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SU(2) vortex configuration in Laplacian center gauge

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Abstract

We study how Laplacian center gauge identifies the vortex content of a thick SU(2) vortex configuration on the lattice. This configuration is a solution of the Yang–Mills classical equations of motion having vortex properties. We find that this gauge fixing procedure cleanly identifies the underlying vortex properties. We also study the monopole content of this configuration detected with this procedure. We obtain two monopole curves lying on the surface of the vortex. © 2001 Elsevier Science B.V. Open access under [CC BY license](https://creativecommons.org/licenses/by/4.0/).

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1. Introduction

Confinement of quarks is still a phenomena not fully understood. Two mechanisms, proposed long time ago, are currently receiving a lot of attention. In the first one [1], confinement is seen as a dual Meissner effect, based in the condensation of magnetic monopoles in the QCD vacuum. In the second one [2], confinement is due to the condensation of vortices. Both pictures of confinement show up in specific partial gauge fixings.

In the dual superconductor picture of confinement, magnetic monopoles appear as defects in the Abelian gauges proposed by 't Hooft [3]. In this case the gauge is fixed up to the Cartan subgroup of the gauge group. Then, monopoles appear at points in space in which the gauge can not be fixed up to the Cartan subgroup,

leaving a gauge freedom larger than the Abelian subgroup. In the vortex picture of confinement vortices are bidimensional objects carrying flux quantized in elements of the center of the group.

Both pictures of confinement receive strong support from lattice results. The dual superconductor picture of confinement is studied by first fixing the lattice configurations to some Abelian gauge, and then, analyzing the Abelian projected configurations. In all the Abelian gauges considered it is found that there is monopole condensation in the confinement phase and there is not in the deconfinement phase [4–6]. The vortex picture of confinement is studied by first fixing the gauge to maximal center gauge and then analyzing the center projected configurations. By doing this it is observed that these projected configurations reproduce the full string tension. Even more, this string tension disappears if the center vortices identified after center projection are removed from the lattice ensemble [7,8]. This phenomena is called center dominance.

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The relevance of center dominance is obscured by the fact that you also obtain the full string tension without doing any gauge fixing [9]. Then, center dominance seems of no physical relevance. Nevertheless, as it is said in [9], the non-triviality of center projection is related to the maximal center gauge fixing because, after doing that, the information about extended physical objects is now encoded in Z_N local observables. So, maximal center gauge is needed to identify the vortex content of the vacuum. Doing this, maximal center gauge fixing and center projection, it is found that the obtained vortex properties extrapolate to the continuum limit, e.g., the ratio of string tension and vortex (area) density is regulator independent [10].

One of the drawbacks of maximal center gauge is that this gauge fixing procedure suffers from the Gribov copies problem. This problem is associated to the structure of the functional to be maximized, because it has many local maximums, and then, the local algorithms used to find the global maximum usually ends in one of these local maximums (Gribov copies). As was pointed out in [11,12] the Gribov copies problem for maximal center gauge is a really severe one. Using a more powerful algorithm (simulated annealing) to find the global maximum of the functional to be maximized, the projected string tension is not in agreement with the physical string tension. This negative result has been addressed in [13,14] arguing that direct maximal center gauge can be understood as a best fit to a given lattice gauge field by a thin vortex configuration [15], being this best fit given by an appropriate choice of the lattice Gribov copy and, with this choice, recovering the nice properties of the maximal center gauge fixing procedure. Laplacian center gauge [16] was proposed as an alternative to the maximal center gauge fixing prescription without the lattice Gribov copies problem. In this case, the vortex properties can be obtained with two different procedures, looking at the points in which the gauge transformation can not be uniquely defined or using the center projection method.

It is the purpose of this article to study how Laplacian center gauge identifies a thick vortex configuration on the lattice. To this end we apply this gauge fixing procedure to a solution of the Yang–Mills equations of motion having vortex properties. After gauge fixing we project to the center of the group, and then check if this solution is seen as a thin vortex in the

projected configuration. We compare the obtained result with the one obtained using maximal center gauge. Finally, we study the monopole and vortex content of this configuration by looking at the singularities of the gauge fixing procedure and see how this result compare with the previous methods.

The layout of the article is the following. In Section 2 we briefly describe both center gauge fixing procedures, maximal center gauge and Laplacian center gauge. In Section 3 we show how a vortex solution appears in these gauge fixing prescriptions. And in Section 4 we present our conclusions.

2. Center gauge fixing

In this section we briefly describe both gauge fixing procedures, maximal center gauge and Laplacian center gauge, for the SU(2) Yang–Mills theory.

The maximal center gauge (MCG) in SU(2) lattice gauge theory is defined as the gauge which brings link variables U as close as possible to elements of its center $Z_2 = \pm 1$. This can be achieved by maximizing the quantity:

$$C = \frac{1}{VD} \sum_{n=1}^V \sum_{\mu=1}^D \frac{1}{4} |\text{Tr} U(n, \mu)|^2, \quad (1)$$

where V is the number of sites on the lattice and D the number of dimensions. The usual procedures to maximize the functional C are local algorithms maximizing this quantity at each lattice point. In Ref. [17] it is described the most used algorithm to perform this maximization.

The Laplacian center gauge (LCG) fixing prescription use the two eigenvectors with lowest eigenvalues, $\psi_1^a(n)$ and $\psi_2^a(n)$, of the Laplacian operator,

$$\mathcal{L}_{nm}^{ab}(R) = \sum_{\mu} (2\delta_{nm}\delta^{ab} - R^{ab}(n, \mu)\delta_{m, n+\hat{\mu}} - R^{ba}(m, \mu)\delta_{n, m+\hat{\mu}}) \quad (2)$$

in presence of a gauge field $R^{ab}(n, \mu)$ in the adjoint representation of the gauge group, to fix completely the gauge up to the center of the SU(2) group. First, the lowest eigenvector, $\psi_1^a(n)$, is rotated to the (σ_3) direction in color space. This step, Laplacian Abelian gauge, fix the gauge up to the Abelian subgroup of

the SU(2) group [18]. This U(1) Abelian freedom is fixed by imposing that the $\psi_2^a(n)$ eigenvector is further rotated to lie in the positive (σ_1, σ_3) half-plane. After these two steps the gauge is completely fixed up to the center degrees of freedom.

3. Gauge fixing of a vortex solution

In this section we study how Laplacian center gauge identifies a thick vortex configuration on the lattice. To this end we apply this gauge fixing procedure to a solution of the Yang–Mills classical equations of motion having the properties of a thick vortex. The layout of this section is the following. First, we review the properties of the solution we are going to work with. This solution was presented in [19]. Second, we show how this solution appears after going to maximal center gauge and center projection. This result was presented in [24] and the solution appears as a thin vortex in the projected configuration. We will compare this result with the one obtained with LCG. Third, we fix the gauge to Laplacian center gauge and then we try to identify vortices in two different ways. First, by looking at the center projected configuration, and second by looking at points in which the gauge transformation is not well defined. Finally, we study the monopole content of this vortex solution.

The configuration we are going to study in maximal and Laplacian center gauge, is a solution of the SU(2) Yang–Mills classical equations of motion, presented in [19]. This solution lives on the four-dimensional torus T^4 , with two large directions, t, x , and two small directions, y, z , satisfies twisted boundary conditions given by the twist vectors $\vec{k} = \vec{m} = (1, 0, 0)$, has action $S = 4\pi^2$ and topological charge $|Q| = 1/2$. We fix the length of the torus in the small directions, y, z , to $l_{\text{small}} = 1$. The length in the large directions, t, x , has to be $l_{\text{large}} \gg l_{\text{small}}$ ($l_{\text{large}} = 4$ is large enough to obtain the desired properties of the solution). Then we have a solution living on a four-dimensional torus T^4 with physical sizes $l_{\text{large}}^2 \times l_{\text{small}}^2 = 4^2 \times 1^2$.

The main properties of the solution are the following. By looking at the action density we can see that it has only one maximum and has a size approximately equal to the size of the torus in the small directions, y and z . The action density goes exponentially to zero in the two large directions, t and x , while in the other two

directions, y and z , never reaches the zero value. This exponential fall off in t and x is the reason why $l_{\text{large}} = 4l_{\text{small}}$ is big enough. And the most important property of this solution is that a square Wilson loop in the xt plane, centered at the maximum of the solution, takes the value -1 for a big enough size of the loop and is almost independent of the yz coordinates [19]. Then, looking at this Wilson loop, we see a bidimensional object (because is independent of the y, z coordinates) carrying flux in an element of the center of the group.

To avoid any complication related to twisted boundary conditions when we gauge fix and center project the configuration, we repeat the solution once on each direction. Then we will have a solution in a four-dimensional torus with physical size $8^2 \times 2^2$, with action $S = 2^4 \times 4\pi^2$, topological charge $|Q| = 2^4 \times 1/2$, and satisfying periodic boundary conditions. Then, we have a solution with 16 maximums in the action density.

To obtain this solution on the lattice we use a cooling algorithm which implements twisted boundary conditions (see [20–23] for details on this procedure). In this article we use three configurations obtained in lattice sizes $N_t \times N_x \times N_y \times N_z$ with $N_t = N_x = 4N_s$ and $N_y = N_z = N_s = 4, 5, 6$. As we fix the length of the torus in the small directions to be $l_{\text{small}} = 1$, the lattice spacing is $a = 1/N_s$. Therefore, we will be looking at the same solution with three different resolutions, $a = 0.25, a = 0.20$ and $a = 0.16$. Once we have these three lattice configurations, we repeat the solution in all directions and we do not need the trick used to implement twisted boundary conditions on the lattice. Then, we have three lattice configurations with lattice sizes $2N_t \times 2N_x \times 2N_y \times 2N_z$ and satisfying periodic boundary conditions. We label these three configurations I, II and III, for the values of $N_s = 4, 5$ and 6 , respectively. From these lattice configurations the field strength $F_{\mu\nu}$ is obtained from the clover average of plaquettes 1×1 and 2×2 , combined in such a way that the discretization errors are $O(a^4)$. And from this $F_{\mu\nu}$ we calculate all other quantities, like the action density or the topological charge.

We show in Figs. 1(a) and 1(b) the action density for the III solution (lattice spacing $a = 0.16$). What we plot is $S(t, x, y, z)$ for fixed values of y and z . In Fig. 1(b) we choose these values to be the maximum of the action density in y, z and in Fig. 1(a) the minimum. We can see that the curves obtained joining

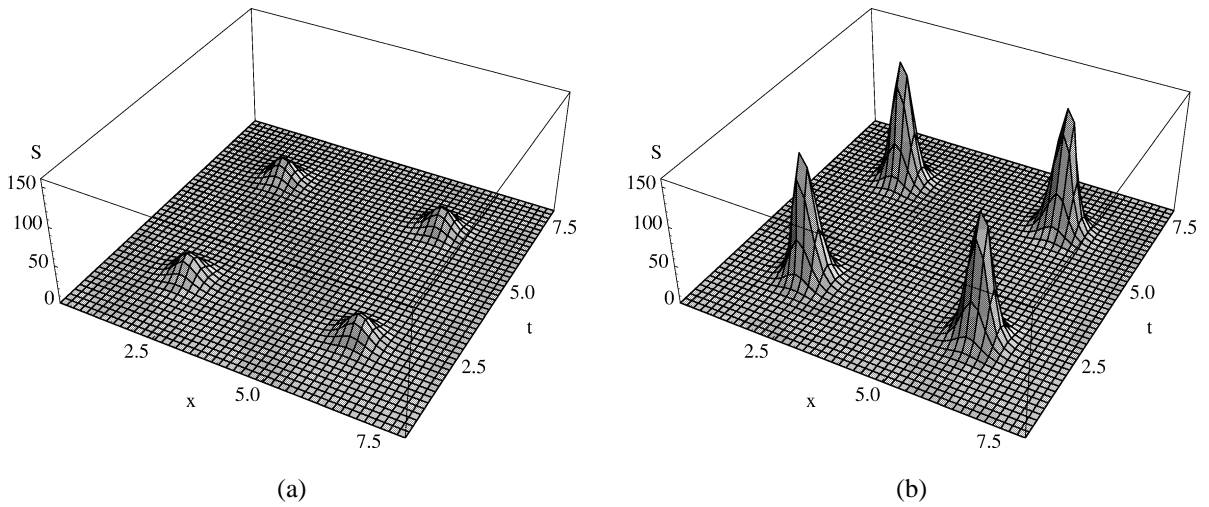


Fig. 1. The action density $S(t, x, y, z)$ for the III solution (lattice spacing $a = 0.16$) is shown as a function of x and t , and for fixed values of the y and z coordinates, in (a), y and z fixed to the minimum in the action density in these coordinates, and in (b), y and z fixed to the maximum.

the data are very smooth and also we can figure out the dependence in y, z , for any value of y, z ; we always have the picture shown in Figs. 1(a) and (b), but changing the height of the peak, going from the maximum value, shown in Fig. 1(b), to the minimum value, shown in Fig. 1(a). We also want to point out that a square Wilson loop centered in one of these maximums, takes the value -1 for a big enough size of the loop, in physical units approximately equal to 2, and this value is almost independent of the y and z coordinates [19].

The first thing we study is the center projected solution after going to maximal center gauge. We use the algorithm presented in [17] to fix the gauge to maximal center gauge. This is a local algorithm which maximizes the functional (1). This procedure has the Gribov copies problem. What we made is repeat the gauge fixing procedure several times and we take the configuration with higher value of C . In fact, the fastest way to get the highest value of C is by going first to Laplacian center gauge and then to maximal center gauge. As a technical detail we say that we stop the gauge fixing procedure when the C quantity is stable up to the eighth significant digit. Once we have the gauge fixed configuration we make the center projection. In this case the Z_2 configuration is quite simple. All plaquettes belonging to the $xy,$

xz, yz, yt and zt planes (planes involving at least one of the small directions) are positives. Only in the xt plane you can find negative plaquettes. There are four P -vortices per xt plane, each one at the same location of the four maximums in the action density. All other plaquettes are positives. So, the vortex solution is seen, in the center projected configuration obtained after fixing to maximal center gauge, as a bidimensional string of negative plaquettes, this string joining the maximums in the action density at each xt plane.

Second, we study this solution in Laplacian center gauge. To fix to Laplacian center gauge we have to calculate the lowest eigenvectors of the Laplacian operator. We use the algorithm presented in [25] to obtain these vectors. We get the three eigenvectors with lowest eigenvalues, and we obtain that in this case the two lowest eigenvalues are degenerated. With these two eigenvectors we fix the gauge to Laplacian center gauge. First, we find the gauge transformation which rotates the first eigenvector to the third direction in color space (σ_3). And then, we find the Abelian gauge transformation which rotates the second vector further to the positive (σ_1, σ_3) half-plane.

We center project the LCG fixed configurations and, as before, study the center projected configuration. We obtain the same structure described before for the center projected configuration obtained after going to

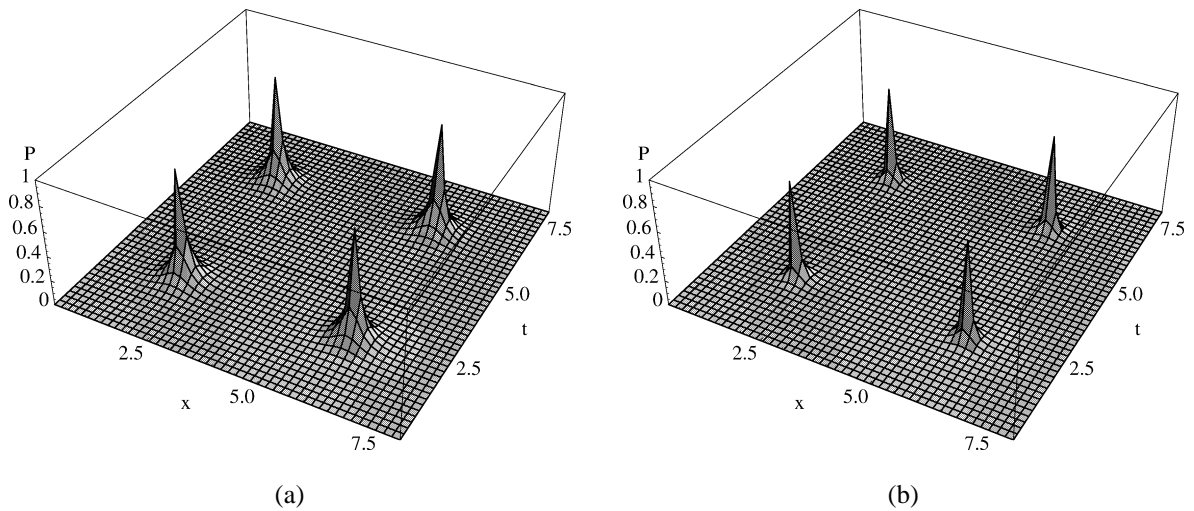


Fig. 2. The cosine of the angle between the two lowest eigenvectors $P(t, x, y, z)$ for the III solution (lattice spacing $a = 0.16$) is shown as a function of x and t , and for fixed values of the y and z coordinates, in (a), y and z fixed to the minimum in the action density in these coordinates, and in (b), y and z fixed to the maximum.

maximal center gauge. So, Laplacian center gauge and center projection clearly identifies the vortex solution as a bidimensional string of P -vortices. If we take the center projected configuration after fixing to Laplacian Abelian gauge, instead of the one after LCG fixing, we do not see any structure unraveling the underlying vortex structure, so the fixing of the $U(1)$ degrees of freedom is crucial to identify the vortex properties. We also want to point out that the same results are obtained if we choose to fix the gauge to LCG linear combinations of the two lowest eigenvectors (these two are degenerated).

Monopoles and vortices can be found in Laplacian center gauge as defects of the gauge fixing procedure. Then, we have to look at the first and second eigenvector and find the points in which you can not build the gauge transformation. In the first step, rotate the first eigenvector to the third direction in color space, we can find a singularity if $\psi_1^a(t, x, y, z) = 0$. This defines lines in four-dimensional space and these lines are identified as monopole lines. In the second step, find the Abelian gauge transformation rotating the second eigenvector further to the positive (σ_1, σ_3) half-plane, there are singularities at points in which the first and second eigenvectors are parallel. This condition

defines surfaces in four-dimensional space and these surfaces are identified as vortex sheets.

As we know that this solution is a thick vortex configuration we look at the cosine of the two lowest eigenvectors, $P(t, x, y, z)$, to see how the vortex background is identified. In Fig. 2(a) we show $P(t, x, y, z)$ as a function of t and x for y and z fixed to the minimum in the action density, and in Fig. 2(b) the same quantity but for y and z fixed to the maximum in the action density. The same picture is obtained for all y, z points. We see that these two vectors are orthogonal at all points of the lattice except on the neighborhood of the maximum in the action density for each y, z point. If at these points the value of P reaches the value 1 then you can not build the gauge transformation. This is the condition defining vortex sheets. If we join the maxima of $P(t, x, y, z)$ for all y, z values we obtain a surface of points with a value for the cosine very close to 1 (always over 0.7). It seems that looking at this quantity we detect a vortex sheet. Note that in the case of degeneration two for the lowest eigenvector, the vortex properties are uniquely determined because if these two vectors are parallel at some point, any linear combination will produce parallel vectors at the same point.

Finally, we study the monopole content of this configuration. By construction, Laplacian gauge magnetic monopoles lie on the center vortices, then, we look at one of the 2×2 yz planes in which the vortex sheet is located (one of the four maximums in the action density for the x, t coordinates). In this case, the monopole pattern obtained depends on the choice of lowest eigenvector. We parameterize the possible choices with an angle, θ , as: $\Psi'_1 = \cos\theta\Psi_1 + \sin\theta\Psi_2$, and we look at the monopole patterns as a function of the θ angle. We always see two monopole curves, each one going between two of the maximums in the action density in the y, z coordinates (positions $y_0, z_0 = 0.5, 1.5$). For a value of θ which can be chosen as the origin of angles $\theta = 0^\circ$, these two curves are straight lines in the y direction ($z = 0.5$ and $z = 1.5$ for $0 \leq y \leq 2$). For $\theta = 45^\circ$ these curves are the two diagonals of this yz plane ($z = y$ and $z = -y + 2$ for $0 \leq y \leq 2$), for $\theta = 90^\circ$ are straight lines in the z direction ($y = 0.5$ and $y = 1.5$ for $0 \leq z \leq 2$), for $\theta = 135^\circ$ are the lines orthogonal to the diagonals ($z = -y + 1$ and $z = y + 1$ for $0 \leq y \leq 1$; $z = -y + 3$ and $z = y - 1$ for $1 \leq y \leq 2$) and for $\theta = 180^\circ$ you recover the picture at $\theta = 0^\circ$. So we can figure out how these curves evolve with θ , the two lines in the y direction seen at $\theta = 0^\circ$ are deformed up to get the two diagonals at $\theta = 45^\circ$, continuing then up two get the two lines in the z direction at $\theta = 90^\circ$, then the lines orthogonal to the diagonals at $\theta = 135^\circ$ and finally getting the starting pattern at $\theta = 180^\circ$.

4. Conclusions

We have studied in this Letter how Laplacian center gauge identifies a SU(2) thick vortex configuration on the lattice. Looking at the center projected configuration obtained after fixing the gauge of a vortex solution to Laplacian Center gauge, we see a bidimensional string of negative plaquettes joining the maximums of the solution in the action density at each x, t plane. This is the same result obtained by looking at the center projected configuration after fixing to maximal center gauge. So both procedures clearly identify the vortex solution as a surface of P -vortices. We have also looked at the other way Laplacian center gauge can locate center vortices, and we have seen that looking at the possible singularities of the gauge

fixing procedure, you obtain the same result as using Laplacian or maximal center gauge and center projection. The candidate points to be singularities of the gauge fixing procedure describe the same surface detected using center projection. Nevertheless, even for this quite simple case in which we know that there is a physical vortex, it is quite difficult to find an interpolation procedure to state that you have an actual singularity: points in which the lowest eigenvector of the \mathbf{L} operator is zero or the two lowest eigenvectors are parallel. This difficulty was previously pointed out in Ref. [26] in which they use the alternative center projection procedure to locate vortices, which we have seen that gives the same results for the vortex solution.

It is worth to stress that, for this particular thick vortex configuration, both procedures to locate vortices, maximal center projection and Laplacian center gauge, either with the collinearity condition of the two lowest eigenvectors or through the center projection method, give the same answer. Nevertheless, this result does not allow us to conclude what is the generic way in which Laplacian center gauge works, specially in realistic SU(2) configurations with strongly overlapping vortices.

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