lepton mass matrices (M_D and M_L) than the latter are to each other. On the other hand, there is the tantalizing fact that $m_u/m_d \sim 1$.

In this Letter we suggest a somewhat new approach which qualitatively explains why $m_t \gg m_b$ but $m_u \sim m_d$. The idea is that there are "underlying" mass matrices (denoted by the superscript zero) whose structure is controlled by $SO(10)$ and which satisfy $M_U^0 \sim M_D^0 \sim M_L^0 \sim M_N^0$ (it is assumed $v_u/v_d \sim 1$), but that a strong mixing of the third family with vectorlike fermions at the GUT scale distorts these underlying mass matrices in such a way that m_b and m_τ are highly suppressed relative to m_t. This distortion does not affect the first family much, so the masses m_u, m_d, and m_e remain of the same order.

The approach we will describe has several other virtues: (a) It can be realized in models with very few parameters. (b) It dovetails with the ideas of Ref. [4] for solving the Strong CP Problem. And (c) it implements the "lopsided" mass matrix approach to explaining large neutrino mixing angles [5,6].

E-mail addresses: smbarr@bxclu.bartol.udel.edu (S.M. Barr), dorsner@physics.udel.edu (I. Dorsner).

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To understand the idea, consider an $SU(5)$ model (which will later be assumed to descend from $SO(10)$) that has three families of fermions in $\mathbf{10}_i + \bar{\mathbf{5}}_i + \mathbf{1}_i$, with mass terms of the form $(M_U^0)_{ij}\mathbf{10}_i\mathbf{10}_j + (M_D^0)_{ij}\mathbf{10}_i\bar{\mathbf{5}}_j + (M_L^0)_{ij}\bar{\mathbf{5}}_i\mathbf{10}_j + (M_N^0)_{ij}\bar{\mathbf{5}}_i\mathbf{1}_j$. The matrices M_D^0 and M_L^0 come from the VEV of a $\bar{\mathbf{5}}$ of Higgs, and M_U^0 and M_N^0 (the neutrino Dirac mass matrix) come from the VEV of a $\mathbf{5}$ of Higgs. Suppose that there are also for each family a vectorlike pair of quark/lepton multiplets, denoted $\bar{\mathbf{5}}'_i$ and $\mathbf{5}'_i$ and having superheavy mass terms $A_{ij}\bar{\mathbf{5}}'_i\mathbf{5}'_j + B_{ij}\bar{\mathbf{5}}_i\mathbf{5}'_j$. ($A_{ij} \sim B_{ij} \sim M_{\rm GUT}$.) There is then mixing between the ordinary three families and the vectorlike fermions, more specifically the mixing is between the $\bar{\mathbf{5}}_i$ and the $\bar{\mathbf{5}}'_i$. (A similar idea, but with mixing among fermions in $\mathbf{10}$s of $SU(5)$ was used in [5]. However, the models proposed there were very different in character from the present models.) With the mass terms specified above, we may write

$$(\bar{\mathbf{5}}\bar{\mathbf{5}}')\mathcal{M}_F\begin{pmatrix}\mathbf{10}\\ \mathbf{5}'\end{pmatrix} = (\bar{\mathbf{5}}_i\bar{\mathbf{5}}'_i)\begin{pmatrix}(M_F^0)_{ij} & B_{ij}\\ 0 & A_{ij}\end{pmatrix}\begin{pmatrix}\mathbf{10}_j\\ \mathbf{5}'_j\end{pmatrix}, \tag{1}$$

where $F = L$ or D, and M_F^0 is either M_L^0 or $(M_D^0)^T$, depending on whether the fermions in $\bar{\mathbf{5}}$ are ℓ_L^- or d_L^c.

In order to find the light fermion mass matrices in the effective low-energy theory, we must do a unitary transformation $\mathcal{M}_F = \mathcal{U}\mathcal{M}_F^0$ that eliminates the off-diagonal block B in the full mass matrix given in Eq. (1). Such a transformation is

$$\mathcal{U} = \begin{pmatrix}\Lambda & -\Lambda x\\ x^\dagger\bar{\Lambda} & \bar{\Lambda}\end{pmatrix}, \tag{2}$$

where $x \equiv BA^{-1}$, $\Lambda \equiv (I + xx^\dagger)^{-1/2}$, and $\bar{\Lambda} = (I + x^\dagger x)^{-1/2}$. (To check the unitarity of $\mathcal{U}$ it is useful to note that $x^\dagger\Lambda = \bar{\Lambda}x^\dagger$ and $x\bar{\Lambda} = \Lambda x$.) This gives the result for the low energy mass matrices

$$M_L = \Lambda M_L^0, \tag{3a}$$

$$M_D = M_D^0\Lambda^T. \tag{3b}$$

Basically, the Hermitian matrix Λ describes the mixing of $\bar{\mathbf{5}}_i$ with $\bar{\mathbf{5}}'_i$. It appears on the left in the equation for M_L since $(M_L)_{ij}$ couples to $\bar{\mathbf{5}}_i\mathbf{10}_j$. It appears on the right in the equation for M_D since $(M_D)_{ij}$ couples to $\mathbf{10}_i\bar{\mathbf{5}}_j$. For the Dirac neutrino masses we have

$$(\bar{\mathbf{5}}\bar{\mathbf{5}}')\mathcal{M}_N\begin{pmatrix}\mathbf{1}\\ 5'\end{pmatrix} = (\bar{\mathbf{5}}_i\bar{\mathbf{5}}'_i)\begin{pmatrix}(M_N^0)_{ij} & B_{ij}\\ 0 & A_{ij}\end{pmatrix}\begin{pmatrix}\mathbf{1}_j\\ \mathbf{5}'_j\end{pmatrix}, \tag{4}$$

giving

$$M_N = \Lambda M_N^0. \tag{5}$$

Since the masses of the up-type quarks come from a $\mathbf{10}_i\mathbf{10}_j$ coupling of the fermions, they are not affected by the mixing of the $\bar{\mathbf{5}}_i$ with the $\bar{\mathbf{5}}'_i$. Consequently,

$$M_U = M_U^0. \tag{6}$$

Before we discuss how the structure we have described can help us explain the magnitudes of quark and lepton masses and mixings, we note that it is exactly the kind of structure that is used in the solution of the Strong CP Problem proposed in [4]. The idea there was the following. Suppose that CP is a symmetry of the Lagrangian that is spontaneously broken, and that the VEV that breaks CP appears in the off-diagonal matrix B in Eq. (1), but not elsewhere in the quark mass matrices. Then M_D^0 and A are real, and it is easily shown that the determinant of the full mass matrix $\mathcal{M}_D$ is therefore real. Also real, of course, is the determinant of M_U. Thus, at tree level, the phase $\bar{\theta}$ is zero. At higher order, these matrices can receive complex corrections that induce a non-vanishing $\bar{\theta}$, but these may be made small. (In SUSY, there can be contributions to the $\bar{\theta}$ parameter that are harder to make small, for example, one-loop corrections to the gluino mass [7]. How large these are depends upon how SUSY is broken. These contributions are not a problem in theories with gauge-mediated SUSY breaking, for example. We imagine that whatever mechanism resolves the usual SUSY flavor and SUSY CP problems will also suppress these extra contributions to $\bar{\theta}$.) On the other hand, since B is a complex matrix, so is the matrix $x = BA^{-1}$ and the matrix $\Lambda = (I + xx^\dagger)^{-1/2}$. Consequently, the mass matrix of the light three families of down-type quarks in the effective low-energy theory, given by $M_D = M_D^0\Lambda^T$, is also complex, which means that in general there is a non-vanishing Kobayashi–Maskawa phase.

In short, the structure in Eq. (1) allows a spontaneously generated phase in the matrix B to contribute

to $\delta_{\rm KM}$ but not at tree level to $\bar{\theta}$. This can also enhance the predictivity of models by reducing the number of parameters, since one can assume that all parameters in M_L^0, M_D^0, M_U^0, M_N^0, and the right-handed Majorana matrix M_R are real, and that the only phase (and only one is needed) comes from Λ. This is the assumption we shall make in the illustrative model we present below.

Returning to the issue of mass and mixing hierarchies, let us assume that the matrix Λ that characterizes the mixing of $\bar{\mathbf{5}}_i$ with $\bar{\mathbf{5}}'_i$, has the form

$$\Lambda \cong \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \lambda t \\ 0 & \lambda t^* & \lambda \end{pmatrix}, \tag{7}$$

where the real parameter $\lambda \ll 1$ and the complex parameter t has magnitude of order one. As we shall see shortly, it is the smallness of λ that gives rise to $m_b, m_\tau \ll m_t$, while the $|t| \sim 1$ explains the large atmospheric neutrino mixing. The phase of t, the only phase in the model, is what produces the KM phase. We shall see later that the form in Eq. (7) is easy to obtain.

To illustrate our basic approach we now present a toy model in which the underlying mass matrices have the following simple "textures":

$$\begin{aligned} M_U^0 &= \begin{pmatrix} 0 & \delta & \delta' \\ \delta & \epsilon_u & 0 \\ \delta' & 0 & 1 \end{pmatrix} m_U, \\ M_D^0 &= \begin{pmatrix} 0 & \delta & 0 \\ \delta & \epsilon_d & 0 \\ 0 & 0 & 1 \end{pmatrix} m_D, \\ M_N^0 &= \begin{pmatrix} 0 & \delta & \delta' \\ \delta & 3\epsilon_u & 0 \\ \delta' & 0 & 1 \end{pmatrix} m_U, \\ M_L^0 &= \begin{pmatrix} 0 & \delta & 0 \\ \delta & 3\epsilon_d & 0 \\ 0 & 0 & 1 \end{pmatrix} m_D, \end{aligned} \tag{8}$$

where $\delta, \delta' \ll \epsilon_u, \epsilon_d \ll 1$. The similarity of these four matrices is assumed to come from $SO(10)$. In $SO(10)$ one would have the $\mathbf{10}_i + \bar{\mathbf{5}}_i + \mathbf{1}_i$ come from a $\mathbf{16}_i$, whereas the extra vectorlike fermions $\bar{\mathbf{5}}'_i + \mathbf{5}'_i$ could come from a $\mathbf{10}_i$.

The textures in Eq. (8) can be obtained from simple $SO(10)$ operators. In particular, we assume that the 33 elements come from a term of the form $h_{33}\mathbf{16}_3\mathbf{16}_3\langle\mathbf{10}_H\rangle$. Thus, what we have called m_U and m_D in Eq. (8) are given by $m_U = h_{33}\langle H_u(\mathbf{10})\rangle$, and $m_D = h_{33}\langle H_d(\mathbf{10})\rangle$. If the two Higgs doublets of the MSSM came purely from the $\mathbf{10}_H$, i.e., if $H_u = H_u(\mathbf{10})$ and $H_d = H_d(\mathbf{10})$, then we would have $m_U/m_D = \tan\beta$. However, one expects in a realistic $SO(10)$ model that H_u and H_d will come from a mixture of several $SO(10)$ Higgs multiplets. Thus, we may write $H_d = \cos\gamma_d H_d(\mathbf{10}) + \sin\gamma_d H_d(\text{other})$ and $H_u = \cos\gamma_u H_u(\mathbf{10}) + \sin\gamma_u H_u(\text{other})$. Inverting these, we obtain $m_D = h_{33} v \cos\gamma_d \cos\beta$ and $m_U = h_{33} v \cos\gamma_u \sin\beta$. Therefore, the usual $\tan\beta$ parameter of the MSSM is given by $\tan\beta = (m_U/m_D)(\cos\gamma_d/\cos\gamma_u)$. From Eq. (8) one sees that the top quark mass is $m_t \cong m_U$. Therefore, $1/\cos\gamma_u \cong h_{33}(v/m_t)\sin\beta$, and we may write $\tan\beta = (m_U/m_D)[\cos\gamma_d(v/m_t) \times \sin\beta]h_{33}$. The expression in the square brackets is less than or equal to 1, and h_{33}, which is a Yukawa coupling in the $SO(10)$ theory, cannot be much larger than 1 without destroying the perturbativity of the theory below the Planck scale. Thus, the value of the parameter m_U/m_D, which can be determined by fitting the quark and lepton masses, puts an upper bound on $\tan\beta$. We shall find that $m_U/m_D \cong 2$, so with $h_{33} = 1.5$ to 2, the value of $\tan\beta$ is consistent with the experimental lower limit of 3 [8]

Now, given Eqs. (3), (7), and (8), one has

$$M_D = \begin{pmatrix} 0 & \delta & \delta\lambda t^* \\ \delta & \epsilon_d & \epsilon_d \lambda t^* \\ 0 & \lambda t & \lambda \end{pmatrix} m_D, \tag{9}$$

and

$$M_L = \begin{pmatrix} 0 & \delta & 0 \\ \delta & 3\epsilon_d & \lambda t \\ \delta\lambda t^* & 3\epsilon_d \lambda t^* & \lambda \end{pmatrix} m_D. \tag{10}$$

Of course, from Eq. (6) one sees that M_U is already given in Eq. (8).

Simply by inspecting these matrices one can observe several significant facts. First, the masses of m_b and m_τ are suppressed by the small parameter λ, whereas m_t is not, so that $m_b, m_\tau \ll m_t$ can be explained without requiring that m_U/m_D be extremely large. Second, the masses of the first family will be almost unaffected by the parameter λ, so that m_d and m_e will not be similarly suppressed compared to m_u. Indeed for m_U/m_D of order one, $m_u \sim m_d$, as observed. Third, there emerges naturally the "lopsided" structure discussed in many recent papers [6]. That is, we see that the 23 element of M_L is much larger than its

32 element, whereas for M_D the opposite is the case. This comes directly from the fact that $M_D = M_D^0 \Lambda^T$ whereas $M_L = \Lambda M_L^0$. This lopsided structure explains why the atmospheric neutrino mixing angle (which gets a contribution from $(M_L)_{23}/(M_L)_{33}$) is of order $|t| \sim 1$, whereas the corresponding quark mixing V_{cb} (which gets a contribution from $(M_D)_{23}/(M_D)_{33}$) is only of order $\epsilon_d |t| \ll 1$.

One can read off from the simple forms in Eqs. (9) and (10) the following approximate relations that hold at the GUT scale:

$$
\begin{aligned}
&m_t \cong m_U, \qquad m_c \cong \epsilon_u m_U, \qquad m_u \cong \bigl(\delta^2/\epsilon_u\bigr) m_U,\\
&m_b \cong \lambda\sqrt{1+|t|^2}\, m_D, \qquad m_s \cong \bigl(\epsilon_d/\sqrt{1+|t|^2}\bigr) m_D,\\
&m_d \cong \bigl(\delta^2/\epsilon_d\bigr) m_D,\\
&m_\tau \cong \lambda\sqrt{1+|t|^2}\, m_D, \qquad m_\mu \cong \bigl(3\epsilon_d/\sqrt{1+|t|^2}\bigr) m_D,\\
&m_e \cong \bigl(\delta^2/3\epsilon_d\bigr) m_D,
\end{aligned}
\tag{11}
$$

and

$$
\begin{aligned}
V_{cb} &\cong (\epsilon_d/\lambda)\left(\frac{t}{1+|t|^2}\right) \cong (m_s/m_b)t,\\
V_{us} &\cong (\delta/\epsilon_d) - (\delta/\epsilon_u)\\
&\cong \sqrt{m_d/m_s}\bigl(1+|t|^2\bigr)^{-1/4} \pm \sqrt{m_u/m_c},\\
V_{ub} &\cong -\delta' + \left(\frac{\delta}{\lambda}\frac{t}{1+|t|^2}\right)\left(1-\frac{\epsilon_d}{\epsilon_u}\right)\\
&\cong -\delta' + V_{us} V_{cb}.
\end{aligned}
\tag{12}
$$

From the form of the M_L (Eq. (10)) and M_N (Eqs. (5) and (8)) it can be seen that the tangent of the atmospheric neutrino mixing angle is controlled by t, which therefore must be of order one. That in turn implies, through the equation for V_{cb}, that $V_{cb} \sim m_s/m_b$. This succesful qualitative relation between the atmospheric neutrino angle, V_{cb} and m_s/m_b is characteristic of lopsided models.

In this model there are eight parameters (m_U/m_D, ϵ_u, ϵ_d, δ, δ', λ, $|t|$, and θ_t) to fit twelve quantities (eight mass ratios of charged leptons and quarks, three CKM angles, and the KM phase). There are therefore four quantitative predictions, which can be taken to be $m_b \cong m_\tau$, $m_s \cong m_\mu/3$, $m_d \cong 3m_e$, and the value of the Cabibbo angle. In addition, the atmospheric angle is predicted to be of order one, though it cannot be predicted more precisely than that without knowing M_R.

We can easily determine the approximate values of most of the parameters of the model from Eqs. (11) and (12). We take the values of the quark and lepton masses and CKM mixings at the GUT scale to be $m_t = 112$ GeV, $m_b = 0.96$ GeV, $m_\tau = 1.16$ GeV, $m_c = 0.27$ GeV, $m_s = 0.015$ GeV, $m_\mu = 0.069$ GeV, $m_u = 0.57$ MeV, $m_d = 0.86$ MeV, $m_e = 0.334$ MeV, $|V_{cb}| = 0.0357$, and $|V_{us}| = 0.222$. These are found by extrapolating experimentally determined central values at low scale [9] to the GUT scale using the following procedure. First, we propagate the masses of light quarks and leptons from 2 GeV scale to M_Z scale using the 3-loop QCD and 1-loop QED renormalization group equations (RGEs). Then, we perform additional running from M_Z to m_t scale using the Standard Model RGEs. (The relevant renormalization-group β functions are summarized in Ref. [10].) Finally, assuming all SUSY particle masses to be degenerate at m_t we run the masses and mixings to the GUT scale $M_{\rm GUT} \approx 2 \times 10^{16}$ GeV using the 2-loop MSSM β functions summarized in Ref. [11]. In the final running we set $\tan\beta = 3$.

The equation for V_{cb} tells us immediately that $|t| \cong V_{cb} m_b/m_s \approx 2$. From Eq. (11) one has that $(m_u m_c)/(m_d m_s) \cong (m_U/m_D)^2 \sqrt{1+|t|^2}$, which implies that $m_U/m_D \approx 2$. The equation $\lambda \cong (m_b/m_t) \times (m_U/m_D)/\sqrt{1+|t|^2}$ then gives $\lambda \approx 10^{-2}$.

The value of $|\epsilon_u|$ is given approximately by $\sqrt{m_c/m_t} \cong 2.4 \times 10^{-3}$. The equation

$$
\epsilon_d \cong \frac{1}{3}(m_\mu/m_\tau)\lambda\bigl(1+|t|^2\bigr)
$$

gives $|\epsilon_d| \approx 10^{-3}$. It is gratifying that ϵ_u and ϵ_d come out to be of the same order. If we choose the relative sign of ϵ_u and ϵ_d to be negative, then we get a good fit to the Cabibbo angle: $V_{us} \cong \sqrt{m_d/m_s}(1+|t|^2)^{-1/4} + \sqrt{m_u/m_c} \cong (0.2)(0.7) + (0.05) = 0.2$.

The value of δ is determined from $\delta^2 \cong m_u m_c/m_t^2$ to be 10^{-4}. Finally, the parameter δ' and the phase of t can be determined from the real and imaginary parts of V_{ub}. Specifically, one has

$$
V_{ub}/(V_{us} V_{cb}) = 1 - \delta' e^{-i\theta_t}/|V_{us} V_{cb}|. \tag{13}
$$

One gets a fairly reasonable fit from the following values of the parameters of the model: $m_U/m_D = 2.03$, $\lambda = 1.03 \times 10^{-2}$, $|t| = 1.45$, $\epsilon_U = 2.38 \times 10^{-3}$, $\epsilon_D = -2.14 \times 10^{-3}$, $\delta = 1.12 \times 10^{-4}$. The resulting masses and mixings and the experimental

Table 1
The values of the quark and charged lepton masses and the CKM angles V_{cb} and V_{us} at the GUT scale in the model (with parameter values given in text), compared to the experimental values extrapolated to the GUT scale. Extrapolation is done taking all SUSY particles to be degenerate at m_t and assuming $\tan\beta = 3$. Masses are given in units of GeV

	Model	Experiment
m_u	0.000587	0.000570
m_c	0.268	0.269
m_t	112	112
m_d	0.00092	0.00086
m_s	0.0238	0.0150
m_b	0.998	0.956
m_e	0.000318	0.000334
m_μ	0.0684	0.0690
m_τ	1.00	1.16
$\|V_{us}\|$	0.19	0.22
$\|V_{cb}\|$	0.032	0.036

values extrapolated to the GUT scale are compared in Table 1. It is apparent that the fit is not completely satisfactory. In particular, the mass of the τ comes out about 15% too small. This is typical of grand unified theories. Simple GUTs generally predict $m_b = m_\tau$ at the GUT scale, whereas the data tend to give m_τ about 15 to 20% larger than m_b. There are a number of ways of improving the agreement, including supposing that m_b gets corrections from sparticle loops. Also off considerably here is m_s. The Georgi–Jarlskog relation $m_s \cong m_\tau/3$ is built into a choice of Clebsch in this toy model. But that relation is known to give a value of m_s that is somewhat large compared to the values favored by recent lattice calculations [12].

While this model does not give a perfect fit, it is simple enough to illustrate the basic idea we are proposing in a transparent way. It seems likely that models based on these ideas can be found that give better fits. Another possibility that might be realized in this approach is a "doubly lopsided" model [13]. One could imagine, for example, that the matrix Λ had the form

$$\Lambda \cong \begin{pmatrix} 1 & 0 & \lambda t_1 \\ 0 & 1 & \lambda t_2 \\ \lambda t_1^* & \lambda t_2^* & \lambda \end{pmatrix}, \tag{14}$$

with $\lambda \ll 1$ and $|t_i| \sim 1$. If the underlying matrix M_L^0 has a hierarchical form, with the 33 element being the largest, then the effective low-energy mass matrix $M_L = \Lambda M_L^0$ would have the doubly lopsided form, with the 13, 23, and 33 elements all being of the same order. This is known to be able to give in a simple way the correct "bi-large" pattern of neutrino mixing angles, with U_{e3} being small [13].

We now turn to the question of whether the form of Λ given in Eq. (7) can arise naturally. Consider the special case where B is diagonal and where the only non-zero elements of A are the diagonal elements and the 23 element: $B_{ij} = b_i\delta_{ij}$, $A_{ij} = a_i\delta_{ij} + a_4\delta_{i2}\delta_{j3}$. Then it is easily found that for $b_1 < a_1$, $b_2 < a_2$, and $b_3 > a_3$, the matrix Λ has the form given in Eq. (7) with $\lambda \cong |a_3|^2/(|a_3|^2 + |b_3|^2)$, and $t = -(b_3^* b_2 a_4)/(a_2|a_3|^2)$. Of course, there are other forms of A and B that also give Eq. (7). Another simple example is that A is diagonal and B has non-zero diagonal elements and 23 element.

In conclusion, we have found a framework that differs from most "texture" models of quark and lepton masses in several respects. First, it can partially explain the fact, usually treated as an accident, that $m_u \sim m_d, m_e$, while also giving $m_t \gg m_b, m_\mu$. This it does, not by requiring $\tan\beta$ to be large, which might be somewhat unnatural, but by mixing the b and τ strongly with vectorlike fermions at the GUT scale. Second, it combines predictive textures with a structure that realizes a non-axion solution to the strong CP problem proposed many years ago [4]. By allowing most of the parameters to be real, even though CP is violated, it has the potential of giving very predictive models. And it gives rise naturally to the "lopsided" kind of structure that has been proposed to explain the largeness of $U_{\mu 3}$ relative to V_{cb} [6].

The toy model we have described illustrates the essential ideas in a transparent way. However, it would

be good to find a model which is more predictive and which does a better job fitting certain quantities, especially m_s. It would also be interesting to investigate further models of this type that are "doubly lopsided" [5,13].

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