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Computational analysis of network survivability with application to power systems

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Abstract

The operability of society's critical infrastructures depends on the availability of electric power. Adverse events (natural disasters, intelligent adversary, etc.) occur rarely, but power system failure under such conditions has typically devastating effects on the economy and lives. A key factor in the system's ability to withstand massive sudden damage caused by adverse events is its topology: the number of system elements that generate and demand power and the connections between them. The topology factor can be quantified by analyzing the impact of all possible combinations of unrecoverable faults (fault scenarios) on the availability and connectivity of system elements. As the number of possible fault scenarios grows as 2^M with increasing number M of system elements, such an analysis becomes a computational challenge for large-scale systems. The paper discusses possibilities of reducing the computational complexity of the problem.

Keywords: Survivability ; power systems ; complex systems ; network topology ; network theory

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1. Introduction

The US President's Commission on Critical Infrastructure Protection [1] identified electric power systems as one of the Nation's eight critical infrastructures (CIs). With the operability of the other seven CIs – telecommunications, natural gas and oil, banking and finance, transportation, water supply systems, government services, and emergency services – depending on the availability of electric power, the resilience and reliability of power systems is of crucial importance. Yet, multiple studies (see, e.g., [2-4]) indicate that the modern electric power infrastructure is not prepared to withstand many forms of large-scale damage caused by natural (hurricanes, earthquakes, floods, wild fires) and man-made malicious (physical destructions or electronic intrusions) physical events. Such events occur rarely, but their impact on the economy and lives is typically devastating.

Faults caused by adverse events are not random and cannot be predicted. The damage they cause is typically several orders of magnitude bigger in scale than damage due to operational faults (manufacturing faults, fatigue cracking, and maloperation) [8] and with limited possibility for repair in a short term. Traditional reliability/availability analysis [5-7] is concerned with the system performance in the presence of operational faults

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that are random, expected to occur within a predicted time interval, and can be repaired within an estimated time. Therefore, approaches different from traditional methods are required to analyze the ability of power systems to withstand massive sudden damage caused by adverse events (hereafter referred to as *survivability*).

In power systems, survivability is associated with the continuation of the generation and distribution of power from generators to loads. The analysis of the propagation of faults initiated by an adverse event within a system brings a useful insight into the system vulnerability [9], but the ultimate question of survivability analysis is whether a system can survive an event. In this regard, the main focus of survivability analysis in application to power systems is the final steady state of a system after all faults (including cascading and secondary) have occurred and before any repair has been accomplished. This state can be evaluated based on i) how much power is available in the system after damage occurred and ii) whether this amount is sufficient to satisfy the existing power demand. Notice that eventually, expected faults under normal operational conditions may result in an unexpected large-scale power system failure such as a blackout. Outcomes (not evolution of damage) of such events are also a subject of survivability analysis.

As many factors [10] influence the system survivability, our analysis has been centered on quantifying the capacity inherent in a system topology to maintain operations after damage occurred (hereafter referred to as *topological survivability*). Indeed, the topology of a power system – the number of loads and power sources (hereafter, generators) included in the system and how they are connected with one another – determines whether power will be available to the loads after damage occurred and, therefore, is a key factor to consider.

Previously, we developed a basic mathematical framework [11] and computational algorithms [12,13] for assessing the topological survivability of a power system with multiple distributed generators and a single load. This approach is applicable when the load represents either an isolated industrial load, or multiple commercial and residential loads interconnected into a single distribution system, or a lower voltage level network. The current paper discusses how survivability analysis can be applied to a power system with a few voltage levels and multiple distributed loads. An issue with the computational analysis of large-scale systems will also be addressed.

2. Mathematical framework

To evaluate the topological survivability of a power system, one needs to determine availability and connectivity of the system elements after a given number of faults occurred in the system and to calculate the power flow available to the loads. As the analysis of topological survivability focuses on the outcome of damage, faults of principal concern are failures of system elements that cannot be recovered in the short term. Therefore, multiple faults are viewed as simultaneous events; only one fault can occur in a given element. Faults in interconnections are not considered as any such faults are equivalent to faults in adjacent elements. Since, one cannot predict what elements and how many of them will be damaged under adverse conditions, the outcome of all possible combinations of faults should be analyzed. A combination of faults is called a *fault scenario*. The total number of fault scenarios N is independent of the system topology and can be easily computed: $N = \sum_m N(m) = 2^M$, where m is the number of faults and $N(m) = M!/m!(M-m)!$ is the number of fault scenarios at a given m.

Each fault scenario results in one of three types of responses from the system depending on the amount of power available to loads after damage: “no response”, reconfiguration or load-shedding, and complete failure. In “no response” scenarios, power flow to loads is preserved. Scenarios, in which power is supplied to loads in reduced amount, are reconfiguration scenarios. If faults completely isolate loads from power sources, the scenario is that of complete failure. The numbers of fault scenarios leading to each response are S, R, and F, respectively.

The total number of fault scenarios leading to each of the three responses can be used to determine the response probability P at a given m: $P(S) = S/N$ (probability of “no response” scenarios), $P(R) = R/N$ (probability of reconfiguration scenarios), and $P(F) = F/N$ (probability of complete failure). The probabilities of the three responses sum to unity at a given m.

In [11] we demonstrated step by step how one can analytically calculate the numbers of fault scenarios S, R, and F and the response probabilities for small topologies containing two and three generators and a single load. As a number of system elements and complexity of the system topology increase, computational analysis becomes the only choice to generate fault scenarios and analyze their impact on system survivability. For real-size systems, even computations may become unfeasible. The computational burden can, to some degree, be overcome by utilizing advanced algorithms and computational techniques [13]. If one can find a way to reduce the number of system elements and disintegrate the system topology into smaller and simpler sub-topologies without losing information on the availability and connectivity of system elements, computational expenses can be reduced even more significantly.

3. Power system representation for survivability analysis

In modern power systems, power is transmitted from the bulk power sources to loads over transmission, subtransmission, and distribution networks [14,15]. Step-up and step-down transformers transfer power from one level of voltage to another. Power may undergo four or five transformations between generator and ultimate user [15]. High-voltage transmission lines can operate at up to 765 kV AC and ~1,000 kV DC [16]. Very large industrial customers may be directly served from the transmission line. The portion of the transmission system that connects the high-voltage substations through step-down transformers to the distribution substations is called the subtransmission network. The subtransmission systems typically range from 69 to 138 kV. Some large industrial loads may be served from the subtransmission line. The distribution system operates at lower voltage levels and is also subdivided into primary and secondary networks depending on voltage level [15]. Some small industrial loads are served directly by the primary feeders. The secondary distribution network reduces the voltage for utilization by commercial and residential loads down to 240/120 V.

As a conequence, even though the geographical layout of electric power systems is two-dimensional, the system topology is three-dimensional, with the additional dimension being the voltage level. The system layers corresponding to different voltage levels consist of two-dimensional individual networks that may or may not be interconnected. Individual networks typically have different topologies and different network characteristics. Power can be supplied to any level directly from power plants and/or from the higher-voltage level network. In a case of distribution networks, power can or will (in next-generation systems) also be supplied from distributed energy resources and storage devices. Each individual network has connections to loads that are individual consumers, distribution networks, and/or connections to the lower voltage level networks. An abstract representation of an electric power system is shown in Fig. 1. Topologies of individual networks (*Transmission* 1, *Transmission* 2, *Subtransmission* 1, …) are not shown. Arrow-headed links show the direction of power flow. Generators (power plants, renewable energy sources, storage, and/or higher-voltage level networks) are shown as circles marked by "~". Loads are shown as circles marked by "-". Dots at the beginning of a link indicate that multiple layers can exist between the layers shown in the figure. Not shown are multiple networks at the same voltage level that are supplied power from the same higher-voltage level network, but not connected with one another.

Figure 1 illustrates the idea that topological survivability of the whole system is determined by topological survivability of individual networks at different voltage levels and by connections existing between these networks. Indeed, removing any layer or connections between any two layers would interrupt bulk power flow to distribution networks. On the other hand, individual networks may survive even if higher- and lower-voltage level networks become unavailable. Thus, a problem of the analysis of topological survivability of a power system can be re-formulated as the analysis of topological survivability of individual networks that constitute the power system. Based on survivability assessment of individual networks, one can compare topological survivability of networks at different voltage levels, identify "weak links" in the whole infrastructure, and suggest design strategies to enhance the system survivability. As the scale of an individual network is smaller than the scale of the whole infrastructure, such an approach is the first step to reduce the complexity of the computational analysis.

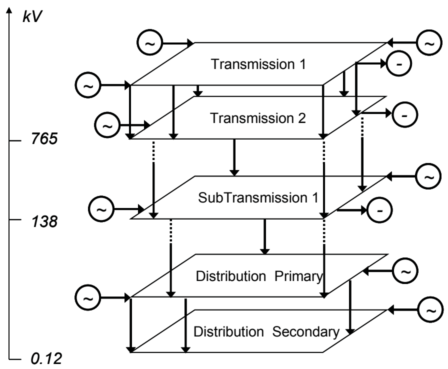


Fig. 1 Power system structure

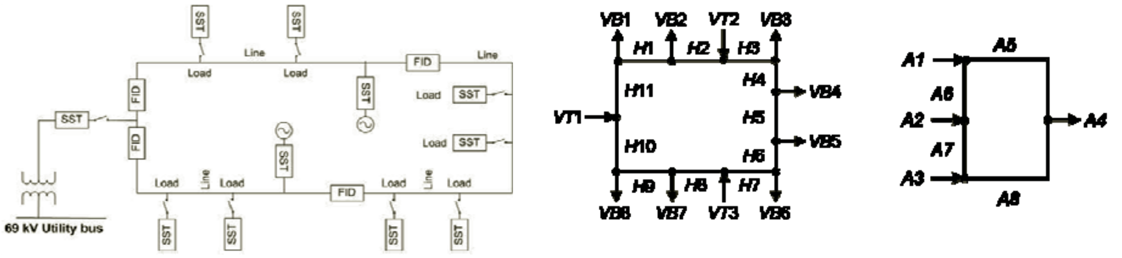
4. Individual network representation

Standard diagrams of power systems contain redundant information that is not necessary for the analysis of the topological survivability. What is of importance for survivability analysis is the number of generators and loads, their locations within the network, their connections with one another, and the amount of power they supply to/demand from the network. Therefore, the next step in reducing the computational complexity of the analysis is to reduce the number of system elements by representing all elements that are connected in series as a single element (hereafter, link). Indeed, a fault in any of the elements connected in series results in interruption of power flow through all of them. Let us consider as an example a possible general configuration of a FREEDM distribution primary system [17]. The main grid (higher-voltage level network) and two distributed energy resources supply power to eight loads connected in a loop (Fig. 2a). In the figure, SST and FID stand for "solid state transformer" and "fault isolation device", respectively. Figure 2b shows the result of transforming Fig. 2a into a diagram in which system elements connected in series are represented by a single link. Links that include generators (hereafter, vertical VT links) and links that include loads (hereafter, vertical VB links) are shown by arrow-headed links. The direction of arrows shows the direction of power flow. Links that simply transfer power from one point to another are called *horizontal links* and are labeled by "H" in Fig. 2b. The total number of elements in this topology is $M = 22$, that is, there are 2^{22} scenarios to analyze.

After the initial number of system elements is reduced, the system topology with multiple generators and distributed loads can be disintegrated into sub-topologies with multiple generators and a single load. Indeed, faults in vertical links connecting other loads to the generator bus cannot interrupt power flow to the load under consideration. Therefore, only those fault scenarios that directly affect power flow to a given load have to be analyzed. Faults that isolate other loads from the system may increase the amount of power available to the load under consideration and thus, increase its chances to survive. This factor can be taken into account by analyzing all loads simultaneously.

As there are eight loads in Fig. 2b, there are eight sub-topologies to consider. However, one can utilize the fact that some loads "see" the system in a similar way. In the topology in Fig. 2b, all eight loads "see" the system as shown in Fig. 2c, where the correspondence of links *A*1-*A*3 to links *VT*1-*VT*3 in Fig. 2b depends on the load under consideration. The load (any of *VB*-links in Fig. 2b) is shown by link *A*4 in Fig. 2c. Links *A*5-*A*8 correspond to horizontal links *H*1-*H*11 in Fig. 2b. Here again, links connected in series are represented by a single link. Table 1 shows the correspondence between links in the topologies shown in Figs. 2b,c for all loads.

Thus, the analysis of the initial topology shown in Fig. 2a is reduced to the analysis of the topology shown in Fig. 2c that has only 8 elements and 256 fault scenarios to consider. The complexity of the problem is reduced drastically. The computational algorithm described in [13] can be used as a core of a computational procedure. A more complex topology with loads connected to the system by multiple links is discussed in [18].



a) b) c)

Fig. 2. A FREEDM distribution primary system (a), its simplified diagram for survivability analysis (b), single-load diagram (c).

5. Conclusions

In this paper, a new approach to reducing the computational complexity of the analysis of topological survivability of large-scale multilayered power systems with multiple generators and distributed loads is discussed. In particular, it is shown that the problem can be reduced to the analysis of individual networks corresponding to a given voltage level. A complex topology of an individual network with multiple generators and multiple loads can be disintegrated into simple topologies with multiple generators and a single load connected to the system by a single link. In such a way, computational expenses can be reduced considerably. The application of the approach is demonstrated for a general configuration of a FREEDM distribution primary system [17]. A similar approach can be applied to any multilayered system with sources and sinks and for the analysis of network characteristics [19].

Table 1

Correspondence of links in Figs. 2b,c for individual loads

Loads, *A*4	*A*1	*A*2	*A*3	*A*5	*A*6	*A*7	*A*8
*VB*1	*VT*1	*VT*3	*VT*2	*H*11	*H*8-*H*10	*H*3-*H*7	*H*1,*H*2
*VB*2	*VT*1	*VT*3	*VT*2	*H*1, *H*11	*H*8-*H*10	*H*3-*H*7	*H*2
*VB*3	*VT*2	*VT*1	*VT*3	*H*3	*H*1,*H*2, *H*11	*H*8-*H*10	*H*4-*H*7
*VB*4	*VT*2	*VT*1	*VT*3	*H*3,*H*4	*H*1,*H*2, *H*11	*H*8-*H*10	*H*5-*H*7
*VB*5	*VT*2	*VT*1	*VT*3	*H*3-*H*5	*H*1,*H*2, *H*11	*H*8-*H*10	*H*6,*H*7
*VB*6	*VT*2	*VT*1	*VT*3	*H*3-*H*6	*H*1,*H*2, *H*11	*H*8-*H*10	*H*7
*VB*7	*VT*3	*VT*2	*VT*1	*H*8	*H*3-*H*7	*H*1,*H*2, *H*11	*H*9,*H*10
*VB*8	*VT*3	*VT*2	*VT*1	*H*8, *H*9	*H*3-*H*7	*H*1,*H*2, *H*11	*H*10

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