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Exclusive J/ψ productions at e^+e^- colliders

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Abstract

Exclusive quarkonium pair production in electron–positron collisions is studied in non-relativistic QCD. The obtained cross section for $J/\psi\ \eta_c$ production in the leading order is confronted against the recent measurements by the Belle Collaboration at KEKB. It is shown that a large renormalization K-factor is necessary to explain the experimental data. We point out that the $J^{PC} = 0^{-+}$ nature of the hadronic systems that are assigned to be η_c should be tested by the triple angular distributions in terms of the scattering angle, and, polar and azimuthal angles of J/ψ into leptons. We further study $J/\psi\ J/\psi$ and $\Upsilon\ \Upsilon$ productions at LEP energies. Although the axial–vector couplings of the Z-boson to charm and bottom quarks allow production of such pairs when one of them is polarized transversally and the other longitudinally, we find that the integrated luminosity at Z pole accumulated by LEP is not large enough to observe the exclusive pair production of quarkonium.

1. Introduction

Quarkonium production and decays have long been considered as an ideal means to investigate the bound-state formation in QCD. Due to its approximately non-relativistic nature, the description of the heavy quark and anti-quark system is one of the simplest applications of QCD. For example, the calculation of quarkonium leptonic decays render experimental results with a high precision, which may play a crucial role in investigating various phenomena such as measuring the parton distribution, detecting the quark–gluon–plasma signal and even new physics.

While the quarkonium physics has been studied for more than twenty years, the recent interest in the field has been focused on the colour-octet scenario [1] which was triggered by the high-p_T J/ψ surplus production discovered by the CDF Collaboration at the Tevatron in 1992 [2–4]. It was proposed based on a novel effective theory, the non-relativistic QCD(NRQCD) [5]. Having achieved the first-step of explaining the CDF data, the colour-octet mechanism (COM) had a strong impact into the quarkonium physics and various efforts have been made to confirm this mechanism. Although the theoretical framework seems to show qualitative agreements with experimental data, there are certain difficulties in the quantitative estimate of the colour-octet contribution [6], in particular, in HERA physics [7]. It was in such cir-

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cumstances that the B factory experiments reported their first result on the prompt charmonium production from e^+e^- collider at $\sqrt{s} = 10.6$ GeV [8,9]. As far as hadronic uncertainty is concerned, the B factories would provide clearer information of the quarkonium production.

The first result for the inclusive $e^+ + e^- \to J/\psi + X$ process from Belle (with 32.7 fb^{-1} data set) indicated a discrepancy from the theoretical prediction [9]. The $e^+ + e^- \to J/\psi + c\bar{c}$ process seemed to dominate the threshold region ($z \to 1$) of the energy spectra of the differential cross section comparing to the COM process $e^+ + e^- \to J/\psi + g$ and colour-singlet process $e^+ + e^- \to J/\psi + gg$, contrary to the theoretical expectation in [10–14]. In their second report (based on 41.8 fb^{-1} data set) [15], the direct measurement of the $e^+ + e^- \to J/\psi + c\bar{c}$ process is presented simultaneously and it is found that experimental result is about 10 times larger than the theoretical prediction for this process. In [15], the total cross section of the exclusive $e^+ + e^- \to J/\psi + \eta_c$ process is found to be

$$\sigma\left(e^+ + e^- \to J/\psi + \eta_c\right) \times \mathcal{B}(\eta_c \to \geqslant 4 \text{ charged}) = \left(0.033^{+0.007}_{-0.006} \pm 0.009\right) \text{pb}. \tag{1}$$

Note that the observed number of events for $J/\psi\ \eta_c$ is (67^{+13}_{-12}). Motivated by this measurement, we investigate $J/\psi\ \eta_c$ production at e^+e^- colliders.

The reminder of the Letter is organized as follows. In Section 2, we will give all the formulae used in our analysis. Motivated by the observed large cross section of the $J/\psi\ \eta_c$ production by Belle, we further consider the heavy quarkonium pair production at LEP energies. For this purpose, we include the formulae for the Z intermediated processes. In Section 3, our numerical results are presented. The double J/ψ production through $\gamma^*\gamma^*$ intermediated states which has been proposed as an explanation of the large cross section of the $J/\psi\ \eta_c$ process [18] is also examined. Finally we give our summary and conclusions in Section 4.

2. Formulae

In this section, we give the formulae which we use in the following sections: the e^+e^- annihilation into a pair of $1S$ charmonium- and bottomonium-states. We use the standard method in our calculation: we start from the double $c\bar{c}$ (or $b\bar{b}$) production amplitudes, and project out the heavy quark and anti-quark pairs into the S-wave states in the colour-singlet (see Fig. 1).

The spin projection operator for the quarkonium production is given by

$$\mathcal{P}_{S,S_z}(P;q) = \sum_{s_1,s_2} v\left(\frac{P}{2} - q; s_2\right)\bar{u}\left(\frac{P}{2} + q; s_1\right) \times \left\langle \frac{1}{2}, s_1; \frac{1}{2}, s_2 \middle| S, S_z \right\rangle, \tag{2}$$

where P and S, S_z are respectively the quarkonium four-momentum, its spin and the z component of the spin; q is the relative momentum of the heavy quarks; and s_1, s_2 represent their spins. In the non-relativistic limit, the covariant forms of the projection operators are very simple:

$$\mathcal{P}_{0,0}(P;0) = \frac{1}{2\sqrt{2}}\gamma_5(\not{P} + M), \tag{3}$$

$$\mathcal{P}_{1,S_z}(P;0) = \frac{1}{2\sqrt{2}}\not{\epsilon}^*(P, S_z)(\not{P} + M), \tag{4}$$

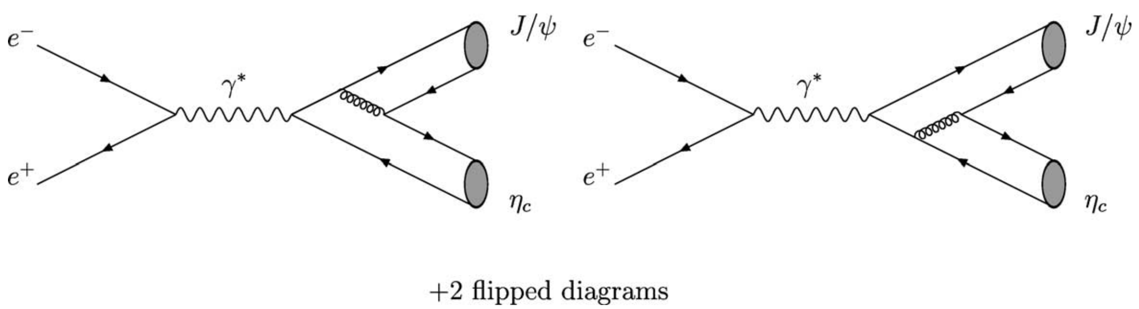

Fig. 1. Feynman diagrams for the $J/\psi\ \eta_c$ production from $e^+ + e^-$ annihilation.

respectively, for the pseudoscalar and the vector quarkonium. Here $\epsilon^\mu(P, S_z)$ denotes the polarization vector of the spin-1 quarkonium state, and $M = 2m$ is the mass of the quarkonium. Projectors (3) and (4) map a $Q\overline{Q}$ pair into the S-wave states. In the rest frame of the vector meson, the polarization vector is given by

$$\begin{aligned}\epsilon_0^\mu &\equiv \epsilon^\mu(P; S_z = 0) = (0, 0, 0, 1),\\ \epsilon_\pm^\mu &\equiv \epsilon^\mu(P; S_z = \pm 1) = (0, \mp, -i, 0)/\sqrt{2}\,,\end{aligned} \tag{5}$$

for the longitudinal and transverse polarizations, respectively. We need to boost the polarization vector from rest frame to the laboratory system along the spin quantization axis so that S_z denotes the helicity (λ) in the laboratory frame. In the frame where the electron beam is along the z-axis and the quarkonium scattering angle is θ, the polarization vectors read

$$\begin{aligned}\epsilon_0^\mu &= \gamma(\beta, \sin\theta, 0, \cos\theta),\\ \epsilon_\pm^\mu &= (0, \mp\cos\theta, -i, \pm\sin\theta)/\sqrt{2},\end{aligned} \tag{6}$$

respectively, where

$$\beta = \sqrt{1 - \frac{4M^2}{s}}, \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}. \tag{7}$$

Now let us show our analytical formulae for the cross sections. The helicity amplitudes for the process $e^+ + e^- \to J/\psi + \eta_c(\Upsilon + \eta_b)$ is given by

$$\begin{aligned}M^\sigma_{\lambda=\pm} &= -\frac{64 g^2 e^2 R_Q(0)^2}{3\pi s^{3/2}}\left[\frac{e_Q}{s} - \frac{v_Q(v_e - \sigma a_e)}{s - m_Z^2 + i m_Z \Gamma_Z}\right]\\ &\quad\times \epsilon^{\alpha\beta\mu\nu}\epsilon^*_{\lambda\alpha} P_{V_Q\beta} P_{\eta_Q\mu} j^\sigma_\nu\,,\end{aligned} \tag{8}$$

$$M^\sigma_{\lambda=0} = 0, \tag{9}$$

where σ denotes the electron helicity in units of $1/2$, λ the J/ψ (Υ) helicity, and Q is c or b. The initial current is given as $j^\nu_\sigma = (0, -i, \sigma, 0)$ and $P^\mu_{V_Q}$ and $P^\mu_{\eta_Q}$ are the four momentum of vector and pseudoscalar mesons, respectively. The vector and axial–vector couplings of the Z-boson are

$$\begin{aligned}v_e &= -\frac{1 - 4\sin^2\theta_w}{4\sin\theta_w\cos\theta_w}, & a_e &= -\frac{1}{4\sin\theta_w\cos\theta_w},\\ v_Q &= \frac{I_3^{QL} - 2e_Q\sin^2\theta_w}{2\sin\theta_w\cos\theta_w}, & a_Q &= \frac{I_3^{QL}}{2\sin\theta_w\cos\theta_w},\end{aligned} \tag{10}$$

where the I_3^{QL} is the third component of the weak isospin of the left-handed quark doublet; e_Q is the Q charge in units of proton charge. The absence of the $\lambda = 0$ amplitudes, the scattering angle dependence of the $\lambda = \pm$ amplitudes as well as their relative sign tell us the spin parity ($J^P = 0^-$) of the hadronic system assigned as η_c.

The above predictions can be tested by experiments through the decay angular distributions of $V_Q \to l^+l^-$, ($V_c = J/\psi,\ V_b = \Upsilon$). Using a definition $D^{\sigma'}_\lambda \equiv M(V_\lambda \to l_{\sigma'}\bar{l}_{\sigma'})$ where σ' is the l^- helicity in the $m_l = 0$ limit ($l = e, \mu$), we obtain

$$D^{\sigma'}_\pm = \sqrt{\frac{3B}{16\pi}}\big(\sigma' \pm \cos\theta^*\big)\frac{1}{\sqrt{2}}e^{\mp i\sigma'\phi^*} \tag{11}$$

for $\lambda = \pm$ and

$$D^{\sigma'}_0 = \sqrt{\frac{3B}{16\pi}}\,\sin\theta^* \tag{12}$$

for $\lambda = 0$. The normalization for the decay amplitudes Eqs. (11) and (12) is

$$\sum_{\sigma'}\int d\cos\theta^*\,d\phi^*\,\big|D^{\sigma'}_\lambda\big|^2 = B \equiv B\big(V_Q \to l^+l^-\big). \tag{13}$$

Here θ^* and ϕ^* are the polar and azimuthal angles of l^- in the V_Q rest frame. The θ^* is measured from the V_Q momentum direction in the e^+e^- collision rest frame, and ϕ^* is measured from the scattering plane ($\phi^* = \pi/2$ is along the $\vec{k} \times \vec{q}$ direction where $\vec{k}$ and $\vec{q}$ are, respectively the electron and V_Q three momenta in the e^+e^- collision c.m. frame). The triple angular distributions are then obtained as

$$\begin{aligned}&\frac{d\sigma}{d\cos\theta\,d\cos\theta^*\,d\phi^*}\\ &\quad= \frac{1}{2s}\frac{1}{4}\sum_{\sigma\sigma'}\bigg|\sum_{\lambda=\pm} M^\lambda_\sigma D^{\sigma'}_\lambda\bigg|^2\frac{\beta}{16\pi}\\ &\quad= \left(\frac{64}{3}\right)^2\frac{\pi\alpha^2\alpha_s^2}{s^3}\\ &\qquad\times\left[\frac{e_Q^2}{s} - \frac{2e_Q v_Q v_e(s - m_Z^2) - s v_Q^2(a_e^2 + v_e^2)}{(s - m_Z^2)^2 + (m_Z\Gamma_Z)^2}\right]\\ &\qquad\times\big|R_Q(0)\big|^4\beta^3\\ &\qquad\times\frac{3B}{16\pi}\big[\big(1 + \cos^2\theta\big)\big(1 + \cos^2\theta^*\big)\\ &\qquad\qquad - \sin^2\theta\sin^2\theta^*\cos 2\phi^*\big],\end{aligned} \tag{14}$$

where the initial beams are unpolarized and final lepton polarizations are unobserved. The $\cos\theta^*$ dependence tells us that only the transversally polarized V_Q are produced and the ϕ^* dependence tells us the relative phase of the $\lambda = +$ and $\lambda = -$ amplitudes. These predictions of the $V_Q\ \eta_Q$ production processes should be tested experimentally. After integrating out the V_Q decay angles and the scattering angle, we find the total cross section

$$\begin{aligned}\frac{d\sigma}{d\cos\theta} &= \left(\frac{64}{3}\right)^2 \frac{\pi\alpha^2\alpha_s^2}{s^3} \\ &\quad\times\left[\frac{e_Q^2}{s} - \frac{2e_Q v_Q v_e(s-m_Z^2) - s v_Q^2(a_e^2+v_e^2)}{(s-m_Z^2)^2 + (m_Z\Gamma_Z)^2}\right] \\ &\quad\times\left|R_Q(0)\right|^4 \beta^3\left(1+\cos^2\theta\right) \qquad (15)\end{aligned}$$

which agrees with the results previously calculated in [16,17].

We note in passing that a pair of V_Q (e.g., $J/\psi + J/\psi$ or $\Upsilon + \Upsilon$) cannot be produced from a single γ^* state because of charge conjugation invariance. Such pairs can be produced via $2\gamma^*$ intermediate states or in the Z-boson decays through its axial–vector couplings to the heavy quarks. The former process has been studied in Ref. [18]. For the latter process, we find the helicity amplitudes

$$\begin{aligned}M^\sigma_{\lambda_1\lambda_2} &= \frac{32g^2e^2a_Q R_Q(0)^2 M}{3\pi s^{3/2}} \frac{(a_e - \sigma v_e)}{(s-m_Z^2 + i m_Z\Gamma_Z)} \\ &\quad\times \epsilon_{\alpha\beta\mu\nu}\epsilon^{*\alpha}_{\lambda_1}\epsilon^{*\beta}_{\lambda_2}\left(P^\mu_{V_Q1} - P^\mu_{V_Q2}\right) j^{\sigma\nu}, \qquad (16)\end{aligned}$$

where σ is the electron helicity (in units of 1/2), λ_1 and λ_2 are the helicities of V_Q with $\cos\theta > 0$ and $\cos\theta < 0$, respectively, $j^\mu_\sigma = (0, i\sigma, 1, 0)$ is the initial e^+e^- current, and $P_{V_Q1,2}$ is the momentum of V_Q's. From this result, one can easily find that the amplitudes for $(\lambda_1,\lambda_2) = (\pm,\pm), (0,0), (\pm,\mp)$ are zero. The absence of the $\lambda_1 = \lambda_2 = \pm$ amplitudes is in accordance with the Yang's theorem (that forbids the transition between the spin-1 state and a pair of identical massless vector bosons), the $\lambda_1 = \lambda_2 = 0$ amplitude is absent due to Bose symmetry [19] and the $\lambda_1 = -\lambda_2 = \pm$ amplitudes vanish because of angular momentum mismatch. It is only a pair of longitudinally and transversally polarized V_Q's that can be produced via the Z-boson exchange in e^+e^- annihilation.

The differential cross section after summing over the V_Q helicities for unpolarized beams is

$$\begin{aligned}\frac{d\sigma}{d\cos\theta} &= \left(\frac{32}{3}\right)^2 \frac{\pi\alpha^2\alpha_s^2}{s^2} \frac{a_Q^2(a_e^2+v_e^2)}{(s-m_Z^2)^2 + (m_Z\Gamma_Z)^2} \\ &\quad\times\left|R_Q(0)\right|^4 \beta^5\left(1+\cos^2\theta\right). \qquad (17)\end{aligned}$$

From Eqs. (9) and (16), we find the Z-boson decay widths:

$$\Gamma(Z\to V_Q\eta_Q) = \left(\frac{64}{3}\right)^2 \frac{2\alpha\alpha_s^2 v_Q^2|R_Q(0)|^4\beta^3}{3m_Z^5}, \qquad (18)$$

$$\Gamma(Z\to V_Q V_Q) = \left(\frac{32}{3}\right)^2 \frac{2\alpha\alpha_s^2 a_Q^2|R_Q(0)|^4\beta^5}{3m_Z^5}. \qquad (19)$$

3. Numerical results

The following input parameters are used in our numerical analysis in this section:

$$\begin{aligned}&\alpha = \frac{1}{137}, \qquad I_c = \frac{1}{2}, \qquad I_b = -\frac{1}{2}, \\ &M_Z = 91.2\ \text{GeV}, \\ &\Gamma_Z = 2.495\ \text{GeV}, \qquad \sin^2\theta_w = 0.231, \\ &\Gamma_{J/\psi\to e^+e^-} = (5.26\pm 0.37)\times 10^{-6}\ \text{GeV}, \\ &\Gamma_{\Upsilon\to e^+e^-} = (1.32\pm 0.05)\times 10^{-6}\ \text{GeV}, \\ &M_{J/\psi} = 3.1\ \text{GeV}, \qquad M_\Upsilon = 9.5\ \text{GeV}. \qquad (20)\end{aligned}$$

Although our analysis is strictly in the leading order of perturbative QCD, we adapt the two-loop running coupling constant of $\overline{\text{MS}}$ scheme in order to define the "K" factor between the leading order prediction and the observed cross section. More specifically, we adopt

$$\frac{\alpha_s}{4\pi}(\mu)_{\overline{\text{MS}}} = \frac{1}{\beta_0\ln(\mu^2/\Lambda^2_{\overline{\text{MS}}})} - \frac{\beta_1\ln\ln(\mu^2/\Lambda^2_{\overline{\text{MS}}})}{\beta_0^3\ln^2(\mu^2/\Lambda^2_{\overline{\text{MS}}})}, \qquad (21)$$

where $\beta_0^{N_f} = (33-2N_f)/3$, $\beta_1^{N_f} = 102 - 10N_f - 8N_f/3$. $\alpha_s(M_Z)_{\overline{\text{MS}}} = 0.118$ corresponds to $\Lambda_{\overline{\text{MS}}} = 0.226$ GeV for $N_f = 5$. We set $\mu = \sqrt{s}/4$ as our leading order estimate, because the invariant mass of the exchanged gluons in the diagram of Fig. 1 is $\mu = \sqrt{s}/2$, and we account for the factor of two mismatch between the momentum scale and the $\overline{\text{MS}}$ scale [20]. The value of the wave function at the origin can be extracted from the leptonic widths $\Gamma(V_Q \to l^+l^-)$:

$$\left|R_c(0)\right|^2 = \frac{9M_{J/\psi}^2}{16\alpha^2}\Gamma_{J/\psi\to e^+e^-}, \tag{22}$$

$$\left|R_b(0)\right|^2 = \frac{9M_{\Upsilon}^2}{4\alpha^2}\Gamma_{\Upsilon\to e^+e^-}. \tag{23}$$

Using the experimental values [21] we obtain

$$\left|R_c(0)\right|^2 = (0.53 \pm 0.04)\ \text{GeV}^3,$$
$$\left|R_b(0)\right|^2 = (5.0 \pm 0.2)\ \text{GeV}^3. \tag{24}$$

First of all, we show our numerical result for the total cross section of $e^+ + e^- \to J/\psi + \eta_c$ which has already been measured by the Belle Collaboration. Using the central values of Eq. (24) and $\Lambda_{\overline{\text{MS}}} = 0.226$ GeV, Eq. (15) gives

$$\sigma\left(e^+ + e^- \to J/\psi + \eta_c\right) = 0.0023\ \text{pb} \tag{25}$$

which is consistent with the previous estimates in [16, 17]. If all the hadronic systems are indeed η_c decay products and if we set

$$\mathcal{B}(\eta_c \to \geqslant 4\ \text{charged hadrons}) = 0.6 \pm 0.1^1 \tag{26}$$

we find

$$K(\sqrt{s} = 10.6\ \text{GeV}) \equiv \frac{\sigma(e^+ + e^- \to J/\psi + \eta_c)_{\text{exp}}}{\sigma(e^+ + e^- \to J/\psi + \eta_c)_{\text{th}}} = (24 \pm 12), \tag{27}$$

where the error includes only experimental ones, Eq. (1) and that in our estimate Eq. (26).

Let us examine if the large K-factor in Eq. (27) can be explained. It is well known that the leptonic width formulae Eqs. (22) and (23) suffers from large NLO corrections. As a result, our estimate of the value of the wave function at the origin could include large error. For instance, one of the potential model calculations give [22,23]

$$\left|R_c(0)\right|^2 = 0.810\ \text{GeV}^3,$$
$$\left|R_b(0)\right|^2 = 6.477\ \text{GeV}^3. \tag{28}$$

By replacing (24) by (28), we have a factor of $(81/53)^2$. If we use $\Lambda_{\overline{\text{MS}}} = 0.296$ GeV for $\alpha_s(M_Z) = 0.123$ and change the scale $\mu = \sqrt{s}/8$ instead of our standard choice of $\mu = \sqrt{s}/4$, we find $\alpha_s(\mu = \sqrt{s}/8) = 0.41$ instead of $\alpha_s(\mu = \sqrt{s}/4) = 0.29$. This gives rise to another factor of $(35/26)^2$. The product of the two factors is about 5.8 which is still significantly smaller than the value Eq. (1) indicated from the experiment. There should be further large contributions in the production amplitude and/or the hadronic system should contain significant amount of non-η_c contributions.

In Ref. [18], it has been shown that the J/ψ pair production process via $\gamma^*\ \gamma^*$ intermediate state ($e^+ + e^- \to \gamma^*\gamma^* \to J/\psi + J/\psi$) should be as large as (or larger than) $e^+ + e^- \to J/\psi + \eta_c$. Because $J/\psi \to$ hadrons cannot be distinguished easily from $\eta_c \to$ hadrons, the Belle data Eq. (1) may contain contributions from J/ψ pair production. We have confirmed the results in [18] and the cross section is found to be

$$\sigma\left(e^+ + e^- \to J/\psi + J/\psi\right) = 0.0027\ \text{pb}. \tag{29}$$

Only a small fraction of the observed cross section of Eq. (1) can come from the J/ψ pair production process, because the normalization of the prediction Eq. (29) has little theoretical uncertainty. We note here that the J/ψ pair production process has the following distribution properties: as is shown in [18], the pair of J/ψ are transversally (either $(+,-)$ or $(-,+)$) polarized and the differential cross section behaves as $(1+\cos^2\theta)/(1-\cos^2\theta)$ while the $J/\psi\ \eta_c$ production behaves as $(1+\cos^2\theta)$ (see Eq. (15)). The absence of interference between $\lambda = +$ and $\lambda = -$ amplitudes predicts that there is no azimuthal angle (ϕ^*) dependence. The overall normalization of the J/ψ pair production contribution should soon be found experimentally once the double leptonic decays of the J/ψ pair are observed.

Our numerical results are summarized in Fig. 2. In this plot, we multiplied the cross section for $J/\psi\ \eta_c$

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[1] We expect that the hadronic final state of η_c decays is not too much different from that of J/ψ decays. We could not find, however, the data on charged particle multiplicity distribution on and off J/ψ resonance. The above estimate has been obtained by using an average of the up, down, and strange quark jet pairs generated by the JETSET Monte Carlo program at the η_c mass.

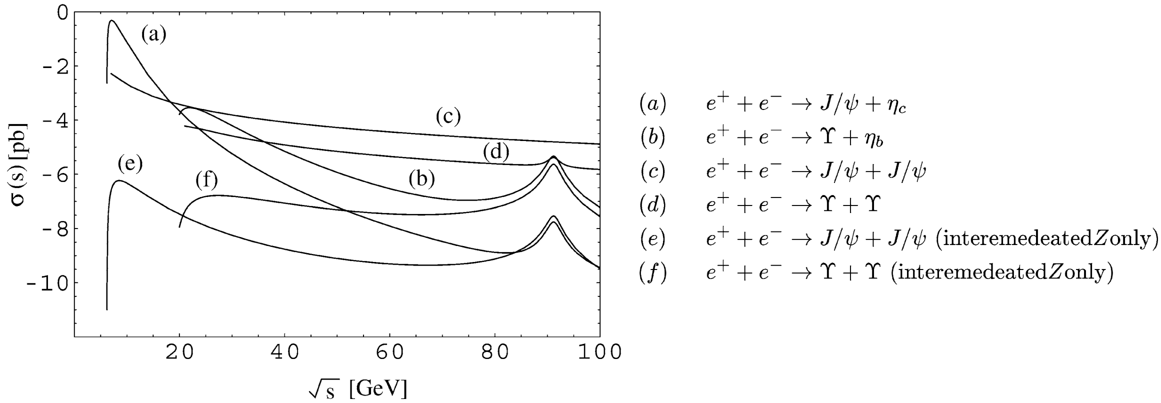

Fig. 2. The $\sqrt{s}$ dependence of the total cross section for the exclusive $1S$ double-quarkonium production (in log-scale) integrated over $|\cos\theta| \leqslant 0.9$ is shown. The K-factor defined in Eq. (30) is multiplied for the $J/\psi\ \eta_c$ and $\Upsilon\ \eta_b$ production processes and the $J/\psi\ J/\psi$ and $\Upsilon\ \Upsilon$ processes from the intermediated Z boson.

production by the K-factor defined

$$K(\sqrt{s}) = K(\sqrt{s_0})\left[\frac{\alpha_s(\sqrt{s/4})}{\alpha_s(\sqrt{s_0/4})}\right]^2. \tag{30}$$

We use the same K factor, with $K(10.6\ \text{GeV}) = 24$, for the single γ^* and Z^* exchange contributions to $J/\psi\ J/\psi$, $\Upsilon\ \eta_b$, and $\Upsilon\ \Upsilon$ production processes, even though the K factor for the bottomonium pair production may be smaller than that for the charmonium pair production. We hoped that we could study $J/\psi\ \eta_c$ production and related processes at Z pole by using the LEP data. However considering the LEP integrated luminosity at Z pole (about 1 fb^{-1}) and the branching ratio of the leptonic decay of J/ψ (0.06), it is unfortunately impossible to observe this process at LEP. We find for the input parameters of Eq. (20) and $K(M_Z) = 9.47$ from Eq. (30) the following branching fractions:

$$B(Z \to J/\psi + \eta_c) = 3.39 \times 10^{-13}, \tag{31}$$

$$B(Z \to J/\psi + J/\psi) = 5.73 \times 10^{-13}, \tag{32}$$

$$B(Z \to \Upsilon + \eta_b) = 9.23 \times 10^{-11}, \tag{33}$$

$$B(Z \to \Upsilon + \Upsilon) = 4.61 \times 10^{-11}. \tag{34}$$

Even with the cut-off of $|\cos\theta| < 0.9$, the J/ψ pair production from two virtual photons dominate over all the other exclusive charmonium and bottomonium pair production processes at all energies except around the B factory energies. This is essentially because the form factor of the exclusive heavy quarkonium production process drops sharply as $s^{-3/2}$ at high energies, as can be seen from Eq. (15)

$$\frac{\sigma(e^+ + e^- \to J/\psi + \eta_c)}{\sigma(e^+ + e^- \to c\bar{c})} \propto s^{-3}. \tag{35}$$

An extra suppression factor of s^{-1} as compared to the high-energy behavior of the light meson pair production processes reflects the non-relativistic constraint that the two constituents should have the same velocity.

4. Conclusions

Exclusive $J/\psi\ \eta_c$ production in e^+e^- collisions at $\sqrt{s} = 10.6$ GeV is studied in view of the recent Belle observation [15]. The observed total cross section turns out to be more than one order of magnitude larger than the naive leading order prediction of non-relativistic QCD, resulting in the huge renormalization factor of $K = 24 \pm 12$, which is consistent with the conclusions in [16,17]. We find that the K factor of up to about 6 can be obtained by taking account of the next-leading-order corrections to the J/ψ leptonic width and by studying the scale dependence of the leading-order prediction. On the other hand, the experimental data may contain contributions from non-η_c origin hadronic events, such as hadrons from J/ψ decays in the J/ψ-pair production via two virtual photon exchange [18], and hadrons from two gluon jets in the color-singlet $J/\psi + gg$ process [10–14]. We propose to use the triple angular distribution of the J/ψ

production and J/ψ decay into charged leptons to test the $J/\psi + \eta_c$ hypothesis. A peculiar azimuthal angle dependence of the lepton distribution about the scattering plane is predicted.

We have also studied $J/\psi + J/\psi$ production via Z-boson exchange and find that a pair of a transversally polarized J/ψ and a longitudinally polarized J/ψ can be produced in Z boson decays via its axial–vector coupling to the charm quark. Unfortunately, the branching fraction of the Z boson decays into $J/\psi + J/\psi$, $J/\psi + \eta_c$, $\Upsilon + \Upsilon$, $\Upsilon + \eta_b$ are all too small to be observed in the LEP data, even with a possible large K factor.

Before closing this report, we note that pair production of S-wave and P-wave charmonium has been studied in Ref. [16,17] and additionally S-wave + D-wave as well as P-wave + P-wave charmonium productions have been studied in Ref. [16]. We confirm their results of $J/\psi\ \chi_{cJ}$ ($J = 0, 1, 2$) production cross sections. Although the perturbative calculation of the $J/\psi\ \eta_c$ production cross section falls short of the observed one, it is still interesting to test whether the ratio among cross sections of all the above processes are consistent with the predictions of NRQCD.

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