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Steady rotation of micropolar fluid sphere in concentric spherical container

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Abstract

The problem of slow steady rotation of a micropolar fluid sphere in concentric spherical container filled with viscous fluid is studied. The appropriate boundary conditions are taken on the surface of the sphere. The hydrodynamic couple and wall correction factor exerted on the micropolar fluid sphere is obtained. The dependence of the wall correction factor on the micropolarity parameter and spin parameter is presented graphically and discussed. The hydrodynamic couple acting on a solid sphere in a cell model and on a solid sphere in an unbounded medium are obtained from the present analysis.
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1. Introduction

Classical theory of Navier-Stokes is not appropriate to describe the behaviour of fluids with microstructure such as animal blood, polymeric suspensions, muddy water and lubricants. In the past few years, there is much attention in the field of fluid mechanics which deal with the micro structures. Eringen [1, 2] proposed the micropolar fluid theory that describes the behaviour of such fluids. The micropolar fluids consist of rigid particles which can rotate with their own spins and micro rotations. These fluids have micro-rotational effects and micro-rotational inertia and support body couples and couple stresses. In the theory of micropolar fluids, the motion of the fluid are described by the classical velocity vector and the microrotation (spin) vector. Micropolar fluids have found in large number of applications in various fields. Among these are lubrication problem, liquid crystals, colloidal suspensions, polymeric additives, occurrence of turbulence etc. Various applications of miropolar fluids are presented in the review article by Ariman et al. [3]. The mathematical theory of equations of micropolar fluids and some of its applications can be found in the textbook by Lukaszewicz [4].

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The torque generated by an incompressible viscous fluid filled between two rotating spheres fixed at the same centre has many applications like fluid gyroscopes, colloidal science and centrifuges. This study is useful in designing and calibration of viscometers [5]. The problems related to this study received considerable interest among researchers due to various applications in engineering and science. Jeffrey [6] was the first who discussed the slow rotation of spheroids in an infinite fluid using curvilinear coordinates. Rao et al. [7] considered the slow steady rotation of a sphere in a micropolar fluid. The problem of rotational motion of an axially symmetric body in a micropolar fluid is analysed by Ramkisson [8]. Dennis et al. [9] studied numerically the problem of steady rotation of a sphere in a viscous fluid and calculated the couple using finite difference like technique for a wide range of Reynolds number. The problem of slow steady rotation of a spheroid (prolate and oblate) in an incompressible micropolar fluid is investigated by Rao and Iyengar [10]. The steady flow of a micropolar fluid flow between two eccentric coaxially rotating spheres was investigated by Kamel and Fong [11] using perturbation techniques. The slow steady rotation of an approximate sphere in an incompressible micropolar fluid is studied by Iyengar and Srinivasacharya [12]. The problem of translational and rotational motion of a spherical porous shell situated at the center of a spherical cavity filled with an incompressible Newtonian fluid was analytically investigated by Keh and Lu [13]. Saad [14] studied the steady translation and rotation of a porous spheroid fixed at the center of spheroidal container. Srinivasacharya and Krishna Prasad [15] analytically solved the slow steady rotation of a composite sphere in a spherical container.

The problem of motion of one fluid passed around in another fluid is of special interest because of its applications in various natural, industrial and biological processes, such as raindrop formation, study of blood flow, liquid-liquid extraction, prediction of atmospheric conditions and sedimentation phenomena. Although, many authors discussed the slow steady rotation of solid spherical or non-spherical particles in viscous and micropolar fluids, the problem of rotation of droplets of one fluid dispersed in another immiscible fluid has not received attention of any author. This motivated us to consider the present study.

In this paper, we consider slow steady rotation of a micropolar fluid sphere in a spherical container containing viscous fluid by applying non zero boundary condition for microrotation vector. The appropriate boundary conditions on the sphere are continuity of velocity components, continuity of stress components and the spin vorticity relation. The hydrodynamic couple and wall correction factor acting on the sphere is obtained and variation of wall correction factor with various parameters is studied.

2. Formulation of the problem

Consider the slow steady rotation of an incompressible micropolar fluid sphere of radius a fixed at the center of a spherical cavity of radius b (See Fig. 1). The gap between the micopolar fluid sphere and the cavity is filled with Newtonian viscous fluid. Assuming that the angular velocity of the micropolar fluid sphere is Ω about the axis of symmetry $\theta = 0$ and the fluid particle is at rest. The angular velocity of the spherical cavity is same as that of the fluid particle in the opposite direction. The region outside and inside the spherical particle are denoted by regions I and II respectively. The equations of motion for region I are

$$\nabla \cdot \vec{q}^{(1)} = 0, \tag{1a}$$

$$\nabla p^{(1)} + \mu_1 \nabla \times \nabla \times \vec{q}^{(1)} = 0, \tag{1b}$$

where $\vec{q}^{(1)}$ is the velocity, $p^{(1)}$ is the pressure and μ_1 is the coefficient of viscosity.
The equations of motion for the region II are the equations governing the steady flow of an incompressible micropolar fluid under Stokesian assumption with the absence of body force and body couple and are given by [2]

$$\nabla \cdot \vec{q}^{(2)} = 0, \tag{2a}$$

$$\nabla p^{(2)} + (\mu_2 + \kappa) \nabla \times \nabla \times \vec{q}^{(2)} - \kappa \nabla \times \vec{\nu} = 0, \tag{2b}$$

$$\kappa \nabla \times \vec{q}^{(2)} - 2\kappa \vec{\nu} - \gamma_0 \nabla \times \nabla \times \vec{\nu} + (\alpha_0 + \beta_0 + \gamma_0) \nabla\nabla \cdot \vec{\nu} = 0, \tag{2c}$$

where $\vec{q}^{(2)}$, $\vec{\nu}$ and $p^{(2)}$ are velocity vector, microrotation vector and pressure, μ_2 is the viscosity coefficient of the classical viscous fluid and $\kappa, \alpha_0, \beta_0$ and γ_0 are the new viscosity coefficients for the micropolar fluids.

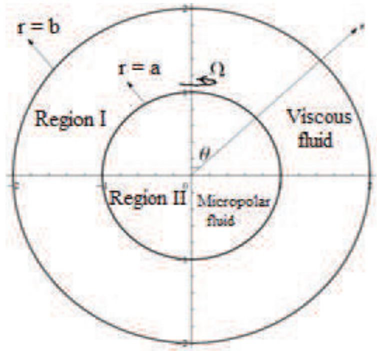

Fig. 1. Geometric sketch for the rotation of a micropolar fluid sphere in a concentric spherical cavity.

The equations for the stress tensor t_{ij} and the couple stress tensor m_{ij} are

$$t_{ij} = -p\,\delta_{ij} + \mu_2\,(q_{i,j} + q_{j,i}) + \kappa\,(q_{j,i} - \epsilon_{ijm}\,\nu_m), \tag{3}$$

$$m_{ij} = \alpha_0\,\nu_{m,m}\,\delta_{ij} + \beta_0\,\nu_{i,j} + \gamma_0\,\nu_{j,i}. \tag{4}$$

where the comma denotes the partial differentiation, δ_{ij} and ϵ_{ijm} are the Kronecker delta and the alternating tensor.

Let (r, θ, ϕ) denote a spherical polar co-ordinate system with origin at the center of a sphere $r = a$. Since the rotation is assumed to be slow, the velocity $\vec{q}$ has its only component along the vector $\vec{e}_\phi$ and the microrotation vector $\vec{\nu}$ lies in the meridian plane. The flow is time independent and all the quantities are independent of ϕ. Thus, we choose the velocity and microrotation vectors as

$$\vec{q}^{(i)} = q_\phi^{(i)}(r,\theta)\,\vec{e}_\phi,\ \ i = 1, 2 \tag{5}$$

$$\vec{\nu} = \nu_r(r,\theta)\,\vec{e}_r + \nu_\theta(r,\theta)\,\vec{e}_\theta \tag{6}$$

The field equations in this case reduce to:
In the viscous region $a \le r \le b$,

$$\frac{\partial p^{(1)}}{\partial r} = 0,\ \ \frac{\partial p^{(1)}}{\partial \theta} = 0 \tag{7}$$

$$L\,q_\phi^{(1)} = 0 \tag{8}$$

and for the micropolar region $r \le a$

$$\frac{\partial p^{(2)}}{\partial r} = 0,\ \ \frac{\partial p^{(2)}}{\partial \theta} = 0 \tag{9}$$

$$L(L - m^2)q_\phi^{(2)} = 0 \tag{10}$$

Assume that $\mathrm{div}\vec{\nu} = \omega(r,\theta)$, $\mathrm{curl}\vec{\nu} = \tau(r,\theta)\,\vec{e}_\phi = -N^{-1}\,Lq_\phi^{(2)}\,\vec{e}_\phi$, we have

$$\left(\nabla^2 - c^2\right)\omega = 0 \tag{11}$$

where

$$m^2 = \frac{a^2\,\kappa(2+\chi)}{\gamma_0(1+\chi)},\ \ c^2 = \frac{2\,\kappa\,a^2}{\alpha_0 + \beta_0 + \gamma_0},\ \ \chi = \frac{\kappa}{\mu_2} \quad \text{and} \quad N = \frac{\chi}{1+\chi}$$

$$L = \nabla^2 - \frac{1}{r^2\sin^2\theta},\ \ \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta}$$

$$\nu_r = \frac{1}{c^2}\frac{\partial\omega}{\partial r} - \frac{\gamma_0}{2\,\kappa}\frac{1}{r}\left(\frac{\partial\tau}{\partial\theta} + \tau\cot\theta\right) + \frac{1}{2\,r}\left(\frac{\partial q_\phi^{(2)}}{\partial\theta} + q_\phi^{(2)}\cot\theta\right) \tag{12}$$

$$\nu_\theta = \frac{1}{c^2}\frac{1}{r}\frac{\partial\omega}{\partial\theta} + \frac{\gamma_0}{2\,\kappa}\left(\frac{\partial\tau}{\partial r} + \frac{\tau}{r}\right) - \frac{1}{2}\left(\frac{\partial q_\phi^{(2)}}{\partial r} + \frac{q_\phi^{(2)}}{r}\right) \tag{13}$$

3. Boundary Conditions

On the surface of micropolar fluid sphere $r = a$, we assume the continuity of velocity components, tangential stresses and spin vorticity relation

1. $q_\phi^{(1)} = q_\phi^{(2)}$
2. $t_{r\phi}^{(1)} = t_{r\phi}^{(2)}$
3. $\nu_r = \frac{s}{2\,r}\left[\frac{\partial q_\phi^{(1)}}{\partial\theta} + q_\phi^{(1)}\cot\theta\right], \quad \nu_\theta = -\frac{s}{2}\left[\frac{\partial q_\phi^{(1)}}{\partial r} + \frac{q_\phi^{(1)}}{r}\right]$
4. On the cell surface $r = b$, $q_\phi^{(1)} = -\Omega\, a\, r\sin\theta$

4. Solution of the problem

The solution of Eqs. (8), (10) and (11) are given respectively

$$q_\phi^{(1)} = \left[A\,r + B\,r^{-2}\right]\sin\theta, \tag{14}$$

$$q_\phi^{(2)} = \left[C\,r + r^{-1/2}\,I_{3/2}(m\,r)\,D\right]\sin\theta, \tag{15}$$

$$\omega(r,\theta) = r^{-1/2}\,I_{3/2}(c\,r)\,E\cos\theta, \tag{16}$$

$$\tau(r,\theta) = -r^{-1/2}\frac{m^2}{N}I_{3/2}(m\,r)\,D\sin\theta. \tag{17}$$

Thus, using the expressions for τ, ω and $q_\phi^{(2)}$ in the equations (12) and (13), the expressions for ν_r and ν_θ are obtained as

$$\nu_r = \left[C + \frac{2}{N}\frac{1}{r^{3/2}}I_{3/2}(m\,r)\,D - \frac{1}{c^2}\frac{1}{r^{3/2}}\left(2\,I_{3/2}(c\,r) - c\,r\,I_{1/2}(c\,r)\right)E\right]\cos\theta \tag{18}$$

$$\nu_\theta = \left[-C + \frac{1}{N}\frac{1}{r^{3/2}}\left(I_{3/2}(m\,r) - m\,r\,I_{1/2}(m\,r)\right)D - \frac{1}{c^2}\frac{1}{r^{3/2}}I_{3/2}(c\,r)\,E\right]\sin\theta \tag{19}$$

5. Torque on the body

The hydrodynamic torque experienced by micropolar fluid sphere in presence of a spherical container is

$$T = 2\,\pi\,a^3\int_0^\pi t_{r\phi}^{(1)}|_{r=1}\sin^2\theta\,d\theta = -4\,\pi\,\mu_1\,\Omega\,a^3\,B = -8\,\pi\,\mu_1\,\Omega\,a^3\,\chi\,(1-s)\Delta_1/\Delta \tag{20}$$

where

$$\Delta_1 = I_{3/2}(c)\left(3\,I_{3/2}(m)(3+2\,\chi) - m\,I_{1/2}(m)(1+\chi)\right) - c\,I_{1/2}(c)\,I_{3/2}(m)(2+\chi) \tag{21}$$

$$\begin{aligned}\Delta = I_{3/2}(c)\Big(2\,m\,I_{1/2}(m)(1+\chi)\,(3\,\lambda(2+\chi) - (1-s)(1-\eta^3)\chi) + 3\,\chi\,I_{3/2}(m)\,(6 + 4\,\chi - s\,\chi - 3\,\lambda(2+\chi)\\ -\ 2\,\eta^3(1-s)(3+2\,\chi))\Big) + c\,I_{1/2}(c)(2+\chi)(-3\,m\,\lambda\,I_{1/2}(m)(1+\chi) + I_{3/2}(m)(\lambda(3+6\,\chi) - (2+s-2\,\eta^3(1-s))\chi))\end{aligned} \tag{22}$$

and $\lambda = \dfrac{2\,\sigma}{1+\chi}$ with $\sigma = \dfrac{\mu_1}{\mu_2}$ i.e., σ is the classical ratio of viscosities between the internal and external fluids, $\eta = \dfrac{a}{b}$. The couple exerting on the sphere in an unbounded medium is

$$T_\infty = -8\,\pi\,\mu_1\,\Omega\,a^3\,\chi(1-s)\Delta_1/\Delta_2 \tag{23}$$

where

$$\begin{aligned}\Delta_2 = I_{3/2}(c)\,(-3\,\chi\,I_{3/2}(m)(-6 + (-4+s)\,\chi + 3\,\lambda(2+\chi)) + 2\,m\,I_{1/2}(m)(1+\chi)((-1+s)\chi + 3\,\lambda(2+\chi)))\\ +\ c\,I_{1/2}(c)(2+\chi)(-3\,m\,\lambda\,I_{1/2}(m)(1+\chi) + I_{3/2}(m)(-(2+s)\chi + \lambda(3+6\chi)))\end{aligned} \tag{24}$$

5.1. Special cases

Case(i) When $s \to 0$ i.e., there is no microrotation at the boundary and $\sigma \to 0$ i.e., rotation of a solid sphere in viscous fluid in Eq. (20), we get the expression for the hydrodynamic couple acting on the solid sphere in a cell model

$$T = -\frac{8\,\pi\,\mu_1\,a^3\,\Omega}{1-\eta^3} \tag{25}$$

This agrees with the result of the rotation of an impermeable solid sphere in a cell model (Keh and Lu [13], Saad [14], Srinivasacharya and Krishna Prasad [15]).
Case(ii) When $\eta \to 0$ in Eq. (25), we get the torque acting on the solid sphere

$$T = -8\,\pi\,\mu_1\,a^3\,\Omega \tag{26}$$

which is a well-known result for slow rotation of a rigid sphere of radius a in a viscous fluid around an axis passing through its center (Kim and Karrila [16], Lamb [17]).
The wall correction factor W_c is defined as the ratio of the actual couple experienced by the particle in the container and the couple on a particle in an infinite expanse of fluid. With the aid of Eqs. (20) and (23) this becomes

$$W_c = \frac{T}{T_\infty} = \frac{\Delta_2}{\Delta} \tag{27}$$

The wall correction factor in case of slow steady rotation of a solid sphere in a spherical container is given by

$$W_c = \frac{1}{1-\eta^3} \tag{28}$$

6. Results and Discussion

The variation of the wall correction factor W_c with the separation parameter η are shown in Figs.2-4 and Table 1 for different values of spin parameter s, micropolarity parameter χ and the classical ratio of viscosities between the internal and external fluid σ. In numerical computation of the wall correction factor, we assumed the value of $\dfrac{\gamma_0}{\mu_2\,a^2} = 0.3$ and $\dfrac{(\alpha_0+\beta_0+\gamma_0)}{\mu_2\,a^2} = 0.4$. The variation of wall correction factor with separation parameter for different values of micropolarity parameter is shown in Fig. 2. It shows that the wall correction factor increases monotonically with increase in micropolarity parameter for fixed values of s and σ. Fig. 3 shows the effect of the spin parameter s on the wall correction factor. The spin parameter ranges over the interval $0 < s < 1$. If $s = 0$, there is no rotation of microelements near the boundary and if $s = 1$, the microrotation is equal to the fluid vorticity at the boundary. It is readily observed from the figure that the wall correction factor with spin condition is less than that of no-spin condition on microrotation. Also, the wall correction factor decreases monotonically with increasing spin parameter for fixed values σ and χ. Fig. 4 shows the variation of wall correction factor with separation parameter for different values of classical ratio of viscosities between the internal and external fluid σ. It is observed from Fig.4 that the wall correction factor for $\sigma = 0$ is greater than that of any value of σ. Also, the wall correction factor decreases with increasing values of σ except $\sigma = 0$ and it is monotonically increases as separation parameter increases. As $s \to 0$ and $\sigma \to 0$, the problem reduces to the steady rotation of a solid sphere in a spherical cavity filled with viscous fluid. Table 1 shows the numerical results of wall correction factor W_c for different values of separation parameter η and $s \to 0$ and $\sigma \to 0$. The results are in good agreement with results obtained by Keh and Lu [13] and Saad [14].

7. Conclusion

The problem of steady rotational motion of a micropolar fluid sphere in a concentric spherical container containing viscous fluid is investigated analytically in this paper by considering non-zero spin boundary condition for the microrotation vector. An expression is obtained for the hydrodynamic torque acting on the sphere and wall correction factor. The wall correction factor increases as the separation parameter increases and decreases as the spin parameter increases. The effect of micropolarity parameter and the classical ratio of viscosities between the internal and external fluid on wall correction factor is also studied.

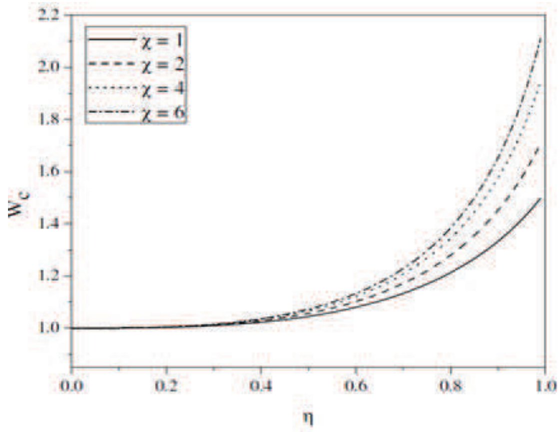

Fig. 2. Variation of W_c with η for various values of χ with $s = 0.2$ and $\sigma = 0.3$.

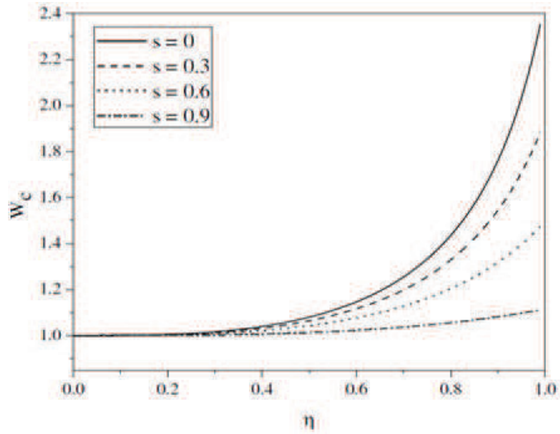

Fig. 3. Variation of W_c with η for various values of s with $\chi = 5$ and $\sigma = 0.3$.

Table 1. Wall correction factor W_c for different values of separation parameter η and viscosity ratio σ for any value of χ and $s = 0$.

	Wall correction factor W_c		
Separation parameter (η)	Present Study $\sigma \to 0$	Solution of Keh and Lu [13]	Solution of Saad [14]
0.1	1.001	1.001	1.001
0.2	1.00806	1.00806	1.00806
0.3	1.02775	1.02775	1.02775
0.4	1.06838	1.06838	1.06838
0.5	1.14286	1.14286	1.14286
0.6	1.27551	1.27551	1.27551
0.7	1.52207	1.52207	1.52207
0.8	2.04918	2.04918	2.04918
0.9	3.69004	3.69004	3.69004

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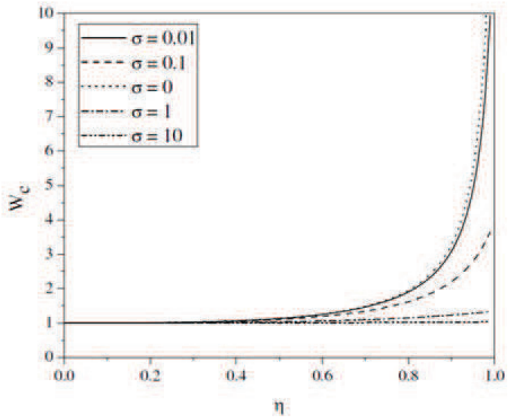

Fig. 4. Variation of W_c with η for various values of σ with $\chi = 5$ and $s = 0.2$.

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