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New complex solutions for some special nonlinear partial differential systems

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ABSTRACT

In this present work, we explore new applications of direct algebraic method for some special nonlinear partial differential systems and equations. Then new types of complex travelling wave solutions are obtained to the Davey–Stewartson system, the coupled Higgs system and the perturbed nonlinear Schrodinger's equation; the balance number of it is not a positive integer.

1. Introduction

The theory of nonlinear dispersive wave motion has recently undergone much study. Phenomena in physics and other fields are often described by nonlinear evolution equations and play a crucial role in applied mathematics and physics. Furthermore, when an original nonlinear equation is directly calculated, the solution will preserve the actual physical characters of solutions [1]. Explicit solutions to the nonlinear equations are of fundamental importance. Various methods for obtaining explicit solutions to nonlinear evolution equations have been proposed. Many explicit exact methods have been introduced in the literature. Among these methods we mainly cite, the extended tanh-function methods [2–5], the F-expansion methods [6,7], the famous Hirota's method [8], the Backlund and Darboux transformation [9–11], the Painleve expansions [12], the homogeneous balance method [13], the Jacobi elliptic function [14,15] and the extended, variational iteration methods [16–18].

The paper is arranged as follows. In Section 2, we describe briefly the direct algebraic method. In Sections 3–5, we apply the method to the Davey–Stewartson system, the nonlinear coupled Higgs system, and the perturbed nonlinear Schrodinger's equation, respectively. In Section 6, some conclusions are given.

2. An analysis of the method

For a given partial differential equation

$$G(u, u_x, u_t, u_{xx}, u_{tt}, \ldots). \tag{1}$$

Our method mainly consists of four steps.

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Step 1: We seek complex solutions of Eq. (1) in the following form:

$$u = u(\xi), \quad \xi = \mathrm{i}k(x - ct), \tag{2}$$

where k and c are real constants. Under transformation (2), Eq. (1) becomes an ordinary differential equation

$$N(u, \mathrm{i}ku', -\mathrm{i}kcu', -k^2u'', \ldots), \tag{3}$$

where $u' = \frac{\mathrm{d}u}{\mathrm{d}\xi}$.

Step 2: We assume that the solution of Eq. (3) is of the form

$$u(\xi) = \sum_{i=0}^{n} a_i F^i(\xi), \tag{4}$$

where a_i $(i = 1, 2, \ldots, n)$ are real constants to be determined later. $F(\xi)$ expresses the solution of the auxiliary ordinary differential equation

$$F'(\xi) = b + F^2(\xi). \tag{5}$$

Eq. (5) admits the following solutions:

$$\begin{gathered} F(\xi) = \begin{cases} -\sqrt{-b}\tanh(\sqrt{-b}\xi), & b \prec 0 \\ -\sqrt{-b}\coth(\sqrt{-b}\xi), & b \prec 0 \end{cases} \\ F(\xi) = \begin{cases} \sqrt{b}\tan(\sqrt{b}\xi), & b \succ 0 \\ -\sqrt{b}\cot(\sqrt{b}\xi), & b \succ 0 \end{cases} \\ F(\xi) = -\frac{1}{\xi}, \quad b = 0. \end{gathered} \tag{6}$$

Integer n in (4) can be determined by considering homogeneous balance [3] between the nonlinear terms and the highest derivatives of $u(\xi)$ in Eq. (3).

Step 3: Substituting (4) into (3) with (5), then the left hand side of Eq. (3) is converted into a polynomial in $F(\xi)$, equating each coefficient of the polynomial to zero yields a set of algebraic equations for a_i, k, c.

Step 4: Solving the algebraic equations obtained in step 3, and substituting the results into (4), then we obtain the exact travelling wave solutions for Eq. (1).

3. Application to the Davey–Stewartson system

Let us first consider the Davey–Stewartson system which has the form

$$\begin{cases} \mathrm{i}q_t + \frac{1}{2}\delta^2(q_{xx} + \delta^2 q_{yy}) + \lambda\,|q|^2\,q - \phi_x q = 0, \\ \phi_{xx} - \delta^2\phi_{yy} - 2\lambda(|q|^2)_x = 0. \end{cases} \tag{7}$$

We use the wave transformation $q = u(\xi)\mathrm{e}^{\mathrm{i}(\alpha x + \beta y + \gamma t)}$, $\phi = v(\xi)$, with wave complex variable $\xi = \mathrm{i}k(x + y - ct)$, where α, β, γ are arbitrary constants. System (7) takes the form as

$$\begin{cases} -\frac{1}{2}\delta^2k^2(1+\delta^2)u'' + k(c - \delta^2\alpha - \delta^4\beta)u' - \left(\alpha + \frac{1}{2}\delta^2\alpha^2 + \frac{1}{2}\delta^4\beta^2\right)u + \lambda u^3 - \mathrm{i}kv'u = 0, \quad \text{(a)} \\ k^2(1-\delta^2)v'' - 2\mathrm{i}k\lambda(u^2)' = 0. \quad \text{(b)} \end{cases} \tag{8}$$

Integrating Eq. (8)(b) once with respect to ξ and setting the constant of integration to be zero, we obtain

$$v' = \frac{2\mathrm{i}\lambda}{k(\delta^2 - 1)}u^2. \tag{9}$$

Substituting (9) into Eq. (8)(a) we have

$$\begin{aligned} &-\frac{1}{2}\delta^2k^3(\delta^4 - 1)u'' + k^2(\delta^2 - 1)(c - \delta^2\alpha - \delta^4\beta)u' \\ &\quad - k^2(\delta^2 - 1)\left(\alpha + \frac{1}{2}\delta^2\alpha^2 + \frac{1}{2}\delta^4\beta^2\right)u + \lambda k(\delta^2 + 1)u^3 = 0. \end{aligned} \tag{10}$$

Considering the homogeneous balance between u'' and u^3 in (10), we required that $3m = m + 2 \Rightarrow m = 1$. So the solution takes the form

$$u = a_1F + a_0. \tag{11}$$

Substituting (11) into Eq. (10) yields a set of algebraic equations for $a_1, a_0, k, \alpha, \beta, \delta$ and c. These equations are found as

$$-a_1\delta^2k^3(\delta^4-1)+a_1^3\lambda k(\delta^2+1)=0, \tag{12}$$

$$a_1k^2(\delta^2-1)(c-\delta^2\alpha-\delta^4\beta)+3a_1^2a_0\lambda k(\delta^2+1)=0, \tag{13}$$

$$-a_1b\delta^2k^3(\delta^4-1)-a_1k(\delta^2-1)\left(\alpha+\frac{1}{2}\delta^2\alpha^2+\frac{1}{2}\delta^4\beta^2\right)+3a_1a_0^2\lambda k(\delta^2+1)=0, \tag{14}$$

$$\frac{1}{2}a_1b\delta^2k^2(\delta^2-1)(c-\delta^2\alpha-\delta^4\beta)-a_0k(\delta^2-1)\left(\alpha+\frac{1}{2}\delta^2\alpha^2+\frac{1}{2}\delta^4\beta^2\right)+a_0^3\lambda k(\delta^2+1)=0. \tag{15}$$

Solving (12)–(15) with Maple package we obtain

$$a_1=\pm\delta k\sqrt{\frac{\delta^2-1}{\lambda}}, \tag{16}$$

$$a_0=\pm\frac{\sqrt{6}}{6}\frac{(-1+\delta^2)\sqrt{(1+\delta^2)(2k^2b\delta^4+\delta^4\beta^2+\delta^2\alpha^2+2k^2b\delta^2+2\alpha)}}{\lambda(1+\delta^2).\sqrt{\frac{-1+\delta^2}{\lambda}}}, \tag{17}$$

$$c=\frac{1}{2}\delta(2\beta\delta^3+2\delta\alpha\pm\sqrt{6}\sqrt{(\delta^2+1)(2k^2b\delta^4+\delta^4\beta^2+\delta^2\alpha^2+2k^2b\delta^2+2\alpha)}), \tag{18}$$

where k, α, β, δ are arbitrary constants.

From (7), (11) and (16)–(18), we obtain the complex travelling wave solutions of (8)(a)–(b) as follows

$$\begin{aligned} u_1 = {} & \pm\delta k\sqrt{\frac{\delta^2-1}{\lambda}}\Bigg[\sqrt{-b}\tanh\Bigg(\sqrt{-b}ik\Bigg(ik\Bigg(x+y \\ & -\frac{1}{2}\delta(2\beta\delta^3+2\delta\alpha\pm\sqrt{6}\sqrt{(\delta^2+1)(2k^2b\delta^4+\delta^4\beta^2+\delta^2\alpha^2+2k^2b\delta^2+2\alpha)})t\Bigg)\Bigg)\Bigg)\Bigg] \\ & \pm\frac{\sqrt{6}}{6}\frac{(-1+\delta^2)\sqrt{(1+\delta^2)(2k^2b\delta^4+\delta^4\beta^2+\delta^2\alpha^2+2k^2b\delta^2+2\alpha)}}{\lambda(1+\delta^2).\sqrt{\frac{-1+\delta^2}{\lambda}}}, \end{aligned}$$

where $b \prec 0$ and k is an arbitrary real constant. So

$$\begin{aligned} q_1 = {} & \Bigg(\pm\delta k\sqrt{\frac{\delta^2-1}{\lambda}}\Bigg[\sqrt{-b}\tanh\Bigg(\sqrt{-b}ik\Bigg(x+y \\ & -\frac{1}{2}\delta(2\beta\delta^3+2\delta\alpha\pm\sqrt{6}\sqrt{(\delta^2+1)(2k^2b\delta^4+\delta^4\beta^2+\delta^2\alpha^2+2k^2b\delta^2+2\alpha)})t\Bigg)\Bigg)\Bigg] \\ & \pm\frac{\sqrt{6}}{6}\frac{(-1+\delta^2)\sqrt{(1+\delta^2)(2k^2b\delta^4+\delta^4\beta^2+\delta^2\alpha^2+2k^2b\delta^2+2\alpha)}}{\lambda(1+\delta^2).\sqrt{\frac{-1+\delta^2}{\lambda}}}\Bigg)e^{i(\alpha x+\beta y+\gamma t)}, \end{aligned}$$

$$\begin{aligned} \phi_1 = {} & \frac{2i\lambda}{k(\delta^2-1)}\int u^2 d\xi = \frac{2i\lambda}{k(\delta^2-1)}\int(-a_1\sqrt{-b}\tanh(\sqrt{-b}\xi)+a_0)^2 d\xi \\ = {} & \frac{2i\lambda}{k(\delta^2-1)}\Bigg(\frac{(\delta k)^2(\delta^2-1)b\tanh(\sqrt{-b}\xi)}{\lambda\sqrt{-b}} \\ & -\frac{1}{12}\frac{\ln(\tanh(\sqrt{-b}\xi)-1)(-1+\delta^2)(2k^2b\delta^4+\delta^4\beta^2+\delta^2\alpha^2+2k^2b\delta^2+2\alpha)}{\sqrt{-b}\lambda(1+\delta^2)} \\ & \pm\ln(\tanh(\sqrt{-b}\xi)-1)\left(\frac{\sqrt{6}}{6}\frac{\delta k(-1+\delta^2)\sqrt{(1+\delta^2)(2k^2b\delta^4+\delta^4\beta^2+\delta^2\alpha^2+2k^2b\delta^2+2\alpha)}}{\lambda(1+\delta^2)}\right) \\ & +\frac{1}{2}\frac{\ln(\tanh(\sqrt{-b}\xi)-1)\,(\delta k)^2\,(\delta^2-1)b}{\lambda\sqrt{-b}} \\ & +\frac{1}{12}\frac{\ln(\tanh(\sqrt{-b}\xi)+1)(-1+\delta^2)(2k^2b\delta^4+\delta^4\beta^2+\delta^2\alpha^2+2k^2b\delta^2+2\alpha)}{\sqrt{-b}\lambda(1+\delta^2)} \end{aligned}$$

$$\pm \ln(\tanh(\sqrt{-b}\xi)+1)\left(\frac{\sqrt{6}}{6}\frac{\delta k(-1+\delta^2)\sqrt{(1+\delta^2)(2k^2b\delta^4+\delta^4\beta^2+\delta^2\alpha^2+2k^2b\delta^2+2\alpha)}}{\lambda(1+\delta^2)}\right)$$
$$-\frac{1}{2}\frac{\ln(\tanh(\sqrt{-b}\xi)+1)\,(\delta k)^2\,(\delta^2-1)b}{\lambda\sqrt{-b}}\Bigg),$$

and

$$u_2=\pm\delta k\sqrt{\frac{\delta^2-1}{\lambda}}\left[\sqrt{-b}\coth\left(\sqrt{-b}ik\left(x+y\right.\right.\right.$$
$$\left.\left.\left.-\frac{1}{2}\delta(2\beta\delta^3+2\delta\alpha\pm\sqrt{6}\sqrt{(\delta^2+1)(2k^2b\delta^4+\delta^4\beta^2+\delta^2\alpha^2+2k^2b\delta^2+2\alpha)})t\right)\right)\right]$$
$$\pm\frac{\sqrt{6}}{6}\frac{(-1+\delta^2)\sqrt{(1+\delta^2)(2k^2b\delta^4+\delta^4\beta^2+\delta^2\alpha^2+2k^2b\delta^2+2\alpha)}}{\lambda(1+\delta^2).\sqrt{\frac{-1+\delta^2}{\lambda}}},$$

where $b \prec 0$ and k is an arbitrary real constant.

$$u_3=\pm\delta k\sqrt{\frac{\delta^2-1}{\lambda}}\left[\sqrt{b}\tan\left(\sqrt{b}ik\left(x+y\right.\right.\right.$$
$$\left.\left.\left.-\frac{1}{2}\delta(2\beta\delta^3+2\delta\alpha\pm\sqrt{6}\sqrt{(\delta^2+1)(2k^2b\delta^4+\delta^4\beta^2+\delta^2\alpha^2+2k^2b\delta^2+2\alpha)})t\right)\right)\right]$$
$$\pm\frac{\sqrt{6}}{6}\frac{(-1+\delta^2)\sqrt{(1+\delta^2)(2k^2b\delta^4+\delta^4\beta^2+\delta^2\alpha^2+2k^2b\delta^2+2\alpha)}}{\lambda(1+\delta^2).\sqrt{\frac{-1+\delta^2}{\lambda}}},$$

where $b \succ 0$ and k is an arbitrary real constant.

$$u_4=\pm\delta k\sqrt{\frac{\delta^2-1}{\lambda}}\left[-\sqrt{b}\tan\left(\sqrt{b}ik\left(x+y\right.\right.\right.$$
$$\left.\left.\left.-\frac{1}{2}\delta(2\beta\delta^3+2\delta\alpha\pm\sqrt{6}\sqrt{(\delta^2+1)(2k^2b\delta^4+\delta^4\beta^2+\delta^2\alpha^2+2k^2b\delta^2+2\alpha)})t\right)\right)\right]$$
$$\pm\frac{\sqrt{6}}{6}\frac{(-1+\delta^2)\sqrt{(1+\delta^2)(2k^2b\delta^4+\delta^4\beta^2+\delta^2\alpha^2+2k^2b\delta^2+2\alpha)}}{\lambda(1+\delta^2).\sqrt{\frac{-1+\delta^2}{\lambda}}},$$

where $b \succ 0$ and k is an arbitrary real constant.

$$u_5=\pm\delta k\sqrt{\frac{\delta^2-1}{\lambda}}\left[\frac{1}{ik\left(x+y-\frac{1}{2}\delta\left(2\beta\delta^3+2\delta\alpha\pm\sqrt{6}\sqrt{(\delta^2+1)(\delta^4\beta^2+\delta^2\alpha^2+2\alpha)}\right)t\right)}\right]$$
$$\pm\frac{\sqrt{6}}{6}\frac{(-1+\delta^2)\sqrt{(1+\delta^2)(\delta^4\beta^2+\delta^2\alpha^2+2\alpha)}}{\lambda(1+\delta^2).\sqrt{\frac{-1+\delta^2}{\lambda}}},$$

where $b=0$.

In these sections, $q_2\cdots q_5$, $\phi_2\cdots\phi_5$ are calculated as q_1, ϕ_1.

4. Application to the coupled Higgs system

The coupled Higgs system reads

$$\begin{cases} u_{tt}-u_{xx}+|u|^2u-2uv=0, & \text{(a)}\\ v_{tt}+v_{xx}-2(|u|^2)_{xx}=0. & \text{(b)}\end{cases} \tag{19}$$

Under the following transformation $u = q(\xi)e^{i(\alpha x+\beta t)}$, $v = w(\xi)$, with wave complex variable $\xi = ik(x-ct)$, Eq. (19)(a)–(b) becomes

$$\begin{cases} k^2(1-c^2)q'' + 2k(c\beta-\alpha)q' + (\alpha^2-\beta^2)q + q^3 - 2qw = 0, & \text{(a)} \\ -k^2(1+c^2)w'' + k^2(q^2)'' = 0. & \text{(b)} \end{cases} \tag{20}$$

Integrating Eq. (19)(b) twice with respect to ξ and setting the constant of integration to be zero, and substituting the yield equation into Eq. (19)(a) we obtain

$$-k^2(1-c^4)q'' + 2k(1+c^2)(\alpha-c\beta)q' + (1+c^2)(\alpha^2-\beta^2)q - (3+c^2)q^3 = 0. \tag{21}$$

Balancing q'' with q^3 in Eq. (21) gives $m = 1$. Therefore, we may choose

$$u = a_1F + a_0. \tag{22}$$

Using the method mentioned above, we obtain the following set of algebraic equations for $a_1, a_0, k, \alpha, \beta, c$:

$$-2a_1k^2(1-c^4) - a_1^3(c^2+3) = 0,$$
$$2ka_1(c^2+1)(\alpha-c\beta) - 3a_1^2a_0(c^2+3) = 0,$$
$$-2a_1bk^2(1-c^4) + a_1(c^2+1)(\beta^2-\alpha^2) - 3a_1a_0^2(c^2+3) = 0,$$
$$2ka_1b(c^2+1)(\alpha-c\beta) + a_0(c^2+1)(\beta^2-\alpha^2) - a_0^3(c^2+3) = 0.$$

From the solution of the system, we can find

$$\begin{aligned} a_1 &= \pm\sqrt{2}k\sqrt{\frac{1}{4}\,\frac{(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})-b^2k^4}{bk^2(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})+3bk^2}}, \\ a_1 &= \pm\sqrt{2}k\sqrt{-\frac{1}{4}\,\frac{(\beta^2-\alpha^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})-b^2k^4}{bk^2(\beta^2-\alpha^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})+3bk^2}}, \end{aligned} \tag{23}$$

$$\begin{aligned} c &= \pm\frac{1}{2}\,\frac{\sqrt{b(\alpha^2-\beta^2+2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})}}{bk}, \\ c &= \pm\frac{1}{2}\,\frac{\sqrt{-b(-\alpha^2+\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})}}{bk}, \end{aligned} \tag{24}$$

with the aid of Maple package. Where k, α, β and a_0 are arbitrary constants.

From (7), (23)–(24) and (22), we obtain the complex travelling wave solutions of (19)(a)–(b) as follows:

For

$$a_1 = \pm\sqrt{2}k\sqrt{\frac{1}{4}\,\frac{(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})-b^2k^4}{bk^2(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})+3bk^2}},$$

$$c = \pm\frac{1}{2}\,\frac{\sqrt{b(\alpha^2-\beta^2+2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})}}{bk}.$$

We have

$$\begin{aligned} q_1 = {} & \pm\sqrt{2}k\sqrt{\frac{1}{4}\,\frac{(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})-b^2k^4}{bk^2(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})+3bk^2}} \\ & \times\left[\sqrt{-b}\tanh\left(\sqrt{-b}ik\left(x\mp\frac{1}{2}\,\frac{\sqrt{b(\alpha^2-\beta^2+2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})}}{bk}t\right)\right)\right]+a_0, \end{aligned}$$

where $b \prec 0$ and k is an arbitrary real constant.

So

$$u_1 = \left[\pm\sqrt{2}k\sqrt{\frac{1}{4}\frac{(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})-b^2k^4}{bk^2(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})+3bk^2}}\right.$$
$$\times\left[\sqrt{-b}\tanh\left(\sqrt{-b}ik\left(x\mp\frac{1}{2}\frac{\sqrt{b(\alpha^2-\beta^2+2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})}}{bk}t\right)\right)\right]$$
$$\left.+a_0\right]e^{i(\alpha x+\beta t)},$$

$$v_1 = \frac{1}{2}k^2\frac{(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})-b^2k^4}{bk^2(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})+3bk^2}$$
$$\times\left[\sqrt{-b}\tanh\left(\sqrt{-b}ik\left(x\mp\frac{1}{2}\frac{\sqrt{b(\alpha^2-\beta^2+2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})}}{bk}t\right)\right)\right]^2$$
$$\pm2\sqrt{2}k\sqrt{\frac{1}{4}\frac{(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})-b^2k^4}{bk^2(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})+3bk^2}}a_0$$
$$\times\left[\sqrt{-b}\tanh\left(\sqrt{-b}ik\left(x\mp\frac{1}{2}\frac{\sqrt{b(\alpha^2-\beta^2+2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})}}{bk}t\right)\right)\right]$$
$$+a_0^2,$$

and

$$q_2 = \pm\sqrt{2}k\sqrt{\frac{1}{4}\frac{(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})-b^2k^4}{bk^2(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})+3bk^2}}$$
$$\times\left[\sqrt{-b}\coth\left(\sqrt{-b}ik\left(x\mp\frac{1}{2}\frac{\sqrt{b(\alpha^2-\beta^2+2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})}}{bk}t\right)\right)\right]+a_0,$$

where $b \prec 0$ and k is an arbitrary real constant.

$$q_3 = \pm\sqrt{2}k\sqrt{\frac{1}{4}\frac{(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})-b^2k^4}{bk^2(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})+3bk^2}}$$
$$\times\left[\sqrt{b}\tan\left(\sqrt{b}ik\left(x\mp\frac{1}{2}\frac{\sqrt{b(\alpha^2-\beta^2+2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})}}{bk}t\right)\right)\right]+a_0,$$

where $b \succ 0$ and k is an arbitrary real constant.

$$q_4 = \pm\sqrt{2}k\sqrt{\frac{1}{4}\frac{(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})-b^2k^4}{bk^2(\alpha^2-\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})+3bk^2}}$$
$$\times\left[-\sqrt{b}\cot\left(\sqrt{b}ik\left(x\mp\frac{1}{2}\frac{\sqrt{b(\alpha^2-\beta^2+2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})}}{bk}t\right)\right)\right]+a_0,$$

where $b \succ 0$ and k is an arbitrary real constant.

If $b = 0$ we do not have new solutions. Also in these sections, $u_2 \cdots u_4$, $v_2 \cdots v_4$ are calculated as u_1, v_1.

Using the method mentioned above we obtain solutions for

$$a_1 = \pm\sqrt{2}k\sqrt{-\frac{1}{4}\frac{(\beta^2-\alpha^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})-b^2k^4}{bk^2(\beta^2-\alpha^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})+3bk^2}},$$

$$c = \pm\frac{1}{2}\frac{\sqrt{-b(-\alpha^2+\beta^2-2a_0^2+\sqrt{\alpha^4-2\alpha^2\beta^2+4\alpha^2a_0^2+\beta^4-4\beta^2a_0^2+4a_0^4+16k^4b^2-8k^2b\beta^2+8k^2b\alpha^2+48k^2ba_0^2})}}{bk}.$$

5. Application to the perturbed nonlinear Schrodinger's equation

The perturbed nonlinear Schrodinger's equation reads

$$iu_t + u_{xx} + \alpha |u|^2 u + i\left[\gamma_1 u_{xxx} + \gamma_2 |u|^2 u_x + \gamma_3(|u|^2)_x u\right] = 0. \tag{25}$$

We may choose the following complex travelling wave transformation:

$$u = \phi(\xi)e^{i(sx-\Omega t)}, \quad \xi = ik(x - ct) \tag{26}$$

where s, Ω, c are constants to be determined later. Using the complex travelling wave solutions (26) we have the nonlinear ordinary differential equation

$$\gamma_1 k^3\phi''' + (-2sk + 3\gamma_1 s^2 kc)\phi' + (-k^2 + 3\gamma_1 sk^2)\phi'' - (k\gamma_2 + 2k\gamma_3)\phi^2\phi' + (\alpha - \gamma_2 s)\phi^3 + (\Omega - s^2 + \gamma_1 s^3)\phi = 0. \tag{27}$$

Considering the homogeneous balance between ϕ''' and ϕ^3 in Eq. (27), we required that $3m = m+3 \Rightarrow m = \frac{3}{2}$. It should be noticed that m is not a positive integer. However, we may still choose the solution of Eq. (8)(b) in the form

$$\phi = AF^{\frac{3}{2}}. \tag{28}$$

So

$$\begin{aligned}
\phi' &= \frac{3}{2}A\left[bF^{\frac{1}{2}} + F^{\frac{5}{2}}\right],\\
\phi'' &= \frac{3}{2}A\left[3bF^{\frac{3}{2}} + \frac{5}{2}F^{\frac{7}{2}} + \frac{1}{2}b^2F^{-\frac{1}{2}}\right],\\
\phi''' &= \frac{3}{2}A\left[\frac{17}{4}b^2\, F^{\frac{1}{2}} + \frac{53}{4}bF^{\frac{5}{2}} - \frac{1}{4}b^3F^{-\frac{3}{2}} + \frac{35}{4}F^{\frac{9}{2}}\right],\\
\phi^2\phi' &= \frac{3}{2}A^3\left[bF^{\frac{7}{2}} + F^{\frac{11}{2}}\right].
\end{aligned} \tag{29}$$

Substituting (28) and (29) into Eq. (27) yields a set of algebraic equations for A, k, s, c, Ω and c. These equations are found as

$$\begin{aligned}
&-\frac{3}{2}A^3b(k\gamma_2 + 2k\gamma_3) + \frac{15}{4}A(-k^2 + 3\gamma_1 sk^2) = 0,\\
&\frac{51}{8}Ab^2\gamma_1 k^3 + \frac{3}{2}Ab(-2sk + 3\gamma_1 s^2k + kc) = 0,\\
&\frac{9}{2}Ab\gamma_1 k^3 + (\Omega - s^2 + \gamma_1 s^3)A = 0,\\
&\frac{159}{8}Ab\gamma_1 k^3 + \frac{3}{2}A(-2sk + 3\gamma_1 s^2k + kc) = 0.
\end{aligned} \tag{30}$$

Solving Eq. (30) with Maple package we obtain

$$\begin{aligned}
\Omega &= -\frac{9}{2}b\gamma_1 k^3 - s^2 + \gamma_1 s^3,\\
c &= -\frac{53}{4}b\gamma_1 k^2 + 2s - 3\gamma_1 s^2,\\
A &= \pm\frac{\sqrt{5}k}{2}\sqrt{\frac{2(-1 + 3\gamma_1 s)}{bk(\gamma_2 + 2\gamma_3)}}.
\end{aligned} \tag{31}$$

From (7), (28) and (31), we obtain the complex travelling wave solutions of (25) as follows

$$\phi_1 = \pm\frac{\sqrt{5}k}{2}\sqrt{\frac{2(-1 + 3\gamma_1 s)}{bk(\gamma_2 + 2\gamma_3)}}\left[\sqrt{-b}\tanh\left(\sqrt{-b}ik\left(x - \left(-\frac{53}{4}b\gamma_1 k^2 + 2s - 3\gamma_1 s^2\right)t\right)\right)\right]^{\frac{3}{2}},$$

where $b \prec 0$ and k is an arbitrary real constant.

Hence

$$\begin{aligned}
u_1 = {} & \pm\frac{\sqrt{5}k}{2}\sqrt{\frac{2(-1 + 3\gamma_1 s)}{bk(\gamma_2 + 2\gamma_3)}}\\
& \times\left[\sqrt{-b}\tanh\left(\sqrt{-b}ik\left(x - \left(-\frac{53}{4}b\gamma_1 k^2 + 2s - 3\gamma_1 s^2\right)t\right)\right)\right]^{\frac{3}{2}} e^{i\left(sx - \left(-\frac{9}{2}b\gamma_1 k^3 - s^2 + \gamma_1 s^3\right)t\right)},
\end{aligned}$$

and

$$\phi_2 = \pm\frac{\sqrt{5}k}{2}\sqrt{\frac{2(-1+3\gamma_1 s)}{bk(\gamma_2+2\gamma_3)}}\left[\sqrt{-b}\coth\left(\sqrt{-b}ik\left(x-\left(-\frac{53}{4}b\gamma_1k^2+2s-3\gamma_1s^2\right)t\right)\right)\right]^{\frac{3}{2}},$$

$$u_2 = \pm\frac{\sqrt{5}k}{2}\sqrt{\frac{2(-1+3\gamma_1 s)}{bk(\gamma_2+2\gamma_3)}} \times\left[\sqrt{-b}\coth\left(\sqrt{-b}ik\left(x-\left(-\frac{53}{4}b\gamma_1k^2+2s-3\gamma_1s^2\right)t\right)\right)\right]^{\frac{3}{2}} e^{i\left(sx-\left(-\frac{9}{2}b\gamma_1k^3-s^2+\gamma_1s^3\right)t\right)},$$

where $b \prec 0$ and k is an arbitrary real constant.

$$\phi_3 = \pm\frac{\sqrt{5}k}{2}\sqrt{\frac{2(-1+3\gamma_1 s)}{bk(\gamma_2+2\gamma_3)}}\left[\sqrt{b}\tan\left(\sqrt{b}ik\left(x-\left(-\frac{53}{4}b\gamma_1k^2+2s-3\gamma_1s^2\right)t\right)\right)\right]^{\frac{3}{2}},$$

$$u_3 = \pm\frac{\sqrt{5}k}{2}\sqrt{\frac{2(-1+3\gamma_1 s)}{bk(\gamma_2+2\gamma_3)}}\left[\sqrt{b}\tan\left(\sqrt{b}ik\left(x-\left(-\frac{53}{4}b\gamma_1k^2+2s-3\gamma_1s^2\right)t\right)\right)\right]^{\frac{3}{2}} e^{i\left(sx-\left(-\frac{9}{2}b\gamma_1k^3-s^2+\gamma_1s^3\right)t\right)},$$

where $b \succ 0$ and k is an arbitrary real constant.

$$\phi_4 = \pm\frac{\sqrt{5}k}{2}\sqrt{\frac{2(-1+3\gamma_1 s)}{bk(\gamma_2+2\gamma_3)}}\left[-\sqrt{b}\cot\left(\sqrt{b}ik\left(x-\left(-\frac{53}{4}b\gamma_1k^2+2s-3\gamma_1s^2\right)t\right)\right)\right]^{\frac{3}{2}},$$

$$u_4 = \pm\frac{\sqrt{5}k}{2}\sqrt{\frac{2(-1+3\gamma_1 s)}{bk(\gamma_2+2\gamma_3)}}\left[\sqrt{b}\cot\left(\sqrt{b}ik\left(x-\left(-\frac{53}{4}b\gamma_1k^2+2s-3\gamma_1s^2\right)t\right)\right)\right]^{\frac{3}{2}} e^{i\left(sx-\left(-\frac{9}{2}b\gamma_1k^3-s^2+\gamma_1s^3\right)t\right)},$$

where $b \succ 0$ and k is an arbitrary real constant. For $b = 0$ we do not have any new solutions.

6. Conclusion

This paper presents a wider applicability for handling nonlinear evolution equations using the new applications of direct algebraic method. An implementation of the new applications of this method is given by applying it to the Davey–Stewartson system, the coupled Higgs system and the perturbed nonlinear Schrodinger's equation; the balance number of it is not a positive integer. In addition, this method is also computerizable, which allows us to perform complicated and tedious algebraic calculation on a computer by the help of symbolic programs such as *Maple*, *Matlab*, and so on.

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