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Magnetic moments of antidecuplet pentaquarks

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Abstract

We analyze the magnetic moments of the exotic pentaquarks of the flavor antidecuplet in the constituent quark model for the cases in which the ground state is in an orbital $L^p = 0^+$ or a $L^p = 1^-$ state. We derive a set of sum rules for the magnetic moments of antidecuplet baryons and their relation with the magnetic moments of decuplet and octet baryons. The magnetic moment of the $\Theta^+(1540)$ is found to be 0.38, 0.09 and 1.05 μ_N for $J^p = 1/2^-$, $1/2^+$ and $3/2^+$, respectively, which is compared with the results obtained in other approaches.

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1. Introduction

The discovery of the $\Theta^+(1540)$ resonance with positive strangeness $S = +1$ by the LEPS Collaboration [1] and its subsequent confirmation by various other experimental collaborations [2] has motivated an enormous amount of experimental and theoretical studies of exotic baryons [3], despite some other reports in which the pentaquark signal is attributed to kinematical reflections from the decay of mesons [4], or in which no evidence has been found for such states [5]. More recently, the NA49 Collaboration [6] reported evidence for the existence of another exotic baryon $\Xi^{--}(1862)$ with strangeness $S = -2$, although this claim has been shown to be, at least partially, inconsistent with the existing data on the spectroscopy of cascade baryons [7]. Results from the CLAS Collaboration are inconclusive for lack of statistics [8]. The Θ^+ and Ξ^{--} resonances are interpreted as $q^4\bar{q}$ pentaquarks belonging to a flavor antidecuplet with quark structure $uudd\bar{s}$ and $ddss\bar{u}$, respectively. In addition, there is now the first evidence [9] for a heavy pentaquark at 3099 MeV in which the antistrange quark in the Θ^+ is replaced by an anticharm quark.

The spin and parity of the Θ^+ have not yet been determined experimentally. The parity of the pentaquark ground state is predicted to be positive by many studies, such as chiral soliton models [10], cluster models [11–13], lattice QCD [14], and various constituent quark models [15]. However, there are also

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many predictions for a negative parity ground state pentaquark from recent work on QCD sum rules [16], lattice QCD [17], quark model calculations [18–20], as well as from a study in the chiral soliton model [21]. Many different proposals have been made to measure the parity in nucleon–nucleon collisions [22] and in photo-production experiments [23–25].

Another unknown quantity is the magnetic moment. Although it may be difficult to determine its value experimentally, it is an essential ingredient in calculations of the photo- and electro-production cross sections [23–25]. Meanwhile, in the absence of experimental information, one has to rely on model calculations. The magnetic moment of the Θ^+ pentaquark has been calculated in a variety of approaches [24–29] ranging from correlated quark models, the chiral soliton model, QCD sum rules and the MIT bag model. It is of interest to compare these results with those for the constituent quark model as well, since different values of the magnetic moments have their consequences for the photo-production cross sections. which in turn may be used to help determine the quantum numbers of the Θ^+ [24,25].

The aim of this Letter is to study the magnetic moment of antidecuplet pentaquarks in the constituent quark model. In general, the pentaquark spectrum contains states of both positive and negative parity. Their precise ordering, in particular the angular momentum and parity of the ground state, depends on the choice of a specific dynamical model and the relative size of the orbital excitations, the spin-flavor splittings and the spin–spin couplings [15,18–20,30]. The present analysis is carried out for antidecuplet pentaquarks of both parities: $J^p = 1/2^-$, $1/2^+$ and $3/2^+$. We derive a set of sum rules for the magnetic moments, and make a comparison with the results obtained in correlated (or cluster) quark models. Some preliminary results of this work have been published in [20]

2. Pentaquark wave functions

We consider pentaquarks to be built of five constituent parts which are characterized by both internal and spatial degrees of freedom. The internal degrees of freedom are taken to be the three light flavors u, d, s with spin $s = 1/2$ and three colors r, g, b. The corresponding algebraic structure consists of the usual spin-flavor and color algebras $SU_{\mathrm{sf}}(6) \otimes SU_{\mathrm{c}}(3)$. In the construction of the classification scheme we are guided by two conditions: the pentaquark wave function should be antisymmetric under any permutation of the four quarks, and should be a color singlet. The permutation symmetry of the four-quark subsystem is characterized by the S_4 Young tableaux [4], [31], [22], [211] and [1111] or, equivalently, by the irreducible representations of the tetrahedral group $\mathcal{T}_d$ (which is isomorphic to S_4) as A_1 (symmetric), F_2, E, F_1 (mixed symmetric) and A_2 (antisymmetric), respectively. For notational purposes we prefer to use the latter to label the discrete symmetry of the pentaquark wave functions. The corresponding dimensions are 1, 3, 2, 3 and 1, respectively. The full decomposition of the spin-flavor states into spin and flavor states $SU_{\mathrm{sf}}(6) \supset SU_{\mathrm{f}}(3) \otimes SU_{\mathrm{s}}(2)$ is given in Table 5 of [19]. The states of a given flavor multiplet can be labeled by isospin I, I_3 and hypercharge Y. It is difficult to distinguish the pentaquark flavor singlets, octets and decuplets from the three-quark flavor multiplets, since they have the same values of the hypercharge Y and isospin projection I_3. The same observation holds for the majority of the states in the remaining flavor states. However, the antidecuplets, the 27-plets and 35-plets contain in addition exotic states which cannot be obtained from three-quark configurations. These states are more easily identified experimentally due to the uniqueness of their quantum numbers. The recently observed Θ^+ and Ξ^{--} resonances are interpreted as pentaquarks belonging to a flavor antidecuplet with isospin $I = 0$ and $I = 3/2$, respectively. In Fig. 1 the exotic states are indicated by a $\bullet$: the Θ^+ is the isosinglet $I = I_3 = 0$ with hypercharge $Y = 2$ (strangeness $S = +1$), and the cascades $\Xi^+_{3/2}$ and $\Xi^{--}_{3/2}$ have hypercharge $Y = -1$ (strangeness $S = -2$) and isospin $I = 3/2$ with projection $I_3 = 3/2$ and $-3/2$, respectively.

A convenient choice to describe the relative motion of the constituent parts is provided by the Jacobi coordinates [31]

$$\vec{\rho}_1 = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2),$$

$$\vec{\rho}_2 = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3),$$

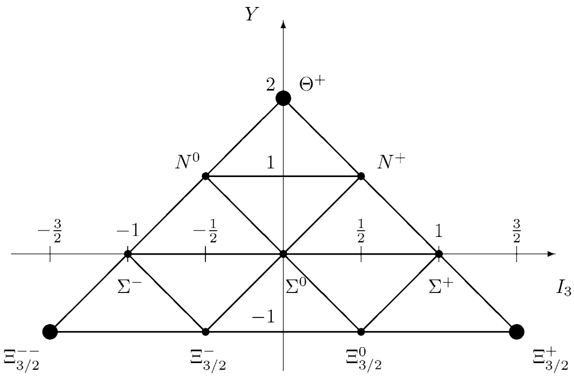

Fig. 1. $SU(3)$ flavor multiplet [33] with E symmetry. The isospin-hypercharge multiplets are $(I, Y) = (0, 2)$, $(\frac{1}{2}, 1)$, $(1, 0)$ and $(\frac{3}{2}, -1)$. Exotic states are indicated with •.

$$\vec{\rho}_3 = \frac{1}{\sqrt{12}}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 - 3\vec{r}_4),$$
$$\vec{\rho}_4 = \frac{1}{\sqrt{20}}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 - 4\vec{r}_5), \tag{1}$$

where $\vec{r}_i$ $(i = 1, \ldots, 4)$ denote the coordinate of the ith quark, and $\vec{r}_5$ that of the antiquark. The last Jacobi coordinate is symmetric under the interchange of the quark coordinates, and hence transforms as A_1 under $\mathcal{T}_d$ $(\sim S_4)$, whereas the first three transform as three components of F_2 [31].

Since the color part of the pentaquark wave function is a [222] singlet and that of the antiquark a [11] anti-triplet, the color wave function of the four-quark configuration is a [211] triplet with F_1 symmetry. The total q^4 wave function is antisymmetric (A_2), hence the orbital-spin-flavor part has to have F_2 symmetry

$$\psi = \left[\psi^{\mathrm{c}}_{F_1} \times \psi^{\mathrm{osf}}_{F_2}\right]_{A_2}. \tag{2}$$

Here the square brackets $[\cdots]$ denote the tensor coupling under the tetrahedral group $\mathcal{T}_d$. The exotic spin-flavor states associated with the S-wave state $L^p_t = 0^+_{A_1}$ belong to the $[f]_t = [42111]_{F_2}$ spin-flavor multiplet [19]. The corresponding orbital-spin-flavor wave function is given by

$$\psi^{\mathrm{osf}}_{F_2} = \left[\psi^{\mathrm{o}}_{A_1} \times \psi^{\mathrm{sf}}_{F_2}\right]_{F_2}, \tag{3}$$

where the orbital wave function depends on the Jacobi coordinates of Eq. (1), $\psi^{\mathrm{o}}_t = \psi^{\mathrm{o}}_t(\vec{\rho}_1, \vec{\rho}_2, \vec{\rho}_3, \vec{\rho}_4)$. A P-wave radial excitation with $L^p_t = 1^-_{F_2}$ gives rise to exotic pentaquark states of the $[51111]_{A_1}$, $[42111]_{F_2}$, $[33111]_E$ and $[32211]_{F_1}$ spin-flavor configurations. They are characterized by the orbital-spin-flavor wave functions

$$\psi^{\mathrm{osf}}_{F_2} = \left[\psi^{\mathrm{o}}_{F_2} \times \psi^{\mathrm{sf}}_t\right]_{F_2}, \tag{4}$$

with $t = A_1$, F_2, E and F_1, respectively. In this Letter, we study the magnetic moments of the lowest pentaquark antidecuplet with positive and negative parity.

3. Magnetic moments

A compilation of theoretical values of the magnetic moments of exotic pentaquarks has been presented in [28,29] for the chiral soliton model, different correlated quark models, the MIT bag model and for QCD sum rules. To the best of our knowledge, the present calculation is the first one for an uncorrelated or constituent quark model. The magnetic moment of a multiquark system is given by the sum of the magnetic moments of its constituent parts

$$\vec{\mu} = \vec{\mu}_{\mathrm{spin}} + \vec{\mu}_{\mathrm{orb}} = \sum_i \mu_i (2\vec{s}_i + \vec{\ell}_i), \tag{5}$$

where $\mu_i = e_i/2m_i$, e_i and m_i represent the magnetic moment, the electric charge and the constituent mass of the ith (anti)quark. The quark magnetic moments μ_u, μ_d and μ_s are determined from the proton, neutron and Λ magnetic moments to be $\mu_u = 1.852$ μ_N, $\mu_d = -0.972\mu_N$ and $\mu_s = -0.613\mu_N$ [32]. The magnetic moments of the antiquarks satisfy $\mu_{\bar{q}} = -\mu_q$.

3.1. Negative parity

We first analyze the negative parity antidecuplet states that are associated with the S-wave state $L^p_t = 0^+_{A_1}$ and belong to the $[f]_t = [42111]_{F_2}$ spin-flavor multiplet. The corresponding pentaquark wave function with angular momentum J and projection $M = J$ is given by Eqs. (2) and (3)

$$\psi_J = \frac{1}{\sqrt{3}}\left[\psi^{\mathrm{o}}_{A_1}\left(\psi^{\mathrm{c}}_{F_{1\lambda}}\psi^{\mathrm{sf}}_{F_{2\rho}} - \psi^{\mathrm{c}}_{F_{1\rho}}\psi^{\mathrm{sf}}_{F_{2\lambda}} + \psi^{\mathrm{c}}_{F_{1\eta}}\psi^{\mathrm{sf}}_{F_{2\eta}}\right)\right]^{(J)}_J. \tag{6}$$

The spin-flavor part can be expressed as a product of the antidecuplet flavor wave function ϕ_E and the

$s=1/2$ spin wave function χ_{F_2}

$$\psi^{\rm sf}_{F_{2\rho}} = -\frac{1}{2}\phi_{E_\rho}\chi_{F_{2\lambda}} - \frac{1}{2}\phi_{E_\lambda}\chi_{F_{2\rho}} + \frac{1}{\sqrt{2}}\phi_{E_\rho}\chi_{F_{2\eta}},$$
$$\psi^{\rm sf}_{F_{2\lambda}} = -\frac{1}{2}\phi_{E_\rho}\chi_{F_{2\rho}} + \frac{1}{2}\phi_{E_\lambda}\chi_{F_{2\lambda}} + \frac{1}{\sqrt{2}}\phi_{E_\lambda}\chi_{F_{2\eta}},$$
$$\psi^{\rm sf}_{F_{2\eta}} = \frac{1}{\sqrt{2}}\phi_{E_\rho}\chi_{F_{2\rho}} + \frac{1}{\sqrt{2}}\phi_{E_\lambda}\chi_{F_{2\lambda}}. \tag{7}$$

The coefficients in Eqs. (6) and (7) are a consequence of the tensor couplings under the tetrahedral group $\mathcal{T}_d$ (Clebsch–Gordon coefficients). The total angular momentum is $J=1/2$. The explicit form of the spin and flavor wave functions is given in the appendix. Since the orbital wave function has $L_t^p = 0^+_{A_1}$, the magnetic moment only depends on the spin part. For the Θ^+, $\Xi^+_{3/2}$ and $\Xi^{--}_{3/2}$ exotic states we obtain

$$\mu_{\Theta^+} = \frac{1}{3}(2\mu_u + 2\mu_d + \mu_s) = 0.38\mu_N,$$
$$\mu_{\Xi^{--}_{3/2}} = \frac{1}{3}(\mu_u + 2\mu_d + 2\mu_s) = -0.44\mu_N,$$
$$\mu_{\Xi^{+}_{3/2}} = \frac{1}{3}(2\mu_u + \mu_d + 2\mu_s) = 0.50\mu_N, \tag{8}$$

in agreement with the results obtained [28] for the MIT bag model [33]. We note, that these results are independent of the orbital wave functions, and are valid for any quark model in which the eigenstates have good $SU_{\rm sf}(6)$ spin-flavor symmetry.

3.2. Positive parity

Next, we study the case of positive parity antidecuplet states. These pentaquark states correspond to a P-wave state $L_t^p = 1^-_{F_2}$ and belong to the $[f]_t = [51111]_{A_1}$ spin-flavor multiplet. The corresponding pentaquark wave function with angular momentum J and projection $M=J$ is given by Eqs. (2) and (4)

$$\psi_J = \frac{1}{\sqrt{3}}\Big[\big(\psi^{\rm o}_{F_{2\rho}}\psi^{\rm c}_{F_{1\lambda}} - \psi^{\rm o}_{F_{2\lambda}}\psi^{\rm c}_{F_{1\rho}} + \psi^{\rm o}_{F_{2\eta}}\psi^{\rm c}_{F_{1\eta}}\big)\psi^{\rm sf}_{A_1}\Big]^{(J)}_J. \tag{9}$$

The spin-flavor part is now a product of the antidecuplet flavor wave function ϕ_E and the $s=1/2$ spin wave function χ_E

$$\psi^{\rm sf}_{A_1} = \frac{1}{\sqrt{2}}(\phi_{E_\rho}\chi_{E_\rho} + \phi_{E_\lambda}\chi_{E_\lambda}). \tag{10}$$

The total angular momentum is $J=1/2,\ 3/2$. The explicit form of the spin and flavor wave functions is given in the appendix. Since the spin of the four-quark system is $s=0$, the spin part of the magnetic moment only depends on the contribution from the antiquark. For Θ^+ state with $J^p = 1/2^+$ we obtain

$$\begin{aligned}\langle\psi_J|\vec{\mu}_{\rm spin}|\psi_J\rangle &= \Big[\big\langle 1,0,\tfrac{1}{2},\tfrac{1}{2}\big|\tfrac{1}{2},\tfrac{1}{2}\big\rangle^2 - \big\langle 1,1,\tfrac{1}{2},-\tfrac{1}{2}\big|\tfrac{1}{2},\tfrac{1}{2}\big\rangle^2\Big]\mu_{\bar{s}}\\ &= \frac{1}{3}\mu_s.\end{aligned} \tag{11}$$

In contrast to the previous case of a negative parity pentaquark, now we also have a contribution from the orbital angular momentum. The orbital excitation with $L_t^p = 1^-_{F_2}$ is an excitation in the relative coordinates of the four-quark subsystem. There is no excitation in the relative coordinate between the four-quark system and the antiquark. Therefore, the orbital part to the magnetic moment is given by

$$\begin{aligned}\langle\psi_J|\vec{\mu}_{\rm orb}|\psi_J\rangle &= \langle\psi_J|\mu_2\,\vec{\ell}_{\rho_1} + \mu_3\,\vec{\ell}_{\rho_2} + \mu_4\,\vec{\ell}_{\rho_3}|\psi_J\rangle\\ &= 3\langle\psi_J|\mu_4\,\vec{\ell}_{\rho_3}|\psi_J\rangle\\ &= \frac{1}{2}\big\langle 1,1,\tfrac{1}{2},-\tfrac{1}{2}\big|\tfrac{1}{2},\tfrac{1}{2}\big\rangle^2(\mu_u+\mu_d)\\ &= \frac{1}{3}(\mu_u+\mu_d).\end{aligned} \tag{12}$$

The total magnetic moment of the Θ^+ state is

$$\mu_{\Theta^+} = \frac{1}{3}(\mu_u + \mu_d + \mu_s) = 0.09\mu_N. \tag{13}$$

The magnetic moments of the exotic pentaquarks of the antidecuplet Θ^+, $\Xi^+_{3/2}$ and $\Xi^{--}_{3/2}$ with angular momentum and parity $J^p = 1/2^+$ are equal[1]

$$\begin{aligned}\mu_{\Xi^{--}_{3/2}} = \mu_{\Xi^{+}_{3/2}} = \mu_{\Theta^+} &= \frac{1}{3}(\mu_u+\mu_d+\mu_s)\\ &= 0.09\mu_N.\end{aligned} \tag{14}$$

The only difference for pentaquarks with angular momentum and parity $J^p = 3/2^+$ is in the angular momentum couplings. As a result we find

$$\mu_{\Theta^+} = \frac{1}{2}(\mu_u + \mu_d - 2\mu_s) = 1.05\mu_N,$$

[1] In this calculation we have used harmonic oscillator wave functions with $N=1$.

Table 1
Magnetic moments of antidecuplet pentaquarks in μ_N for $J^p = \frac{1}{2}^-$, $\frac{1}{2}^+$ and $\frac{3}{2}^+$

	$J^p = \frac{1}{2}^-$	$J^p = \frac{1}{2}^+$	$J^p = \frac{3}{2}^+$
Θ^+	$\frac{1}{9}(6\mu_u + 6\mu_d + 3\mu_s)$	$\frac{1}{3}(\mu_u + \mu_d + \mu_s)$	$\frac{1}{2}(\mu_u + \mu_d - 2\mu_s)$
N^0	$\frac{1}{9}(5\mu_u + 6\mu_d + 4\mu_s)$	$\frac{1}{3}(\mu_u + \mu_d + \mu_s)$	$\frac{1}{2}(\mu_d - \mu_s)$
N^+	$\frac{1}{9}(6\mu_u + 5\mu_d + 4\mu_s)$	$\frac{1}{3}(\mu_u + \mu_d + \mu_s)$	$\frac{1}{2}(\mu_u - \mu_s)$
Σ^-	$\frac{1}{9}(4\mu_u + 6\mu_d + 5\mu_s)$	$\frac{1}{3}(\mu_u + \mu_d + \mu_s)$	$\frac{1}{2}(-\mu_u + \mu_d)$
Σ^0	$\frac{1}{9}(5\mu_u + 5\mu_d + 5\mu_s)$	$\frac{1}{3}(\mu_u + \mu_d + \mu_s)$	0
Σ^+	$\frac{1}{9}(6\mu_u + 4\mu_d + 5\mu_s)$	$\frac{1}{3}(\mu_u + \mu_d + \mu_s)$	$\frac{1}{2}(\mu_u - \mu_d)$
$\Xi^{--}_{3/2}$	$\frac{1}{9}(3\mu_u + 6\mu_d + 6\mu_s)$	$\frac{1}{3}(\mu_u + \mu_d + \mu_s)$	$\frac{1}{2}(-2\mu_u + \mu_d + \mu_s)$
$\Xi^{-}_{3/2}$	$\frac{1}{9}(4\mu_u + 5\mu_d + 6\mu_s)$	$\frac{1}{3}(\mu_u + \mu_d + \mu_s)$	$\frac{1}{2}(-\mu_u + \mu_s)$
$\Xi^{0}_{3/2}$	$\frac{1}{9}(5\mu_u + 4\mu_d + 6\mu_s)$	$\frac{1}{3}(\mu_u + \mu_d + \mu_s)$	$\frac{1}{2}(-\mu_d + \mu_s)$
$\Xi^{+}_{3/2}$	$\frac{1}{9}(6\mu_u + 3\mu_d + 6\mu_s)$	$\frac{1}{3}(\mu_u + \mu_d + \mu_s)$	$\frac{1}{2}(\mu_u - 2\mu_d + \mu_s)$
$\sum_{i\in\overline{10}} \mu_i$	$\frac{50}{9}(\mu_u + \mu_d + \mu_s)$	$\frac{10}{3}(\mu_u + \mu_d + \mu_s)$	0

$$\mu_{\Xi^{--}_{3/2}} = \frac{1}{2}(-2\mu_u + \mu_d + \mu_s) = -2.64\mu_N,$$
$$\mu_{\Xi^{+}_{3/2}} = \frac{1}{2}(\mu_u - 2\mu_d + \mu_s) = 1.59\mu_N. \tag{15}$$

3.3. Sum rules

The results obtained for the magnetic moments are valid for any constituent quark model in which the eigenstates have good $SU_{sf}(6)$ spin-flavor symmetry. In Table 1, we present the magnetic moments of all antidecuplet pentaquarks for the three different combinations of angular momentum and parity discussed in the previous section, i.e., $J^p = 1/2^-$, $1/2^+$ and $3/2^+$. In all three cases, the magnetic moments satisfy the generalized Coleman–Glashow sum rules [34,35]

$$\begin{aligned}
&\mu_{\Theta^+} + \mu_{\Xi^{+}_{3/2}} = \mu_{N^+} + \mu_{\Sigma^+},\\
&\mu_{\Theta^+} + \mu_{\Xi^{--}_{3/2}} = \mu_{N^0} + \mu_{\Sigma^-},\\
&\mu_{\Xi^{--}_{3/2}} + \mu_{\Xi^{+}_{3/2}} = \mu_{\Xi^{-}_{3/2}} + \mu_{\Xi^{0}_{3/2}},
\end{aligned} \tag{16}$$

and

$$2\mu_{\Sigma^0} = \mu_{\Sigma^-} + \mu_{\Sigma^+} = \mu_{N^0} + \mu_{\Xi^{0}_{3/2}} = \mu_{N^+} + \mu_{\Xi^{-}_{3/2}}. \tag{17}$$

The same sum rules hold for the chiral quark-soliton model in the chiral limit [26]. In addition, there exist interesting sum rules that relate the magnetic moments of the antidecuplet pentaquarks to those of the decuplet and octet baryons. For the case of negative parity pentaquarks, we use Eq. (8) to obtain the sum rules

$$\begin{aligned}
&\mu_{\Theta^+} - \mu_{\Xi^{--}_{3/2}}\\
&\quad = \frac{1}{9}(\mu_{\Delta^{++}} - \mu_{\Omega^-})\\
&\quad = \frac{1}{12}(2\mu_p + \mu_n + \mu_{\Sigma^+} - \mu_{\Sigma^-} - \mu_{\Xi^0} - 2\mu_{\Xi^-}),\\
&\mu_{\Theta^+} - \mu_{\Xi^{+}_{3/2}}\\
&\quad = \frac{1}{9}(\mu_{\Delta^-} - \mu_{\Omega^-})\\
&\quad = \frac{1}{12}(\mu_p + 2\mu_n - \mu_{\Sigma^+} + \mu_{\Sigma^-} - 2\mu_{\Xi^0} - \mu_{\Xi^-}),\\
&\mu_{\Xi^{+}_{3/2}} - \mu_{\Xi^{--}_{3/2}}\\
&\quad = \frac{1}{9}(\mu_{\Delta^{++}} - \mu_{\Delta^-})\\
&\quad = \frac{1}{12}(\mu_p - \mu_n + 2\mu_{\Sigma^+} - 2\mu_{\Sigma^-} + \mu_{\Xi^0} - \mu_{\Xi^-}).
\end{aligned} \tag{18}$$

The first sum rule is similar, but not identical, to the result obtained in the chiral quark-soliton model[26].[2] For the positive parity pentaquarks, the results of

[2] Note that in Eq. (19) of Ref. [26] a term $-3v_2/8$ is missing. The present result of Eq. (18) is obtained for $v_2 = 40(w - v)/3$ which, for the three fits considered in [26], would correspond to numerical values of $v_2 = 4.91$, 4.37 and 4.00, respectively, quite close to the value of $v_2 = 5$ used in the calculation of the magnetic moments in [26].

Eq. (15) can be used to obtain the sum rules that relate the magnetic moments of the $J^p = 3/2^+$ antidecuplet pentaquarks to those of the decuplet baryons

$$\mu_{\Theta^+} - \mu_{\Xi^{--}_{3/2}} = \frac{9}{2}(\mu_{\Delta^{++}} - \mu_{\Omega^-}),$$
$$\mu_{\Theta^+} - \mu_{\Xi^{+}_{3/2}} = \frac{9}{2}(\mu_{\Delta^{-}} - \mu_{\Omega^-}),$$
$$\mu_{\Xi^{+}_{3/2}} - \mu_{\Xi^{--}_{3/2}} = \frac{9}{2}(\mu_{\Delta^{++}} - \mu_{\Delta^-}). \tag{19}$$

In the limit of equal quark masses $m_u = m_d = m_s = m$, the magnetic moments of the antidecuplet pentaquark states (denoted by $i \in \overline{10}$) become proportional to the electric charges

$$\mu_i = \frac{1}{9}\frac{1}{2m}Q_i, \quad \text{for } J^p = 1/2^-,$$
$$\mu_i = \frac{1}{2}\frac{1}{2m}Q_i, \quad \text{for } J^p = 3/2^+, \tag{20}$$

compared to

$$\mu_i = \frac{1}{2m}Q_i, \tag{21}$$

for the decuplet baryons ($i \in 10$). For the exotic pentaquarks, Eq. (20) implies $\mu_{\Xi^{--}_{3/2}} = -2\mu_{\Xi^{+}_{3/2}} = -2\mu_{\Theta^+}$. For angular momentum and parity $J^p = 1/2^+$, the magnetic moments vanish in the limit of equal quark masses due to a cancellation between the spin and orbital contributions. For all three cases, the sum of the magnetic moments of all members of the antidecuplet vanishes identically

$$\sum_{i\in\overline{10}} \mu_i = 0. \tag{22}$$

4. Discussion

The magnetic moments for negative parity $J^p = 1/2^-$ pentaquarks of Eq. (8) are typically an order of magnitude smaller than the proton magnetic moment, whereas for positive parity $J^p = 1/2^+$ they are even smaller due to a cancellation between orbital and spin contributions, see Eq. (14). The largest values of the magnetic moment are obtained for $J^p = 3/2^+$ pentaquarks, but they are still smaller than the proton value. The magnetic moment of the $\Theta(1540)$ in the constituent quark model is found to be 0.38, 0.09 and 1.05 μ_N for $J^p = 1/2^-$, $1/2^+$ and $3/2^+$, respectively.

Table 2
Comparison of magnetic moments in μ_N of exotic antidecuplet pentaquarks with angular momentum and parity $J^p = \frac{1}{2}^-$

		$J^p = \frac{1}{2}^-$		
Method	Ref.	Θ^+	$\Xi^{+}_{3/2}$	$\Xi^{--}_{3/2}$
Present		0.38	0.50	−0.44
MIT bag	[28]	0.37	0.45	−0.42
JW diquark	[24]	0.49		
KN bound state	[24]	0.31		
Cluster	[25]	0.60		
QCD sum rules	[27]	0.12 ± 0.06[a]		

[a] Absolute value.

In Tables 2 and 3 we present a comparison with other theoretical predictions for the magnetic moments of exotic pentaquarks with negative and positive parity, respectively.

The first estimate of the Θ^+ magnetic moment was made by Nam, Hosaka and Kim in a study of photo-production reactions [24]. They used the diquark model of [11] (JW) to estimate the anomalous magnetic moment as $\kappa = -0.7$ for positive and -0.2 for negative parity. For the Θ^+ as a KN bound state, they obtained $\kappa = -0.4$ for positive and -0.5 for negative parity. In all cases, the spin is $J = 1/2$.

In [26], Kim and Praszałowicz investigated the magnetic moments of the baryon antidecuplet in the chiral soliton model in the chiral limit (χQCD). The spin and parity are $J^p = 1/2^+$. The Θ^+ magnetic moment was found to be 0.12, 0.20 or 0.30 μ_N, depending on three different ways to determine the parameters. The magnetic moments of the exotic cascade pentaquarks are obtained from the proportionality of the magnetic moments of the antidecuplet baryons to the electric charge. We note that in this calculation no $SU(3)$ symmetry breaking effects were taken into account, unlike the other approaches discussed in this section.

Also Zhao used the diquark model of [11] to obtain an anomalous magnetic moment $\kappa = -0.87$ for a positive parity Θ^+ pentaquark [25]. For the case of negative parity, the magnetic moment was estimated from the sum of $u\bar{s}$ and udd clusters to be 0.60 μ_N.

Huang et al. used light cone QCD sum rules to extract the absolute value of the $\Theta(1540)$ magnetic moment as 0.12 ± 0.06 μ_N [27]. In this calculation, the Θ^+ was assumed to be an isoscalar with spin $J = 1/2$, no assumption was made of its parity.

Table 3
Comparison of magnetic moments in μ_N of exotic antidecuplet pentaquarks with angular momentum and parity $J^p = \frac{1}{2}^+$ and $\frac{3}{2}^+$

	$J^p = \frac{1}{2}^+$				$J^p = \frac{3}{2}^+$			
Method	Ref.	Θ^+	$\Xi^+_{3/2}$	$\Xi^{--}_{3/2}$	Ref.	Θ^+	$\Xi^+_{3/2}$	$\Xi^{--}_{3/2}$
Present		0.09	0.09	0.09		1.05	1.59	−2.64
JW diquark	[25,28]	0.08	−0.06	0.12	[29]	1.01	1.22	−2.43
SZ diquark	[28]	0.23	0.33	−0.17	[29]	1.23	1.85	−2.84
KL cluster	[28]	0.19	0.13	−0.43	[29]	0.84	0.89	−1.20
χQSM	[26]	0.12	0.12	−0.24				
χQSM	[26]	0.20	0.20	−0.40				
χQSM	[26]	0.30	0.30	−0.60				
JW diquark	[24]	0.18						
KN bound state	[24]	0.36						
QCD sum rules	[27]	0.12 ± 0.06[a]						

[a] Absolute value.

Finally, there are two studies in which the magnetic moments of exotic antidecuplet baryons are calculated for spin $J = 1/2$ [28] and $J = 3/2$ [29] for a variety of models of pentaquarks: the diquark–diquark–antiquark models of [11] and [12] (SZ), as a diquark–triquark bound state [13] (KL), and the MIT bag model [33]. In the latter case, the parity is negative, whereas in all others it is positive.

Although there is some variation in the numerical values obtained for different models of pentaquarks, generally speaking, the predictions for the magnetic moment of the Θ^+ are relatively close, especially in comparison with the magnetic moment of the proton they are all small. For the case of negative parity pentaquarks, our results are identical to those derived for the MIT bag model [28]. The small difference in the numerical values is due to the values of the quark magnetic moments μ_q used in the calculations. For positive parity pentaquarks with $J^p = 1/2^+$ the values of the magnetic moments are suppressed due to cancellations between the spin and orbital contributions. Our predictions for the magnetic moments of the exotic pentaquarks with $J^p = 3/2^+$ are in qualitative agreement with those of the diquark models of [11] and [12], but differ somewhat from the ones for the diquark–triquark cluster model of [13].

5. Summary and conclusions

In this Letter, we have analyzed the pentaquark magnetic moments of the lowest flavor antidecuplet for both positive and negative parity in the constituent quark model. The resulting magnetic moments were obtained in closed analytic form, which made it possible to derive generalized Coleman–Glashow sum rules for the antidecuplet magnetic moments, as well as sum rules connecting the magnetic moments of antidecuplet pentaquarks to those of decuplet and octet baryons. The numerical values are in qualitative agreement with those obtained in other approaches, such as correlated quark models, QCD sum rules, MIT bag model and the chiral soliton model.

In conclusion, the spectroscopy of exotic baryons will be a key testing ground for models of baryons and their structure. Especially the measurement of the angular momentum and parity of the $\Theta^+(1540)$ may help to distinguish between different models and to gain more insight into the relevant degrees of freedom and the underlying dynamics that determines the properties of exotic baryons. The magnetic moment is an important ingredient for the calculation of the total and differential cross sections for photo- and electro-production which have been proposed as a tool to help determine the quantum numbers of the Θ^+ pentaquarks. The values of the magnetic moments presented here, together with those of [28,29], may be used as an input for such calculations.

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Appendix A. Spin wave functions

The spin wave function with $s = 1/2$ and F_2 symmetry is a combination of the spin wave function for the four-quark system with [31] and $s = 1$ and that of the antiquark with $s = 1/2$. We start by decoupling the spin of the antiquark

$$\chi_{F_{2\alpha}} = \left|[31], 1, \tfrac{1}{2}; \tfrac{1}{2}, \tfrac{1}{2}\right\rangle_{F_{2\alpha}} = \sqrt{\frac{2}{3}}\left|[31], 1, 1\right\rangle_{F_{2\alpha}} \downarrow - \sqrt{\frac{1}{3}}\left|[31], 1, 0\right\rangle_{F_{2\alpha}} \uparrow, \tag{A.1}$$

with $\alpha = \rho, \lambda, \eta$. The $\uparrow$ and $\downarrow$ represent the spin of the antiquark. The spin wave functions of the four-quark system are given by

$$\begin{aligned}
\left|[31], 1, 1\right\rangle_{F_{2\rho}} &= -\frac{1}{\sqrt{2}}|\downarrow\uparrow\uparrow\uparrow - \uparrow\downarrow\uparrow\uparrow\rangle, \\
\left|[31], 1, 1\right\rangle_{F_{2\lambda}} &= -\frac{1}{\sqrt{6}}|\downarrow\uparrow\uparrow\uparrow + \uparrow\downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow\uparrow\rangle, \\
\left|[31], 1, 1\right\rangle_{F_{2\eta}} &- \frac{1}{2\sqrt{3}}|\downarrow\uparrow\uparrow\uparrow + \uparrow\downarrow\uparrow\uparrow \\
&\quad + \uparrow\uparrow\downarrow\uparrow - 3\uparrow\uparrow\uparrow\downarrow\rangle.
\end{aligned} \tag{A.2}$$

The states with other values of the projection m_s can be obtained by applying the lowering operator in spin space.

The spin wave function with $s = 1/2$ and E symmetry is a combination of the spin wave function for the four-quark system with [22] and $s = 0$ and that of the antiquark with $s = 1/2$

$$\chi_{E_\alpha} = \left|[22], 0, \tfrac{1}{2}; \tfrac{1}{2}, \tfrac{1}{2}\right\rangle_{E_\alpha} = \left|[22], 0, 0\right\rangle_{E_\alpha} \uparrow, \tag{A.3}$$

with $\alpha = \rho, \lambda$. In this case, the spin wave functions of the four-quark system are given by

$$\begin{aligned}
\left|[22], 0, 0\right\rangle_{E_\rho} &= -\frac{1}{2}|\downarrow\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow\uparrow - \downarrow\uparrow\downarrow\uparrow\rangle, \\
\left|[22], 0, 0\right\rangle_{E_\lambda} &= -\frac{1}{2\sqrt{3}}|\downarrow\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow\downarrow - 2\uparrow\uparrow\downarrow\downarrow \\
&\quad + \uparrow\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow\uparrow - 2\downarrow\downarrow\uparrow\uparrow\rangle.
\end{aligned} \tag{A.4}$$

Appendix B. Flavor wave functions

The flavor wave functions for the antidecuplet Θ^+ pentaquark with $I = I_3 = 0$ are given by

$$\begin{aligned}
\phi_{E_\rho} &= -\frac{1}{2}(duud - udud + uddu - dudu)\bar{s}, \\
\phi_{E_\lambda} &= -\frac{1}{2\sqrt{3}}(duud + udud - 2uudd + uddu \\
&\quad + dudu - 2dduu)\bar{s}.
\end{aligned} \tag{B.1}$$

The flavor states with other values of the isospin I, its projection I_3 and hypercharge Y can be obtained by applying the ladder operators in flavor space and using the phase convention of De Swart [36].

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