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Numerical Model of Lateral Electric Field Excited Resonator on Piezoelectric Plate Bordered with Viscous and Conductive Liquid

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Abstract

The numerical method of calculation of characteristics of lateral electric field excited resonator contacting with viscous and conducting liquid and results of these calculations are described. The method based on finite element analysis allows to find the distribution of mechanical and electrical fields in piezoelectric plate and liquid and to calculate the frequency dependencies of electrical impedance and admittance of resonator. It has been shown that values of real parts of impedance and admittance on resonant frequencies unambiguously correspond to viscosity and conductivity of liquid.

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1. Introduction

In recent years the designers of liquid sensors have given the special attention to piezoelectric lateral electric field excited resonators. There are a vast number of papers which describe these resonators with most of them cited in [1, 2]. The potentials of the resonators as liquid sensors for measuring the viscosity, permittivity, and electrical conductivity have been demonstrated in Refs. [1, 3]. However by now the rigorous methods of theoretical analysis of such resonators contacting with liquid are absent. In this connection it is urgent to develop a method of

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calculation of resonator characteristics, which will allow to choose the optimal crystallographic orientation of resonator plate, the position and shape of electrodes, and also the way of suppression of parasitic oscillations.

This paper describes the numerical method of calculation of characteristics of lateral electric field excited resonator contacting with viscous and conducting liquid. Resonator represents thin rectangular piezoelectric plate with two electrodes on the lower side which are surrounded with viscoelastic damping layer [2]. The upper side of plate contacts with liquid layer of finite thickness with given mechanical and electrical characteristics. The method is based on finite element analysis, which is described in detail in [4] and allows to find the distribution of mechanical and electrical fields in plate and liquid, and also to calculate the frequency dependencies of electrical impedance and admittance of resonator.

2. Theoretical model of resonator

This paper concerns with two-dimensional model of resonator. The basis of resonator is represented by piezoelectric plate bounded in directions $x$ and $y$. Two infinitely thin electrodes $e_1$ and $e_2$ are set on the lower side of the plate. These electrodes produce the electric field which harmonically oscillates with frequency $\omega$. The part of electrodes and space around them are covered by special damping layer. The upper side of the plate is contacted with layer of liquid of finite thickness. In order to consider the distribution of electric field the model involves the sufficiently large round region of space around the plate on the external border $\Gamma$ of which the electrical potential is equal zero (Fig.1). Along the axis $z$ the plate, electrodes, and liquid layer are considered to be infinite.

So the model under study consists of three regions: 1 – piezoelectric plate, 2 – layer of viscous and conducting liquid, and 3 – ambient space. Physical problems are solved independently in each region and then their solutions are joined on boundaries by using boundary conditions.

In region 1 we solve the problem of propagation of harmonic acoustic wave in piezoelectric medium which is accompanied by electric field. As is well known [5] the mechanical displacement and electrical potential within the piezoelectric plate should satisfy the motion and Laplace’s equations:

$$c_{ijkl} \frac{\partial^2 u_i}{\partial x_j \partial x_k} + \epsilon_{ijk} \frac{\partial \phi}{\partial x_j} + \rho \omega^2 u_i = 0, \quad \epsilon_{ijkl} \frac{\partial^2 u_i}{\partial x_j \partial x_k} - \epsilon_{ikj} \frac{\partial \phi}{\partial x_j} = 0. \quad (1)$$

Here $u_i$ is mechanical displacement, $\phi$ is electrical potential, $\rho$ is density of plate, $c_{ijkl}$, $\epsilon_{ijk}$, $\epsilon_{jk}$ are tensors of elastic, piezoelectric, and dielectric constants. On the electrodes $e_1$ and $e_2$ placed on the lower side of plate we assign the values of electrical potential as $\phi_{e_1} = -1$, $\phi_{e_2} = +1$. On the boundaries $d_1$ and $d_2$ covered by damping layer the boundary condition is written as $T_{j}n_{j} = -\omega Z u_i$ where $T_{j}$ is tensor of mechanical stress, $n_{j}$ is unit normal to the surface, $Z$ is mechanical impedance of damping layer, and $i$ is imaginary unit.
In the region 2 we simultaneously solve two problems: the problem of propagation of acoustic wave in liquid and the problem of distribution of electrical charge produced by electric field. As is well known [6] the acoustic field in liquid medium may be described by single wave equation respectively to pressure:

$$\frac{1}{\rho_v} \nabla^2 p + \frac{k^2}{\rho_v} p = 0$$

(2)

Here $p$ is pressure; $p = p_0 + p_a$, where $p_0$ is constant (atmospheric) pressure and $p_a$ is acoustic pressure; $k = \omega c_v$ is wave number of acoustic wave; $c_v = c (1 + i \omega \eta / (\rho c^2)^{1/2}$ and $\rho_v = \rho c^2 / c_v^2$, where $c$ is sound velocity in liquid, $\rho$ is liquid density, and $\eta$ is liquid viscosity. The upper side is mechanically free: $p = p_0$. On the lateral sides of the region 2 we use the condition of fixed boundary $n \nabla p = 0$, where $n$ is outside normal to boundary (walls of liquid container). The boundary conditions on the interface between liquid and piezoplate have the standard form $n (1/\rho_v) \nabla p = - n \omega^2 u$ and $F = n p$. Here $u$ is mechanical displacement in plate and $F$ is load (force per unit area) experienced by the plate.

The distribution of electrical charge in conducting medium produced by electric field is described by the following system of equations

$$\varepsilon_0 \frac{\partial \varphi}{\partial x} \delta_{ij} + q = 0, \quad -\sigma_{ij} \frac{\partial^2 \varphi}{\partial x \partial x_j} + d_{ij} \frac{\partial^2 q}{\partial x \partial x_j} + I \omega q = 0.$$  

(3)

Here $q$ is bulk density of charge, $\sigma_{ij}$ is conductivity tensor, $d_{ij}$ is tensor of diffusion coefficients. The boundary conditions include continuity of electrical potential and normal component of electrical displacement and the absent of electrical current through upper and lower side of liquid.

In the region 3 we solve the problem of distribution of electric potential in the space:

$$\varepsilon_0 \delta_{ij} \frac{\partial \varphi}{\partial x} \delta_{ij} = 0$$

(4)

where $\varepsilon_0$ is permittivity of vacuum, $\delta_{ij}$ is Kronecker delta. The boundary conditions correspond to zero potential on the outside boundary of region $\varphi^1 = 0$ and continuity of potential and normal components of electrical displacement on other boundaries.

The exciting electric voltage between electrodes is alternating and changes harmonically with frequency $\omega$. Due to absence of other source of oscillations and linearity of the problem the solution also is harmonic. So the considered problem comes to the system of differential equations [7]

$$L(u, \varphi, q) - f = 0$$

(5)

where $L$ is differential operator and $f$ is given value. The solution of this problem means the finding of functions $u(x,y)$, $\varphi(x,y)$, and $q(x,y)$, which satisfy equations (1-4) and boundary conditions for given value $\omega$. This problem may be solved with the help of the finite element analysis by using Galerkin’s method for derivation of equations for each element.

3. Results of simulation of the resonator with viscous and conducting liquids

The aforementioned method allowed to calculate the frequency dependencies of real and imaginary parts of electrical impedance and admittance of the resonator loaded by viscous and conducting liquids. The calculations were carried out for the resonator based on plate of X-cut of lithium niobate. The values of thickness of plate and liquid layer were equal 0.5 and 2 mm, respectively. The width of electrodes was equal 5 mm with the width of gap of 2 mm. It has been found that the frequencies of resonances practically not depend from liquid viscosity, however the frequency of parallel resonance decreases by 0.2% when the conductivity of liquid increases. The dependencies of
real parts of electrical impedance $R$ and admittance $G$ on viscosity (Fig.2) and conductivity (Fig.3) correspond to these resonance frequencies were calculated.

![Fig. 2. The dependencies of resonant frequencies (a), real parts of electrical impedance (b) and admittance (c) on liquid viscosity. Solid line corresponds to the parallel resonance, dashed line corresponds to the series resonance.](image)

![Fig. 3. The dependencies of resonant frequencies (a), real parts of electrical impedance (b) and admittance (c) on liquid conductivity. Solid line corresponds to the parallel resonance, dashed line corresponds to the series resonance.](image)

One can see that these dependencies have the monotonic character that allows to build the calibration curves of the liquid sensor for determination of viscosity or conductivity of liquid by measuring the electrical impedance or admittance on resonant frequencies. The change of these values may be used as informative parameters of such sensors.

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**References**


