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## A numerical resolution of the current-voltage equation for a real photovoltaic cell

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### Abstract

Solving the characteristic equation of the real photovoltaic cell that contains two variables is to find for each value of the voltage of the cell the value of the electric current generated by this cell. Since the complex shape of this equation does not permit to resolve it analytically despite that we know the intervals of solutions, the interval of solutions of the electric current varies from the value 0 to the value  $I_{sc}$  (the short-circuit current). The interval of solutions of the voltage varies from the value 0 to the value  $V_{oc}$  (the open circuit voltage), from the advantage of knowing the intervals of solutions, we can solve this equation with a numerical model.

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*Keywords:* Cell junction temperature; Current-voltage characteristic; fill factor; irradiance; Matlab program; Nominal operating cell temperature; Photocurrent; Saturation current

### 1. Introduction

It is convenient to use a model for the description of the relationship between current and voltage, which is able to describe the solar cell in the generation region as well as the breakdowns at positive and negative voltages. The photovoltaic cells current–voltage mathematical description is usually defined by a coupled nonlinear equation, difficult to solve using analytical methods [1, 2]. Modeling photovoltaic cell, photovoltaic arrays and modules requires a set of equations characterizing all elements of the system studied. At this level, the condition that it is necessary is to know the input criteria (basic data), that is to say, weather data for the studied site, the data for use, and data on equipments [3, 4].

**Nomenclature**

A	Diode ideality factor of a solar cell, between 1 and 5 in practice [5]
$E_c$	Irradiance ( $W/m^2$ ).
I	Current of the cell (A)
$I_{ph}$	Photocurrent (A)
$I_{sat}$	Saturation current of the diode (A)
$I_{sc}$	Short-circuit current (A)
k	$1.38 \times 10^{-23}$ Boltzmann constant
NOCT	Nominal operating cell temperature ( $^{\circ}C$ )
PV	Photovoltaic
q	$1.602 \times 10^{-19}$ electron charge (C)
$R_p$	Resistance in parallel (shunt) to the diode ( $\Omega$ )
$R_s$	Series resistance ( $\Omega$ )
$T_a$	Ambient temperature (K)
$T_c$	Junction Temperature (K)
V	Voltage of the cell (Volt)
$V_{oc}$	Open circuit voltage (Volt)

**2. Mathematical model of photovoltaic cell**

Modeling of a photovoltaic cell can be performed at different levels of complexity. It is about getting an equivalent electrical circuit of the photovoltaic cell. There are several models from which we choose the rather complex circuit, as shown in Fig. 1[6].

This model has a source of current  $I_{ph}$ , which depends on the solar radiation and temperature, a diode in parallel in which the reverse saturation current depends on temperature and a series resistor  $R_s$ , which represents the effect of the internal resistance and the contacts of the cell. We can introduce more complexity in the model by incorporating further a resistor in parallel with the diode  $R_p$ , which represented the effect of the current which crosses through the crystal.

The equations 1 and 2 which describe the curve I-V of a real PV cell are [6]:

$$I = I_{ph} - I_{sat} \times \left( \exp \left( q \times \frac{(V + R_s \times I)}{A \times k \times T_c} \right) - 1 \right) - \frac{(V + R_s \times I)}{R_p} \quad (1)$$

$$T_c - T_a = \frac{NOCT - 20}{800} \times E_c \quad (2)$$

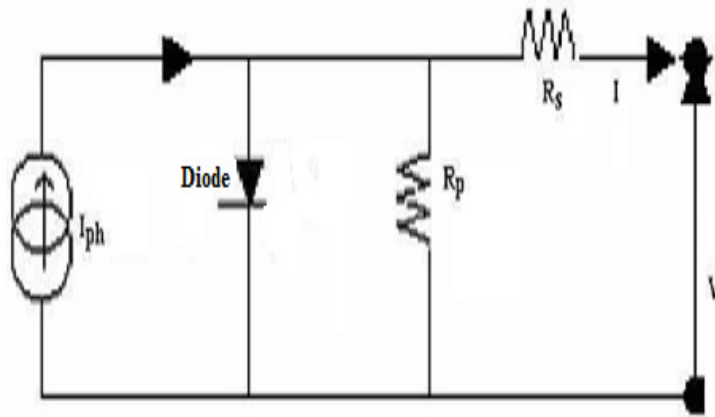


Fig. 1 Equivalent circuit of a photovoltaic cell [6]

### 3. Characterization of current-voltage equation

#### 3.1 Short-circuit current

This is the current  $I_{sc}$  for which the voltage of the photovoltaic cell is zero. In the ideal case (no  $R_s$ ,  $R_p$  infinite), this current  $I_{sc}$  merges with the photocurrent  $I_{ph}$ . For  $V = 0$  we obtain the following equation 3:

$$I_{sc} = I_{ph} - I_{sat} \times \left( \exp \left( q \times \frac{R_s \times I_{sc}}{A \times k \times T_c} \right) - 1 \right) - \frac{R_s \times I_{sc}}{R_p} \quad (3)$$

#### 3.2 Open-Circuit voltage

This is the voltage  $V_{oc}$  to which the current supplied by the photovoltaic cell is zero (the maximum voltage of a solar cell or a PV generator). For  $I = 0$  we obtain the following equation 4:

$$0 = I_{ph} - I_{sat} \times \left( \exp \left( q \times \frac{V_{oc}}{A \times k \times T_c} \right) - 1 \right) - \frac{V_{oc}}{R_p} \quad (4)$$

#### 3.3 Power, fill factor and performance of a photovoltaic cell

The interesting part of the current-voltage characteristic for the user, is the one that generates energy. Therefore it will be neither at the point of open-circuit voltage, nor at the short-circuit current, which does not generate any power, which is the product of current by voltage. We have plotted in Fig. 2 the characteristic under a solar light and theoretical curves of constant power (with dotted) [7].

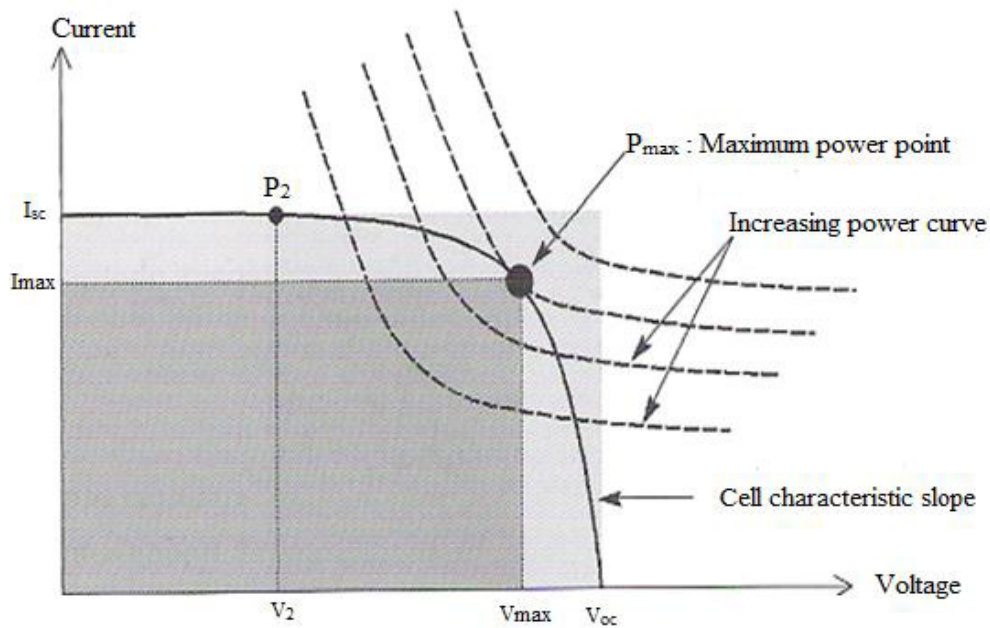


Fig. 2 Maximum power on a current-voltage characteristic [7]

The  $P_{max}$  point located in the inflexion of the characteristic curve, the power of the solar cell is the maximum for involved irradiance. The maximum power point is associated to the maximum voltage  $V_{max}$  and the maximum current  $I_{max}$ . We note however that the solar cell may also be requested to lower power, for example at a voltage lower than  $V_{max}$  (point  $P_2$  in Fig. 2). It is clear that the curve is more square, the maximum power  $P_{max}$  is more raised. Using the equation 5, we evaluate this property by the fill factor [7].

$$ff = \frac{P_{max}}{V_{oc} \times I_{sc}} \tag{5}$$

Energy efficiency is defined as the ratio between the maximum power produced and the power of the solar radiation that reaches the photovoltaic cell.

If  $S$  is the surface of the cell and  $E_c$  is the irradiance ( $W/m^2$ ), the equation 6 calculates the energy efficiency [7].

The  $P_{max}$  point located in the inflexion of the characteristic curve, the power of the solar cell is the maximum for involved irradiance. The maximum power point is associated to a maximum voltage  $V_{max}$

$$Nc = \frac{P_{max}}{E_c \times S} \tag{6}$$

This efficiency is often measured in baseline conditions, that is to say, under the sunshine of  $1000 W/m^2$ , the temperature of  $25^\circ C$  and the spectrum AM1, 5. These conditions are called Standard Test Conditions (STC).

#### 4. Resolution of the current-voltage equation for a real photovoltaic cell using the Matlab program

Fig. 3 shows the I-V curve of a real photovoltaic cell. A reference cell was used to adjust the power at the output of the solar simulator. It was produced and calibrated at the Fraunhofer Institute for Solar Energy Systems for a power of  $100 \text{ mW/cm}^2$ . For such a power, the voltage at the output of the reference cell is  $31.56 \text{ mV} \pm 2.0\%$ . Required adjustments (xenon lamp power and distance between the Fresnel lens and the sample) were made to reproduce this value. After calibration, the facility was used to plot the I-V characteristics under irradiance of solar cells [8].

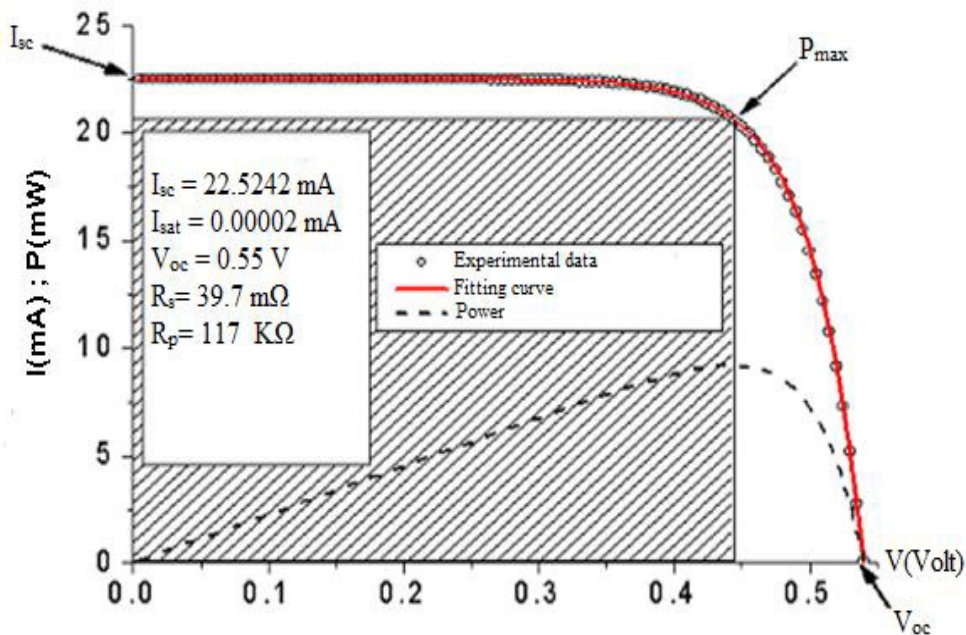


Fig. 3 I-V curve of a real photovoltaic cell [8]

However, the curve profiles obtained were influenced by the vibrations generated by the seven fans used for cooling. In addition, the minimum pitch of measurement is  $20 \text{ mV}$ . For cells with  $I_{sc}$  or low  $V_{oc}$ , such a change in voltage is insufficient to have a good accuracy [8].

Fig. 4 shows the flowchart which permits to:

1. Calculate the junction temperature  $T_c$  using the equation 2.
2. Calculate the saturation current  $I_{sat}$  and the photocurrent  $I_{ph}$  using the equations 3 and 4.

$$I_{sat} = \frac{I_{cc} \times \left(1 + \frac{R_s}{R_p}\right) - \frac{V_{co}}{R_p}}{\exp\left(\frac{q \times V_{co}}{A \times k \times T_c}\right) - \exp\left(\frac{q \times R_s \times I_{cc}}{A \times k \times T_c}\right)} \quad (7)$$

$$I_{ph} = I_{sat} \times \left( \exp\left(q \times \frac{V_{co}}{A \times k \times T_c}\right) - 1 \right) + \frac{V_{co}}{R_p} \quad (8)$$

3. Solve the equation 1 and plot the current-voltage curve and the power-voltage curve. The program which we established can directly solve the equation 1, but in the form of two equations:

$$X(V, I) = I_{sat} \times \left( \exp\left(q \times \frac{(V + R_s \times I)}{A \times k \times T_c}\right) - 1 \right) \quad (9)$$

$$Y(V, I) = I_{ph} - \frac{V}{R_p} - \left( \frac{R_s}{R_p} + 1 \right) \times I \quad (10)$$

4. Find the maximum power, then the maximum voltage and the maximum current without calculation.
5. Calculate the fill factor and energy efficiency.
6. Introduce the following data:
  - $I_{sc}$ : Short-circuit current (Ampere)
  - $V_{oc}$ : Open-circuit voltage (volts).
  - $R_s$ : Series resistance ( $\Omega$ ).
  - $R_p$ : Parallel resistor (shunt) to the diode ( $\Omega$ ).
  - $k$ :  $1.38 \times 10^{-23}$  J / K, Boltzmann constant.
  - $q$ :  $1.602 \times 10^{-19}$  electron charge (C).
  - $T_c$ : Junction Temperature (K).
  - A: Ideality factor of the solar cell, between 1 and 5 in practice.
  - NOCT: Nominal operating cell temperature.
  - $T_a$ : Ambient temperature.
  - $E_c$ : Irradiance ( $W/m^2$ ).
  - S: Surface of the cell ( $m^2$ ).

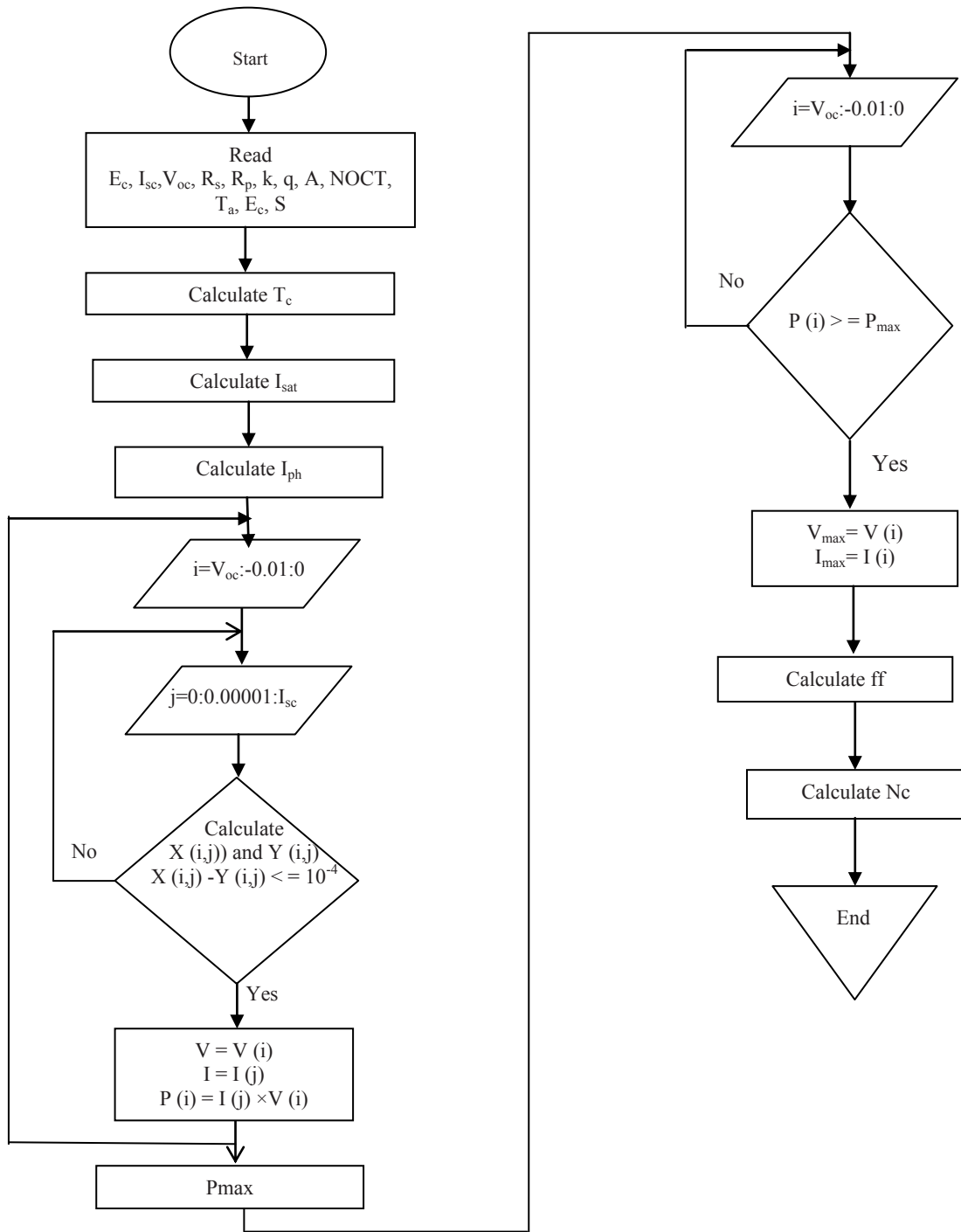


Fig. 4 Flowchart for the calculation of the current-voltage for a real photovoltaic cell

Using the data used in the Fig. 3, we obtained the results that appear in Fig. 5 and 6:

$E_c = 1000 \text{ W/m}^2$ ,  $I_{sc} = 0.0225242 \text{ A}$ ,  $V_{oc} = 0.55 \text{ V}$ ,  $R_s = 0.0397 \text{ } \Omega$ ,  $R_p = 117,000 \text{ } \Omega$ ,  $\text{NOCT} = 45 \text{ } ^\circ \text{ C}$ ,  $S = 0.0001 \text{ m}^2$ ,  $A = 1.6$ ,  $k = 1.38 \times 10^{-23} \text{ J/K}$ ,  $q = 1.602 \times 10^{-19} \text{ C}$ ,  $T_a = 298 \text{ K}$

The Matlab program has calculated the junction temperature ( $T_c = 329.25 \text{ K}$ ), the saturation current ( $I_{sat} = 1.246 \times 10^{-7}$ ) and the photocurrent ( $I_{ph} = 0.022524 \text{ A}$ ) using the following equations 2, 7 and 8.

Then, using the equations 9 and 10, the Matlab program computes and compares for each voltage value  $V$ , ranging between 0 and  $V_{oc}$ , the value of the current  $I$  ranging between 0 and  $I_{sc}$ , so that after replacing these values in the equations 9 and 10, we will obtain a difference less or equal to  $10^{-4}$ .

Once the  $I$  value is obtained for each  $V$  value, the Matlab program calculates the power corresponding to these two values, and when it calculates the whole powers, it searches the maximum power and then, it will find the maximum voltage and the maximum current. After that, it traces the I-V and P-V curves and indicates on these curves the maximum power.

And in the end, the Matlab program calculates the fill factor ( $ff = 0.73095$ ) and the energy efficiency ( $N_c = 9.0552\%$ ), using the equations 5 and 6.

According to the results obtained by Matlab program and those in Fig. 3, they are identical especially concerning the maximum voltage and maximum current. The I-V curve of the real photovoltaic cell in the Fig. 3 which is obtained by experimental data and the I-V curve in the Fig. 6 plotted by the Matlab program are identical.

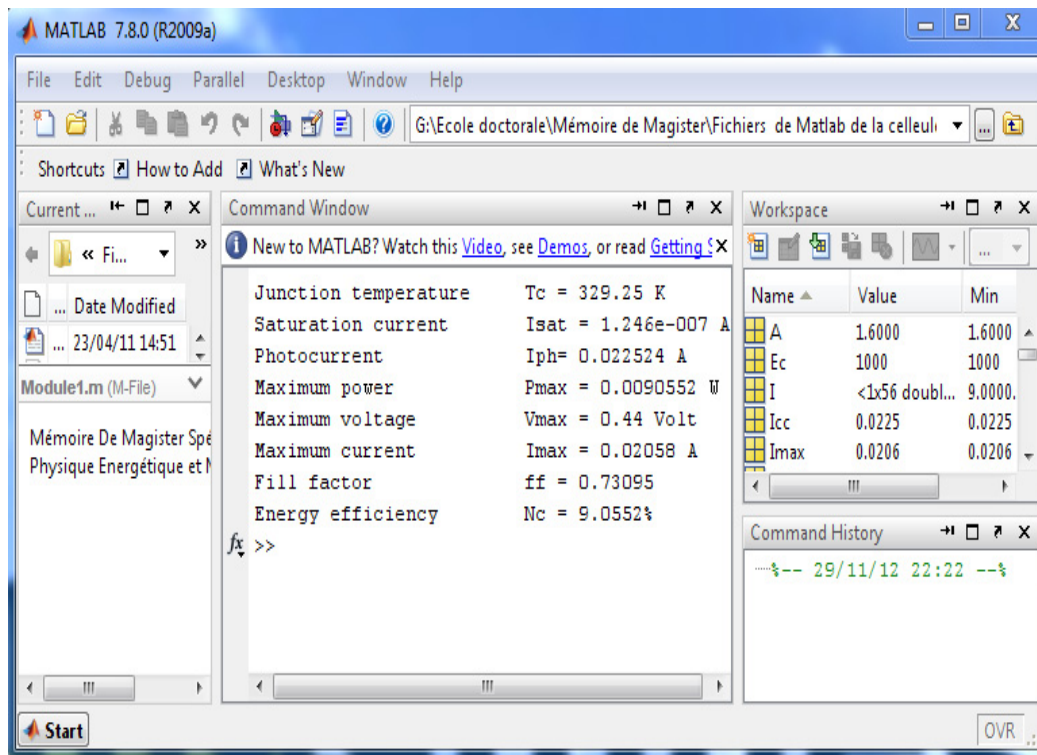


Fig. 5 Matlab calculation of the characteristics of a real photovoltaic cell



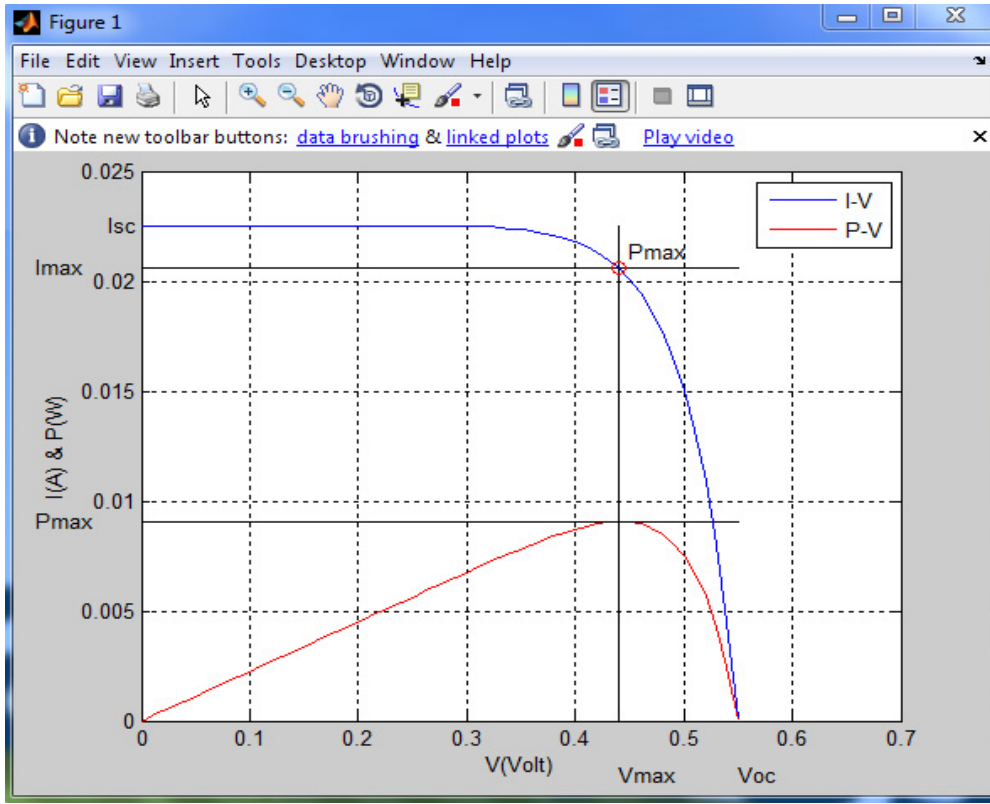


Fig. 6 I-V and P-V curves of a real photovoltaic cell

## 5. Conclusion

The construction of numerical model can cope with different configurations of basic units as well as other mismatch effects. In this paper, a numerical resolution of the current-voltage equation for a real photovoltaic cell and calculation certain of its characteristics are presented. This equation is a nonlinear equation and complex, and we used the Matlab program that could calculate all the solutions in a very short time.

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