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Massive gravitational waves from $f(R)$ theories of gravity: Potential detection with LISA

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ABSTRACT

This Letter is a generalization of previous results on gravitational waves (GWs) from $f(R)$ theories of gravity. In some previous papers, particular $f(R)$ theories have been linearized for the first time in the literature. Now, the process is further generalized, showing that every $f(R)$ theory can be linearized producting a third massive mode of gravitational radiation. In this framework, previous results are particular cases of the more general problem that is discussed in this Letter. The potential detectability of such massive GWs with LISA is also discussed with the auxilium of longitudinal response functions.

Recently, the data analysis of interferometric GWs detectors has been started (for the current status of GWs interferometers see [1–8]) and the scientific community aims at a first direct detection of GWs in next years.

Detectors for GWs will be important for a better knowledge of the Universe and either to confirm or ruling out the physical consistency of General Relativity or any other theory of gravitation [9–14].

An early, but quite interesting, paper on other theories of gravity was written by Wagoner [28].

In this Letter, the production of a hypothetical massive component of gravitational radiation which arises from a general $f(R)$ theory of gravity is shown. The presence of the mass could also have important applications in cosmology because the fact that GWs can have mass could give a contribution to the dark matter of the Universe.

The first and simplest $f(R)$ theory of gravity was proposed by Starobinsky [15], who discussed the action

$$S=\int d^4x\sqrt{-g}\left(R+\alpha R^2\right)+\mathcal{L}_m. \tag{1}$$

The production and the potential detection of GWs from this theory has been analysed in [16]. In [17], it has been also shown that, from this particular linearized theory, it is possible to obtain an oscillating model of Universe.

Another example is the action

$$S=\int d^4x\sqrt{-g}\left(R+R^{-1}\right)+\mathcal{L}_m. \tag{2}$$

This action has been analysed in a cosmological context in [18], while the GWs case has been analysed in [19]. This kind of theory could, in principle, be connected with the Dark Matter problem too [20]. Criticisms on $f(R)$ theories of gravity arises from the fact that lots of such theories can be excluded by requirements of Cosmology and Solar System tests [24,25]. It is important to emphasize that the theory of Eq. (2), differently from the theory of the action (1) is not in conflict with such constrains [24,25]. In this context, even the theory

$$S=\int d^4x\sqrt{-g}f_0R^{1+\varepsilon}+\mathcal{L}_m, \tag{3}$$

which has been discussed in [25], is quite interesting.

Equations (1), (2) and (3) are particular choices in respect to the well-known canonical one of general relativity (the Einstein–Hilbert action [21,22]) which is

$$S=\int d^4x\sqrt{-g}\,R+\mathcal{L}_m, \tag{4}$$

where R is the Ricci scalar curvature.

Now, we will analyse the general case, i.e.

$$S=\int d^4x\sqrt{-g}f(R)+\mathcal{L}_m, \tag{5}$$

where $f(R)$ is a generic high order theory of gravity.

Of course, the cases which have been analysed in [16] and in [19] are particular cases of the more general case that we are going to analyse now.

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As we will interact with gravitational waves, i.e. the linearized theory in vacuum, $\mathcal{L}_m = 0$ will be put and the pure curvature action

$$S = \int d^4x\sqrt{-g}\,f(R) \tag{6}$$

will be considered.

By varying the action (6) in respect to $g_{\mu\nu}$ (see Refs. [16,17,19] for a parallel computation) the field equations are obtained (note that in this Letter we work with $G = 1$, $c = 1$ and $\hbar = 1$):

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - f'(R)_{;\mu;\nu} + g_{\mu\nu}\Box f'(R) = 0 \tag{7}$$

which are the modified Einstein field equations. $f'(R)$ is the derivative of f in respect to the Ricci scalar. Writing down, explicitly, the Einstein tensor, Eqs. (7) become

$$G_{\mu\nu} = \frac{1}{f'(R)}\left\{\frac{1}{2}g_{\mu\nu}\left[f(R) - f'(R)R\right] + f'(R)_{;\mu;\nu} - g_{\mu\nu}\Box f'(R)\right\}. \tag{8}$$

Taking the trace of the field equations (8) one gets

$$3\Box f'(R) + Rf'(R) - 2f(R) = 0, \tag{9}$$

and, with the identifications [23]

$$\Phi \to f'(R) \quad \text{and} \quad \frac{dV}{d\Phi} \to \frac{2f(R) - Rf'(R)}{3} \tag{10}$$

a Klein–Gordon equation for the effective Φ scalar field is obtained:

$$\Box\Phi = \frac{dV}{d\Phi}. \tag{11}$$

To study gravitational waves, the linearized theory has to be analyzed, with a little perturbation of the background, which is assumed given by a Minkowskian background plus $\Phi = \Phi_0$, i.e. we are linearizing into a background with constant curvature [19,24]. We also assume Φ_0 to be a minimum for V:

$$V \simeq \frac{1}{2}\alpha\delta\Phi^2 \;\Rightarrow\; \frac{dV}{d\Phi} \simeq m^2\delta\Phi, \tag{12}$$

and the constant m has mass dimension.

Putting

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu}, \\ \Phi &= \Phi_0 + \delta\Phi, \end{aligned} \tag{13}$$

to first order in $h_{\mu\nu}$ and $\delta\Phi$, calling $\tilde{R}_{\mu\nu\rho\sigma}$, $\tilde{R}_{\mu\nu}$ and $\tilde{R}$ the linearized quantity which correspond to $R_{\mu\nu\rho\sigma}$, $R_{\mu\nu}$ and R, the linearized field equations are obtained [16,17,19]:

$$\begin{aligned} \tilde{R}_{\mu\nu} - \frac{\tilde{R}}{2}\eta_{\mu\nu} &= (\partial_\mu\partial_\nu h_f - \eta_{\mu\nu}\Box h_f), \\ \Box h_f &= m^2 h_f, \end{aligned} \tag{14}$$

where

$$h_f \equiv \frac{\delta\Phi}{\Phi_0}. \tag{15}$$

$\tilde{R}_{\mu\nu\rho\sigma}$ and Eqs. (14) are invariants for gauge transformations [16,17,19]

$$\begin{aligned} h_{\mu\nu} &\to h'_{\mu\nu} = h_{\mu\nu} - \partial_{(\mu}\epsilon_{\nu)}, \\ \delta\Phi &\to \delta\Phi' = \delta\Phi; \end{aligned} \tag{16}$$

then

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{h}{2}\eta_{\mu\nu} + \eta_{\mu\nu}h_f \tag{17}$$

can be defined, and, considering the transformation for the parameter ϵ^μ,

$$\Box\epsilon_\nu = \partial^\mu\bar{h}_{\mu\nu}, \tag{18}$$

a gauge parallel to the Lorenz one of electromagnetic waves can be chosen:

$$\partial^\mu\bar{h}_{\mu\nu} = 0. \tag{19}$$

In this way, field equations read like

$$\Box\bar{h}_{\mu\nu} = 0, \tag{20}$$

$$\Box h_f = m^2 h_f. \tag{21}$$

Solutions of Eqs. (20) and (21) are plan waves [16,17,19]:

$$\bar{h}_{\mu\nu} = A_{\mu\nu}(\vec{p})\exp(ip^\alpha x_\alpha) + \text{c.c.}, \tag{22}$$

$$h_f = a(\vec{p})\exp(iq^\alpha x_\alpha) + \text{c.c.}, \tag{23}$$

where

$$\begin{aligned} k^\alpha &\equiv (\omega, \vec{p}), \quad \omega = p \equiv |\vec{p}|, \\ q^\alpha &\equiv (\omega_m, \vec{p}), \quad \omega_m = \sqrt{m^2 + p^2}. \end{aligned} \tag{24}$$

In Eqs. (20) and (22) the equation and the solution for the standard waves of General Relativity [21,22] have been obtained, while Eqs. (21) and (23) are respectively the equation and the solution for the massive mode (see also [16,17,19]).

The fact that the dispersion law for the modes of the massive field h_f is not linear has to be emphasized. The velocity of every "ordinary" (i.e. which arises from General Relativity) mode $\bar{h}_{\mu\nu}$ is the light speed c, but the dispersion law (the second of Eq. (24)) for the modes of h_f is that of a massive field which can be discussed like a wave-packet [16,17,19]. Also, the group-velocity of a wave-packet of h_f centered in $\vec{p}$ is

$$\vec{v}_G = \frac{\vec{p}}{\omega}, \tag{25}$$

which is exactly the velocity of a massive particle with mass m and momentum $\vec{p}$.

From the second of Eqs. (24) and Eq. (25) it is simple to obtain:

$$v_G = \frac{\sqrt{\omega^2 - m^2}}{\omega}. \tag{26}$$

Then, wanting a constant speed of the wave-packet, it has to be [16,17,19]

$$m = \sqrt{1 - v_G^2}\,\omega. \tag{27}$$

Now, the analysis can remain in the Lorenz gauge with transformations of the type $\Box\epsilon_\nu = 0$; this gauge gives a condition of transversality for the ordinary part of the field: $k^\mu A_{\mu\nu} = 0$, but does not give the transversality for the total field $h_{\mu\nu}$. From Eq. (17) it is

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{\bar{h}}{2}\eta_{\mu\nu} + \eta_{\mu\nu}h_f. \tag{28}$$

At this point, if being in the massless case [16,17,19], it could been put

$$\begin{aligned} \Box\epsilon^\mu &= 0, \\ \partial_\mu\epsilon^\mu &= -\frac{\bar{h}}{2} + h_f, \end{aligned} \tag{29}$$

which gives the total transversality of the field. But in the massive case this is impossible. In fact, applying the Dalembertian operator to the second of Eqs. (29) and using the field equations (20) and (21) it results

$$\Box\epsilon^{\mu}=m^2h_f, \tag{30}$$

which is in contrast with the first of Eqs. (29). In the same way, it is possible to show that it does not exist any linear relation between the tensorial field $\bar{h}_{\mu\nu}$ and the massive field h_f. Thus a gauge in which $h_{\mu\nu}$ is purely spatial cannot be chosen (i.e. it cannot be put $h_{\mu 0}=0$, see Eq. (28)). But the traceless condition to the field $\bar{h}_{\mu\nu}$ can be put:

$$\begin{aligned}&\Box\epsilon^{\mu}=0,\\&\partial_{\mu}\epsilon^{\mu}=-\frac{\bar{h}}{2}.\end{aligned} \tag{31}$$

These equations imply

$$\partial^{\mu}\bar{h}_{\mu\nu}=0. \tag{32}$$

To save the conditions $\partial_{\mu}\bar{h}^{\mu\nu}$ and $\bar{h}=0$ transformations like

$$\begin{aligned}&\Box\epsilon^{\mu}=0,\\&\partial_{\mu}\epsilon^{\mu}=0,\end{aligned} \tag{33}$$

can be used and, taking $\vec{p}$ in the z direction, a gauge in which only A_{11}, A_{22}, and $A_{12}=A_{21}$ are different to zero can be chosen. The condition $\bar{h}=0$ gives $A_{11}=-A_{22}$. Now, putting these equations in Eq. (28), it results

$$h_{\mu\nu}(t,z)=A^{+}(t-z)e^{(+)}_{\mu\nu}+A^{\times}(t-z)e^{(\times)}_{\mu\nu}+h_f(t-v_Gz)\eta_{\mu\nu}. \tag{34}$$

The term $A^{+}(t-z)e^{(+)}_{\mu\nu}+A^{\times}(t-z)e^{(\times)}_{\mu\nu}$ describes the two standard polarizations of gravitational waves which arise from General Relativity, while the term $h_f(t-v_Gz)\eta_{\mu\nu}$ is the massive field arising from the generic high order $f(R)$ theory. In other words, the function $f'(R)$ of the Ricci scalar generates a third massive polarization for gravitational waves which is not present in standard General Relativity. Note that the line element (34) has been obtained in both of Refs. [16] and [19] starting by the actions (1) and (2) respectively. Here we have shown that such a line element is characteristic of *every* $f(R)$ theory of gravity.

The analysis of the two standard polarization is well known in the literature [2,3,21,22]. For a pure polarization arising from the $f(R)$ theory Eq. (34) can be rewritten as

$$h_{\mu\nu}(t-v_Gz)=h_f(t-v_Gz)\eta_{\mu\nu} \tag{35}$$

and the correspondent line element is the conformally flat one

$$ds^2=\left[1+h_f(t-v_Gz)\right]\left(-dt^2+dz^2+dx^2+dy^2\right). \tag{36}$$

In [19] it has been shown that in this kind of line element the effect of the mass is the generation of a *longitudinal* force (in addition to the transverse one) while in the limit $m\to 0$ the longitudinal force vanishes.

Now, before starting the analysis, it has to be discussed if there are phenomenological limitations to the mass of the GW [19,24]. A strong limitation arises from the fact that the GW needs a frequency which falls in the frequency-range for both of earth based and space based gravitational antennas, that is the interval 10^{-4} Hz $\leqslant f\leqslant 10$ KHz [1,4–8,26,27]. For a massive GW, from [12,14,16,19] it is:

$$2\pi f=\omega=\sqrt{m^2+p^2}, \tag{37}$$

were p is the momentum. Thus, it needs

$$0\ \text{eV}\leqslant m\leqslant 10^{-11}\ \text{eV}. \tag{38}$$

A stronger limitation is given by requirements of cosmology and Solar System tests on extended theories of gravity. In this case it is [24]

$$0\ \text{eV}\leqslant m\leqslant 10^{-33}\ \text{eV}. \tag{39}$$

For these light scalars, their effect can be still discussed as a coherent GW.

The frequency-dependent response function, for a massive mode of gravitational radiation, has been obtained in [19] for the particular case $f(R)=R+R^{-1}$. Here the computation will be performed with another treatment and the results will be applied to LISA, following the advice in [24].

Eq. (36) can be rewritten as

$$\left(\frac{dt}{d\tau}\right)^2-\left(\frac{dx}{d\tau}\right)^2-\left(\frac{dy}{d\tau}\right)^2-\left(\frac{dz}{d\tau}\right)^2=\frac{1}{1+h_f}, \tag{40}$$

where τ is the proper time of the test masses.

From Eqs. (36) and (40) the geodesic equations of motion for test masses (i.e. the beam-splitter and the mirrors of the interferometer), can be obtained

$$\begin{aligned}&\frac{d^2x}{d\tau^2}=0,\\&\frac{d^2y}{d\tau^2}=0,\\&\frac{d^2t}{d\tau^2}=\frac{1}{2}\frac{\partial_t(1+h_f)}{(1+h_f)^2},\\&\frac{d^2z}{d\tau^2}=-\frac{1}{2}\frac{\partial_z(1+h_f)}{(1+h_f)^2}.\end{aligned} \tag{41}$$

The first and the second of Eqs. (41) can be immediately integrated obtaining

$$\frac{dx}{d\tau}=C_1=\text{const}, \tag{42}$$

$$\frac{dy}{d\tau}=C_2=\text{const}. \tag{43}$$

In this way Eq. (40) becomes

$$\left(\frac{dt}{d\tau}\right)^2-\left(\frac{dz}{d\tau}\right)^2=\frac{1}{1+h_f}. \tag{44}$$

If we assume that test masses are at rest initially we get $C_1=C_2=0$. Thus we see that, even if the GW arrives at test masses, we do not have motion of test masses within the x–y plane in this gauge. We could understand this directly from Eq. (36) because the absence of the x and of the y dependences in the metric implies that test masses momentum in these directions (i.e. C_1 and C_2 respectively) is conserved. This results, for example, from the fact that in this case the x and y coordinates do not explicitly enter in the Hamilton–Jacobi equation for a test mass in a gravitational field [2].

Now we will see that, in presence of the GW, we have motion of test masses in the z direction which is the direction of the propagating wave. An analysis of Eqs. (41) shows that, to simplify equations, we can introduce the retarded and advanced time coordinates (u,v):

$$\begin{aligned}&u=t-v_Gz,\\&v=t+v_Gz.\end{aligned} \tag{45}$$

From the third and the fourth of Eqs. (41) we have

$$\frac{d}{d\tau}\frac{du}{d\tau}=\frac{\partial_v[1+h_f(u)]}{(1+h_f(u))^2}=0. \tag{46}$$

This equation can be integrated obtaining

$$\frac{du}{d\tau} = \alpha, \tag{47}$$

where α is an integration constant. From Eqs. (44) and (47), we also get

$$\frac{dv}{d\tau} = \frac{\beta}{1+h_f}, \tag{48}$$

where $\beta \equiv \frac{1}{\alpha}$, and

$$\tau = \beta u + \gamma, \tag{49}$$

where the integration constant γ corresponds simply to the retarded time coordinate translation u. Thus, without loss of generality, we can put it equal to zero. Now let us see what is the meaning of the other integration constant β. We can write the equation for z from Eqs. (47) and (48):

$$\frac{dz}{d\tau} = \frac{1}{2\beta}\left(\frac{\beta^2}{1+h_f} - 1\right). \tag{50}$$

When it is $h_f = 0$ (i.e. before the GW arrives at the test masses) Eq. (50) becomes

$$\frac{dz}{d\tau} = \frac{1}{2\beta}\left(\beta^2 - 1\right). \tag{51}$$

But this is exactly the initial velocity of the test mass, thus we have to choose $\beta = 1$ because we suppose that test masses are at rest initially. This also imply $\alpha = 1$.

To find the motion of a test mass in the z direction we see that from Eq. (49) we have $d\tau = du$, while from Eq. (48) we have $dv = \frac{d\tau}{1+h_f}$. Because it is $v_G z = \frac{v-u}{2}$ we obtain

$$dz = \frac{1}{2v_G}\left(\frac{d\tau}{1+h_f} - du\right), \tag{52}$$

which can be integrated as

$$\begin{aligned} z &= z_0 + \frac{1}{2v_G}\int\left(\frac{du}{1+h_f} - du\right) \\ &= z_0 - \frac{1}{2v_G}\int_{-\infty}^{t-v_G z}\frac{h_f(u)}{1+h_f(u)}du, \end{aligned} \tag{53}$$

where z_0 is the initial position of the test mass. Now the displacement of the test mass in the z direction can be written as

$$\begin{aligned} \Delta z &= z - z_0 \\ &= -\frac{1}{2v_G}\int_{-\infty}^{t-v_G z_0 - v_G\Delta z}\frac{h_f(u)}{1+h_f(u)}du \\ &\simeq -\frac{1}{2v_G}\int_{-\infty}^{t-v_G z_0}\frac{h_f(u)}{1+h_f(u)}du. \end{aligned} \tag{54}$$

We can also rewrite the results in function of the time coordinate t:

$$\begin{aligned} x(t) &= x_0, \\ y(t) &= y_0, \\ z(t) &= z_0 - \frac{1}{2v_G}\int_{-\infty}^{t-v_G z_0}\frac{h_f(u)}{1+h_f(u)}d(u), \\ \tau(t) &= t - v_G z(t). \end{aligned} \tag{55}$$

Calling l and $L+l$ the unperturbed positions of the beam-splitter and of the mirror and using the third of Eqs. (55) the varying position of the beam-splitter and of the mirror are given by

$$\begin{aligned} z_{BS}(t) &= l - \frac{1}{2v_G}\int_{-\infty}^{t-v_G l}\frac{h_f(u)}{1+h_f(u)}d(u), \\ z_M(t) &= L + l - \frac{1}{2v_G}\int_{-\infty}^{t-v_G(L+l)}\frac{h_f(u)}{1+h_f(u)}d(u). \end{aligned} \tag{56}$$

But we are interested in variations in the proper distance (time) of test masses, thus, in correspondence of Eqs. (56), using the fourth of Eqs. (55) we get

$$\begin{aligned} \tau_{BS}(t) &= t - v_G l - \frac{1}{2}\int_{-\infty}^{t-v_G l}\frac{h_f(u)}{1+h_f(u)}d(u), \\ \tau_M(t) &= t - v_G L - v_G l - \frac{1}{2}\int_{-\infty}^{t-v_G(L+l)}\frac{h_f(u)}{1+h_f(u)}d(u). \end{aligned} \tag{57}$$

Then the total variation of the proper time is given by

$$\Delta\tau(t) = \tau_M(t) - \tau_{BS}(t) = v_G L - \frac{1}{2}\int_{t-v_G l}^{t-v_G(L+l)}\frac{h_f(u)}{1+h_f(u)}d(u). \tag{58}$$

In this way, recalling that in the used units the unperturbed proper distance (time) is $T = L$, the difference between the total variation of the proper time in presence and the total variation of the proper time in absence of the GW is

$$\begin{aligned} \delta\tau(t) &\equiv \Delta\tau(t) - L \\ &= -L(v_G + 1) - \frac{1}{2}\int_{t-v_G l}^{t-v_G(L+l)}\frac{h_f(u)}{1+h_f(u)}d(u). \end{aligned} \tag{59}$$

This quantity can be computed in the frequency domain, defining the Fourier transform of h_f as

$$\tilde{h}_f(\omega) = \int_{-\infty}^{\infty} dt\, h_f(t)\exp(i\omega t), \tag{60}$$

and using the translation and derivation Fourier theorems, obtaining

$$\begin{aligned} \delta\tilde{\tau}(\omega) &= L\left(1 - v_G^2\right)\exp\left[i\omega L(1+v_G)\right] \\ &\quad + \frac{L}{2\omega L(v_G^2-1)^2}\left[\exp[2i\omega L](v_G+1)^3\left(-2i + \omega L(v_G - 1)\right)\right. \\ &\quad + 2L\exp\left[i\omega L(1+v_G)\right]\left(6iv_G + 2iv_G^3 - \omega L + \omega L v_G^4\right) \\ &\quad \left. + L(v_G+1)^3\left(-2i + \omega L(v_G+1)\right)\right]\tilde{h}_R. \end{aligned} \tag{61}$$

A "signal" can be also defined:

$$\begin{aligned} \tilde{S}(\omega) &\equiv \frac{\delta\tilde{\tau}(\omega)}{L} \\ &= \left(1 - v_G^2\right)\exp\left[i\omega L(1+v_G)\right] \\ &\quad + \frac{1}{2\omega L(v_G^2-1)^2}\left[\exp[2i\omega L](v_G+1)^3\left(-2i + \omega L(v_G - 1)\right)\right. \\ &\quad + 2\exp\left[i\omega L(1+v_G)\right]\left(6iv_G + 2iv_G^3 - \omega L + \omega L v_G^4\right) \\ &\quad \left. + (v_G+1)^3\left(-2i + \omega L(v_G+1)\right)\right]\tilde{h}_R. \end{aligned} \tag{62}$$

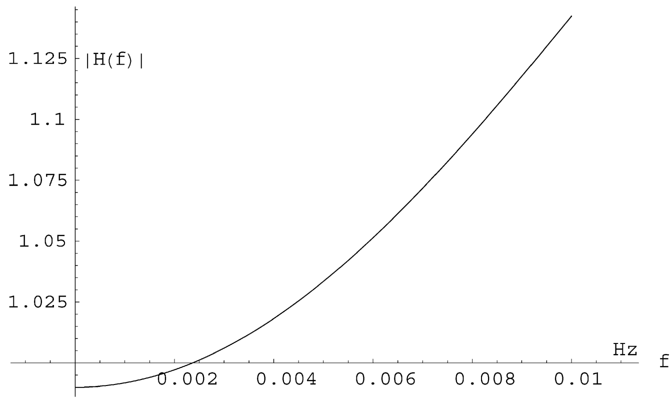

Fig. 1. The longitudinal response function (63) of an arm of LISA for $v_G = 0.1$ (non-relativistic case).

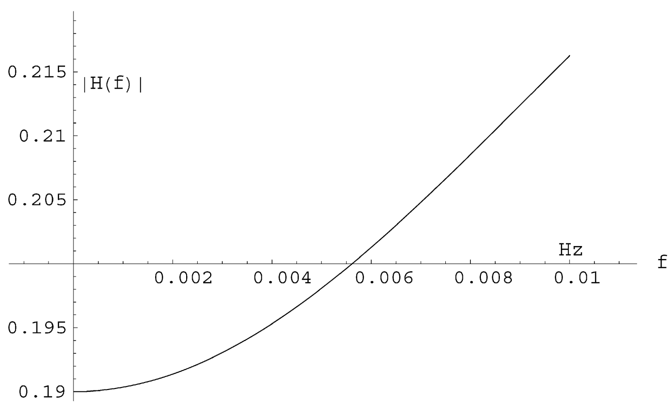

Fig. 2. The longitudinal response function (63) of an arm of LISA for $v_G = 0.9$ (relativistic case).

Then the function

$$\begin{aligned}\Upsilon_l(\omega) \equiv{}& \left(1 - v_G^2\right)\exp\left[i\omega L(1 + v_G)\right] \\ &+ \frac{1}{2\omega L(v_G^2 - 1)^2}\left[\exp[2i\omega L](v_G + 1)^3\left(-2i + \omega L(v_G - 1)\right)\right. \\ &+ 2\exp\left[i\omega L(1 + v_G)\right]\left(6iv_G + 2iv_G^3 - \omega L + \omega L v_G^4\right) \\ &\left.+ (v_G + 1)^3\left(-2i + \omega L(v_G + 1)\right)\right], \qquad (63)\end{aligned}$$

is the response function of an arm of the interferometer located in the z-axis, due to the longitudinal component of the massive gravitational wave arising from the high order gravity theory and propagating in the same direction of the axis.

For $v_G \to 1$ it is $\Upsilon_l(\omega) \to 0$. Such a response function has been obtained in [19] too, but with a different kind of analysis.

In Figs. 1 and 2 are shown the response functions (63) for an arm of LISA ($L = 5 \times 10^6$ km) [26,27] for $v_G = 0.1$ (non-relativistic case) and $v_G = 0.9$ (relativistic case). We see that in the non-relativistic case the signal is stronger as it could be expected (for $m \to 0$ we expect $\Upsilon_l(\omega) \to 0$).

It is very important to emphasize that, differently from the response functions of massless gravitational waves, this longitudinal response function increases with frequency, i.e., the presence of the mass prevents signal to drop off the regime in the high-frequency portion of the sensitivity band. Thus, considering such a high-frequency portion of the sensitivity band becomes fundamental if LISA would detect massive GWs arising from $f(R)$ theories of gravity which are not banned by requirements of Cosmology and Solar System tests [24,25], like, for example, the two theories arising from the actions (2) and (3).

Conclusions

This Letter has been a generalization of previous results on gravitational waves (GWs) from $f(R)$ theories of gravity. In some previous papers, particular $f(R)$ theories have been linearized for the first time in the literature. Now, the process has been further generalized, showing that every $f(R)$ theory can be linearized producting a third massive mode of gravitational radiation. In this framework, previous results are particular cases of the more general problem that has been discussed in this Letter. The potential detectability of such massive GWs with LISA has been also discussed with the auxilium of longitudinal response functions.

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