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ABSTRACT

We consider nontrivial critical models in $d = 6 + \epsilon$ spacetime dimensions with anticommuting scalars transforming under the symplectic group $\text{Sp}(N)$. These models are nonunitary, but the couplings are real and all operator dimensions are positive. At large N we can take $\epsilon \rightarrow 1$ consistently with the loop expansion and thus provide evidence that these theories may be used to define critical models in $d = 7$. The relation of these theories to critical $\text{Sp}(N)$ theories, defined similarly to the well-known critical $\text{O}(N)$ theories, is examined, and some similarities are pointed out.

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1. Introduction

Conformal field theories (CFTs) in $d = 2$ spacetime dimensions are abundant and their properties have been studied extensively. As we consider higher spacetime dimensions it becomes harder to find nontrivial CFTs, and if we require supersymmetry it becomes impossible beyond $d = 6$ [1]. In this short paper we will give up on some basic properties of CFTs in $d \leq 6$ in order to provide evidence for the existence of (perhaps unconventional) interacting CFTs in $d = 7$.

Our considerations are inspired by recent work of Fei et al., who analyzed $\text{O}(N)$ and $\text{Sp}(N)$ models in $d = 6 - \epsilon$ in great detail [2,3]. For the $\text{Sp}(N)$ case they worked with the Lagrangian

$$\mathcal{L} = \frac{1}{2} \Omega_{ij} \partial^\mu \chi^i \partial_\mu \chi^j + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma + \frac{1}{2} g \Omega_{ij} \chi^i \chi^j \sigma + \frac{1}{6} h \sigma^3, \quad (1)$$

where Ω_{ij} is the invariant symplectic matrix. The scalar fields χ are anticommuting, and so the theory is not unitary, while the scalar field σ is commuting. We point out that in $d = 6 + \epsilon$ these $\text{Sp}(N)$ models have UV fixed points at real values of the couplings and with positive operator dimensions. The corresponding theories have a potential that is unbounded from below, but within perturbation theory the vacuum configuration $\chi = \sigma = 0$ is stable. This is similar to the situation in [2].

A critical theory with $\text{O}(N)$ symmetry and commuting scalars that can formally be defined in any d also exists, and its central charge has been computed analytically in d at leading order

in $1/N$ [4,5]. If we send $N \rightarrow -N$ in the answer, then this gives the central charge of the corresponding critical theory with $\text{Sp}(N)$ symmetry and anticommuting scalars. For $d = 7$ the value of the central charge indicates the presence of an interacting fixed point.

It is not clear if the corresponding CFT is related to the theory defined at the critical point of (1) we discuss in this paper. In the context of the $1/N$ expansion one can instead study the (Euclidean) theory

$$\mathcal{L}_{\text{sym}} = \frac{1}{2} \Omega_{ij} \partial^\mu \chi^i \partial_\mu \chi^j + \frac{1}{4} \lambda (\Omega_{ij} \chi^i \chi^j)^2. \quad (2)$$

(For details of the corresponding $\text{O}(N)$ theory at large N the reader is referred to [6].) Typically one does this in $d = 4 - \epsilon$ dimensions, but one can also use $d = 4 + \epsilon$. In the latter case we can take $\epsilon \rightarrow 3$ at large N and we have a UV fixed point as well. This fixed point is nonunitary due to the anticommuting scalars, and there is a violation of the unitarity bound for χ . Nevertheless it has real coupling λ and positive operator dimensions at large N , so it is very similar in nature to the nontrivial fixed point of (1).

The paper is organized as follows. In the next section we analyze the fixed points of (1) borrowing heavily on results of [2, 7,3]. In Section 3 we consider the central charge of the critical $\text{Sp}(N)$ models in $d > 6$, and we speculate on the relation of the corresponding CFT to the theory at the critical point of (1). We make some comments on the sphere free energy and the F -theorem [8–10] in Section 4, and we conclude in Section 5.

2. Fixed points

In $d = 6 + \epsilon$ and at one loop the beta functions for g and h are [11,2,3,12]

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$$\beta_g = \frac{\epsilon}{2} g - \frac{1}{12} \frac{1}{64\pi^3} g((N+8)g^2 + 12gh - h^2), \quad (3a)$$

$$\beta_h = \frac{\epsilon}{2} h + \frac{1}{4} \frac{1}{64\pi^3} (4Ng^3 - Ng^2h - 3h^3). \quad (3b)$$

In the large- N limit we have

$$\beta_{g, N \gg 1} = \frac{\epsilon}{2} g - \frac{1}{12} \frac{1}{64\pi^3} Ng^3, \quad (4a)$$

$$\beta_{h, N \gg 1} = \frac{\epsilon}{2} h + \frac{1}{4} \frac{1}{64\pi^3} Ng^2(4g - h), \quad (4b)$$

and it is easy to find nontrivial fixed points. For these fixed points higher loop corrections in (4a) and (4b) can be neglected consistently at large N , for, due to the interactions in (1), each higher order in perturbation theory generates contributions to the beta function that are at most linear in N . One nontrivial fixed point at large N is at

$$g_* = 8\sqrt{6}\pi^{3/2} \sqrt{\frac{\epsilon}{N}}, \quad h_* = 6g_*, \quad (5)$$

and there is also an equivalent fixed point at $(-g_*, -h_*)$. Since $\epsilon, N > 0$ these fixed points occur for real values of the couplings. Corrections in powers of $1/N$ that give solutions to $\beta_g = \beta_h = 0$ can also be computed and give [2]

$$g_* = 8\sqrt{6}\pi^{3/2} \sqrt{\frac{\epsilon}{N}} \left(1 - \frac{22}{N} + \dots\right),$$

$$h_* = 48\sqrt{6}\pi^{3/2} \sqrt{\frac{\epsilon}{N}} \left(1 - \frac{162}{N} + \dots\right), \quad (6)$$

while higher loop corrections have been considered in [11,12,7]. Note that the next-to-leading-order result in $1/N$ in (6) is sensitive to higher-loop corrections in the limit $\epsilon \rightarrow 1$ [7]. These corrections are $\mathcal{O}(\epsilon^{3/2})$ in (6).

The eigenvalues of the stability matrix at the fixed point (5) are negative (both equal to $-\epsilon$), so these are UV fixed points. The trivial fixed point is an IR fixed point. A ‘‘UV completion’’ of the nontrivial fixed points, so that they appear as IR fixed points of another theory needs to be considered, although at this point the existence of such a ‘‘UV completion’’ is unclear. Following the example of [2] one may speculate that this may be found starting from a theory in $d = 8 - \epsilon$.

The anomalous dimensions of the fields at one loop are given by [11,2,3,12]

$$\gamma_\chi = \frac{1}{6} \frac{1}{64\pi^3} g^2, \quad (7a)$$

$$\gamma_\sigma = -\frac{1}{12} \frac{1}{64\pi^3} (Ng^2 - h^2). \quad (7b)$$

At the fixed point (6) we have

$$\gamma_{\chi*} = \frac{\epsilon}{N} \left(1 - \frac{44}{N} + \dots\right),$$

$$\gamma_{\sigma*} = -\frac{\epsilon}{2} + \frac{40\epsilon}{N} \left(1 - \frac{170}{N} + \dots\right). \quad (8)$$

Neglecting higher-loop effects (of $\mathcal{O}(\epsilon^2)$ in (8)) and using the result up to order $1/N$ we see, in the limit $\epsilon \rightarrow 1$, that the anomalous dimension of σ is negative if $N \geq 80$. In that case the unitarity bound is violated for σ . Nevertheless, the violation is mild and the dimension of σ is positive. If three-loop effects are taken into account then [7]

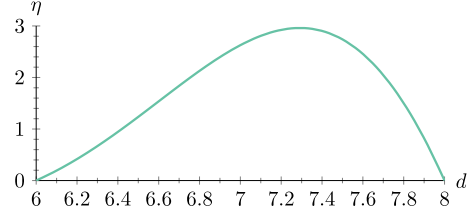


Fig. 1. Plot of the function $\eta(d)$ defined in (11) for $6 \leq d \leq 8$.

$$\gamma_{\chi*} = \frac{\epsilon}{N} \left(1 + \frac{11}{12} \epsilon - \frac{13}{144} \epsilon^2 + \mathcal{O}(\epsilon^3)\right) + \mathcal{O}\left(\frac{1}{N^2}\right),$$

$$\gamma_{\sigma*} = -\frac{\epsilon}{2} + \frac{40\epsilon}{N} \left(1 + \frac{13}{15} \epsilon - \frac{11}{180} \epsilon^2 + \mathcal{O}(\epsilon^3)\right) + \mathcal{O}\left(\frac{1}{N^2}\right). \quad (9)$$

With the result leading in $1/N$ and neglecting the $\mathcal{O}(\epsilon^4)$ contributions we see that in the limit $\epsilon \rightarrow 1$ the anomalous dimension of χ is positive for all positive N , while that of σ is positive for positive $N \leq \frac{1300}{9} \approx 144.4$.

The result (9) can be improved for results analytic in d for the anomalous dimensions of χ and σ at large N also exist [13–15,2]. At leading order in $1/N$ they are

$$\gamma_{\chi*} = \frac{1}{N} \eta,$$

$$\gamma_{\sigma*} = -\frac{d-6}{2} + \frac{1}{N} \frac{4(d-1)(d-2)}{d-4} \eta, \quad (10)$$

where

$$\eta = -\frac{2^{d-3}(d-4)\Gamma(\frac{d-1}{2})}{\pi^{3/2}\Gamma(\frac{1}{2}d+1)} \sin \frac{\pi d}{2}. \quad (11)$$

The function $\eta = \eta(d)$ is plotted in Fig. 1 in the region $6 \leq d \leq 8$. As we see η is zero at $d = 6, 8$. This suggests that χ becomes free in $d = 8$, while σ has $\Delta_\sigma = 2$, well below the unitarity bound. The positivity of η in the region $6 < d < 8$ indicates that if we trust the $1/N$ expansion at N not too large then the dimension of σ may not violate the unitarity bound for $6 < d < 8$. For $d = 7$, for example, the unitarity bound for σ is violated if $N > \frac{8192}{7\pi^2} \approx 118.6$.

One can also consider operator mixing between $\Omega_{ij}\chi^i\chi^j$ and σ^2 . In the large- N limit the results can again be borrowed from [3]. From the equation of motion of σ it is clear that there is one linear combination of $\Omega_{ij}\chi^i\chi^j$ and σ^2 that is a descendant of σ . Its scaling dimension is $\Delta_\sigma + 2$. The other independent combination is a primary with scaling dimension $\Delta = d - 2 - \frac{100\epsilon}{N}$.

We note here that for $N = 2$ there are fixed points of (3a) and (3b) with a symmetry enhancement from $\text{Sp}(2)$ to the orthosymplectic supergroup $\text{OSp}(1|2)$ [3]. Considering $\epsilon \rightarrow 1$ this points to the existence of $\text{OSp}(1|2)$ symmetric CFTs in $d = 7$, although here we are no longer in the large- N limit.

Our discussion provides evidence for the existence of nontrivial CFTs in $d = 7$. In the large- N limit the $\epsilon \rightarrow 1$ limit of our $d = 6 + \epsilon$ results can be taken consistently with neglecting higher-loop effects. While the CFTs for which we find evidence are not unitary, the violation of unitarity is not due to imaginary couplings.¹ Distinctively, all operator dimensions are positive, although that of σ violates the unitarity bound at large N . This violation is mild, but it still shows that not all states in the theory have positive norm. As we see from (10) $\Delta_\sigma \rightarrow 2$ as $N \rightarrow \infty$ in all d .

¹ This would be the case if we had $\mathcal{O}(N)$ symmetry with commuting scalars ϕ^i in (1) in $d = 6 + \epsilon$.

3. Central charge

The two-point function of the stress-energy tensor can be defined as

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0) \rangle = C_T \frac{I_{\mu\nu\rho\sigma}(x)}{x^{2d}}, \quad (12)$$

with

$$I_{\mu\nu\rho\sigma}(x) = \frac{1}{2} (I_{\mu\rho}(x)I_{\nu\sigma}(x) + I_{\mu\sigma}(x)I_{\nu\rho}(x)) - \frac{1}{d} \delta_{\mu\nu} \delta_{\rho\sigma},$$

$$I_{\mu\nu}(x) = \delta_{\mu\nu} - 2 \frac{x_\mu x_\nu}{x^2}. \quad (13)$$

The central charge of the critical $O(N)$ theory with commuting scalars is given by [4,5]

$$C_T = \frac{d\Gamma^2(\frac{1}{2}d)}{4(d-1)\pi^d} \left(N + \theta\eta + \mathcal{O}\left(\frac{1}{N}\right) \right), \quad (14)$$

where

$$\theta = 4 \frac{\mathcal{C}(\frac{1}{2}d)}{d+2} + 2 \frac{d^2 + 6d - 8}{d(d-2)(d+2)}, \quad (15)$$

η is given in (11), and

$$\mathcal{C}(x) = \psi(3-x) + \psi(2x-1) - \psi(x) - \psi(1),$$

$$\psi(x) = \Gamma'(x)/\Gamma(x). \quad (16)$$

Setting $d = 6 - \epsilon$, expanding in ϵ and taking $\epsilon \rightarrow 0$ we find

$$C_{T, d=6} = \frac{6}{5\pi^6} (N+1), \quad (17)$$

and so in $d = 6$ we get a result for the central charge consistent with $N+1$ free scalars. This was discussed in [2] as a nice check of their proposal for the UV completion of the $d = 5$ critical $O(N)$ theory.

If N scalars are anticommuting then $N \rightarrow -N$ in (17). In the remainder of this section we send $N \rightarrow -N$ and apply (14) in $d > 6$. This corresponds to the critical $Sp(N)$ symmetric theory with anticommuting scalars.

From (14) we can see that, if we neglect $1/N$ corrections, then C_T is negative for all integer $N \geq 2$ in the limit $\epsilon \rightarrow 1$, for then

$$C_{T, d=7} = -\frac{525}{512\pi^6} \left(N - \frac{516608}{33075\pi^2} \right). \quad (18)$$

This result indicates the presence of an interacting fixed point. It is not clear if this fixed point is related to the $\epsilon \rightarrow 1$ limit of the $d = 6 + \epsilon$ fixed point of (1) discussed here. Nevertheless, if we identify it with the fixed point of (2) in the context of the $1/N$ expansion, then we observe some similarities: both fixed points are nonunitary with violations of the unitarity bounds for operator dimensions, and they both have real couplings and positive operator dimensions.

Let us also consider $d = 8$ where we get

$$C_{T, d=8} = -\frac{72}{7\pi^8} (N+4), \quad (19)$$

indicating a free theory there. This theory in $d = 8 - \epsilon$ may have an IR fixed point. This may coincide with the UV fixed point of (1) in $d = 7$, something that would provide its UV completion.

For the critical $O(N)$ models we see a shift of N by 1 in (17) and by -4 in (19). Although $\eta(d)$ is zero for even d , there is an obvious pole at $d = 2$ in (14), while at even $d \geq 6$ $\mathcal{C}(\frac{1}{2}d)$ has a pole due to $\psi(3 - \frac{1}{2}d)$. Consequently, N is shifted in (14) in $d = 2$

and in even $d \geq 6$. The analytic expression that gives the shift of N in even $d \geq 2$ is

$$N_{\text{shift}} = -\frac{(d-4)\Gamma(d-1)}{\Gamma(\frac{1}{2}d)\Gamma(\frac{1}{2}d+2)} \cos \frac{\pi d}{2}. \quad (20)$$

This is an integer for any even $d \geq 2$.

4. Sphere free energy and the F -theorem

At the interacting fixed point in $d = 6 + \epsilon$ we can compute the sphere free energy using the results of [16]. If Z_{S^d} is the partition function on the d -dimensional sphere, then for

$$F = -\log Z_{S^d}, \quad \tilde{F} = -F \sin \frac{\pi d}{2}, \quad (21)$$

we have, for the interacting theory,

$$F = F_{\text{free}} + \frac{1}{8640} \frac{1}{64\pi^3} (h_*^2 - 3Ng_*^2) + \mathcal{O}(\epsilon^2), \quad (22)$$

and

$$\tilde{F} = \tilde{F}_{\text{free}} + \frac{\pi}{17280} \frac{h_*^2 - 3Ng_*^2}{64\pi^3} \epsilon + \mathcal{O}(\epsilon^3), \quad (23)$$

where $F_{\text{free}} = (1-N)F_s$ with F_s the value of F for a free conformal scalar. In our examples in $d = 6 + \epsilon$ the IR theory is free and the UV interacting. As a result, in the large- N limit we get

$$F_{\text{UV}} < F_{\text{IR}}, \quad \tilde{F}_{\text{UV}} < \tilde{F}_{\text{IR}}, \quad (24)$$

for a flow between these two theories. One can also verify that (24) holds for any $N \geq 2$ [3]. Note that results analytic in d also exist here [16].

As we see the F -theorem is violated both for F and for \tilde{F} in $d = 6 + \epsilon$. This does not raise concerns for the validity of the F -theorem since the theory we are considering is not unitary. A closely related example, where the F -theorem for F is violated but, contrary to our case, that for \tilde{F} holds despite the violation of unitarity, was encountered in [3].

5. Conclusion

In this short paper we provided evidence for the existence of nonunitary UV fixed points in $d = 6 + \epsilon$ dimensions, and suggested that these fixed points survive in $d = 7$. The important distinguishing feature of the fixed points we propose is that the critical values of the couplings are real and the operator dimensions are positive. The absence of unitarity in these models is due to the presence of anticommuting scalars and, at large N , due to the violation of the unitarity bound for the scalar operator σ .

We also considered the critical $Sp(N)$ models and saw that their central charge indicates the existence of an interacting CFT in $d = 7$. Additionally, we saw that fixed points of (2) in the $1/N$ expansion in $d = 7$ have similar properties with the fixed points of (1) in $d = 7$. One cannot conclusively determine the relation of these fixed points at this point, but it is tempting to suggest that they may be equivalent. Further support to this may come from a possible UV completion of these UV fixed points starting from a theory in $d = 8 - \epsilon$.

It is important to investigate the way in which the theories proposed here may play a role in the higher-spin [17–20] dS/CFT correspondence [21,22]. These theories fall in the class of models with weakly broken higher-spin symmetry, since all higher-spin currents are nearly conserved in the large- N limit. In $d = 3$ this, along with the fact that in the AdS/CFT correspondence [23–25] conserved vectors of the boundary theory correspond to gauge

fields in the bulk, led to the conjecture that the singlet sector of critical $O(N)$ models is dual to interacting higher-spin theory in $d = 4$ [26]. Since the Vasiliev equations are known for all d [20] a similar result may apply for the $d = 7$ models discussed here.

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