A novel probabilistic fatigue assessment tool and its application to an offshore riser joint

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Abstract

Although the complete stress field of a structure is available after finite element analysis, traditional S-N based fatigue assessment is merely based on the (peak) stress cycles at the critical location of the structure and generally deterministic in its nature. This is true both for fatigue assessment by “hand” and by post-processor. At the design stage, fatigue-crack growth is generally limited to “worst-case” cracks. This paper gives a brief presentation of a novel tool for probabilistic fatigue assessment, LINKpfat, both capable of performing weakest-link fatigue analysis and fatigue-crack growth analysis based on computationally efficient determination of stress-intensity factors by means of weight functions.

Both the traditional peak-stress module and the weakest-link module of LINKpfat are applied to the fatigue assessment of an offshore riser joint with moderate stress concentrations for three different load cases. It turns out that the peak-stress analysis may overestimate the load capability of the riser with about 35% by neglecting the size effect. The latter is consistently accounted for by the weakest-link module.

Keywords: Fatigue-crack growth; post-processor; probabilistic; threaded connection; weakest-link.
1. Introduction

Traditionally, fatigue design of engineering structures is based on S-N data from constant-amplitude fatigue testing of smooth specimens subjected to axial stress cycles. For components with initial manufacturing defects or service-induced cracks, it is common to make a fatigue assessment based on the growth of a (worst-case) fatigue-crack. Both these approaches are deterministic in the sense that they use specific load histories and materials data. However, there is presently a trend in fatigue design towards methods predicting the probability that a structure reaches a prescribed life.

Invariably, the starting point of the various approaches for fatigue assessment is a finite element analysis (FEA) of the structure under investigation. Thus, the complete stress field is available to the designer. However, traditional criteria merely refer to the peak-stress cycle, possibly modified with respect to the stress gradient [1]. This is also true for traditional finite-element based fatigue post-processors [2–4]. These are generally conceived as ‘word-for-word’ implementations of standard design methods and data. No doubt, this brings considerable improvements to fatigue analyses in terms of speed and reproducibility, in particular in designing for variable amplitude loading and multiaxial stress cycles. However, the mere implementation of empirical models and standards does not make use of the potential of computer-based methods for analysing more physically based fatigue models taking the complete stress field into account.

Examples of the latter are (i) weakest-link type S-N based assessment [5–13] and (ii) fatigue-crack growth analysis by means of stress-intensity factors based on weight functions [14–17]. These methods have been implemented in LINKpfat, a novel probabilistic fatigue post-processor, which will be briefly described in the following. The weakest-link module of the post-processor will be illustrated through its application to an offshore application.

2. Probabilistic fatigue post-processor

Two innovative capabilities have been introduced in LINKpfat. One is based on weakest-link fatigue theory accounting for the interplay between the level and the distribution of stress [7]. The other is fatigue-crack growth analysis based on the computationally efficient calculation of stress-intensity factors by means of weight-functions [14].

LINKpfat is an FEA post-processor in the sense that it can import the stress field from the FEA of a structure for subsequent fatigue assessment. The stress analysis is carried out for the crack-free structure under some “unit load”, e.g., the bending moment on an offshore riser, and then multiplied by the time history of the load. Thus, the stress history can be computed for any point of the structure.

2.1. Weakest link

The post-processor supports plane (triangular, quadrilateral) and three-dimensional (tetrahedral, hexahedral) elements. The implemented multiaxial-stress criteria are the classical maximum principal stress, Sines and Crossland criteria and the critical-plane criteria due to Dang Van, Findley, Matake and McDiarmid. “Quality zones” with different material properties are supported.

Beside a traditional “local-stress” module (based on “hot-spot” or peak stress), LINKpfat offers a weakest-link module assuming the probability of survival of a component to be the product of the probabilities of survival of the elements of the finite-element model [7]. The probability of survival of the $i$th finite element is a function of the stress cycle, the mean and scatter of the fatigue strength and the size of the element, viz.,

$$P_{s_{v_i}} = 2 \int_{r_0}^{\infty} \left( \frac{\sigma_i(x)}{\sigma_\alpha} \right)^{\beta_\sigma} \frac{dV}{V_0},$$

where $\sigma_\alpha$ signifies the median fatigue limit for a homogeneously stressed reference specimen of volume $V_0$, and $\beta_\sigma$ the shape parameter (increases with decreasing scatter). From this may be deduced the stress amplitude
According to weakest-link theory, a homogeneously stressed reference specimen subjected to this “fatigue-effective” stress amplitude will have the same probability of survival as the actual component. For a material without scatter, the life prediction becomes equal to that of the local-stress module. Weakest-link theory accounts for the statistical influence of the size of the most highly stressed region in a rational manner; cf. Section 3. Thus, if the volume subjected to high stresses in the component is small, compared to the size of the reference fatigue test specimen, the “fatigue-effective” stress amplitude will be well below the peak-stress amplitude and vice versa. Surface as well as volume formulations of weakest-link theory are supported by LINKpfat.

2.2. Fatigue crack growth

In practice, fatigue initiates at a “defect” at the surface or in the volume of a component. Typical defects are non-metallic inclusions, pores, cavities, lack of fusion, machining marks or corrosion pits.

The “single-defect” module of LINKpfat [14] predicts the fatigue life of a component based on the growth of a crack from the contour of a given material defect (neglecting crack initiation). The crack plane is chosen to be perpendicular to the direction of the maximum principal stress at the initiating defect. Embedded elliptical cracks and semi-elliptical surface cracks, as shown in Fig. 1, are considered. The stress intensity factor is determined by means of the corresponding weight function and the stress field of the crack-free component. The required Gauss integration of the normal stress over the crack surface is automatically performed by the post-processor. Presently only three-dimensional (tetrahedral, hexahedral) elements are supported.

A further step towards a realistic fatigue life prediction is the analysis of cracks growing from the randomly distributed defects of a component [15–17]. LINKpfat identifies the most critical defect by considering the size and location of each defect as well as the local state of stress. The number of defects in each finite element is randomly drawn from a Poisson distribution, the size of each defect from an extreme-value distribution. The distribution parameters may be estimated by means of X-ray computed tomography data [16]. By repeating this process for a large number of nominally equal components (Monte-Carlo simulation), the fatigue life distribution of the component is obtained. Again, both volume and surface formulations of random-defect analysis are supported by LINKpfat. The latter has proven useful in the assessment of components subjected to corrosion fatigue [17].

\[
\sigma_{\text{WL}} = \left( \int_V \sigma_a \cdot dV/V_0 \right)^{1/2}.
\]

Fig. 1. Semi-elliptical crack at root of notch.

Fig. 2. Riser joint (dimensions in mm) subjected to internal pressure, bending, and tension.
3. Application of weakest-link post-processor to offshore riser joint

Fatigue analysis of offshore components and systems is normally performed based on the S-N approach as outlined in DNV-RP-C203 [18]. Stress raisers, e.g., grooves, fillets and holes, and welds are classified as fatigue “hot spots”. For each critical location, an appropriate S-N curve from DNV-RP-C203 is selected based on factors such as NDE, inspectability, surface finish, material strength, environment (air, cathodic protection or free corrosion) and, if relevant, weld class. An equivalent or “fatigue-effective” uniaxial stress range at each fatigue hot spot is then determined from a finite element analysis using the local stress approach and the normal stress criterion. This stress range is entered into the selected S-N curve in order to estimate the number of cycles to fatigue failure. The S-N curves of DNV-RP-C203 are mean minus two standard deviation curves, so-called design curves. The fatigue capacity curve (see below) can be obtained by repeating this process for several external load levels. The calculated fatigue capacity of complex equipment, such as riser connectors, is normally validated by means of full-scale fatigue testing.

The external loading on the equipment is established from a global riser analysis. The load history is converted into a load histogram by use of the rainflow cycle-counting algorithm. The fatigue life is then estimated using the above determined fatigue capacity curve (Δp-N for pressure cycling, ΔT-N for cyclic tension and ΔM-N for cyclic bending), load histogram and fatigue damage calculated by means of the Palmgren-Miner linear damage rule. The allowable fatigue life is determined by dividing the estimated fatigue life with an appropriate fatigue safety factor (3 for equipment that can be inspected and 10 for equipment that cannot be inspected). The inspection interval for inspectable equipment is set to 1/10 of the estimated fatigue life.

DNV-RP-C203 is considered to yield conservative fatigue life estimates for equipment where the fatigue capacity is limited by fatigue cracking from a local stress raiser controlled by the maximum principal stress (normal stress criterion). The degree of conservatism is found to increase with increasing stress concentration factors. Examples are given by threaded riser connectors with sharp corners at thread roots [9, 13]. Full-scale fatigue tests of threaded specimens revealed excellent agreement with weakest-link life predictions [13].

For equipment with sharp notches, the weakest-link method will give a fatigue-effective stress that is much smaller than the fatigue stress obtained by the local stress (peak-stress) approach [6, 8, 11, 12]. The reason for this is that the highly stressed volume is much smaller than that of the smooth fatigue test specimen (reference specimen) used to establish the design S-N curve.

Table 1. Fatigue-effective stress ranges for riser joint.

<table>
<thead>
<tr>
<th>Loading</th>
<th>Method</th>
<th>Multiaxial stress criterion: Normalised value in % (stress range in MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure cycling Δp=138 MPa</td>
<td>LSA</td>
<td>100% (316) 128% (404) 128% (404) 132% (418) 132% (417) 133% (422) 131% (415)</td>
</tr>
<tr>
<td></td>
<td>WEAK</td>
<td>123% (389) 122% (492) 122% (493) 122% (509) 121% (503) 121% (509) 121% (502)</td>
</tr>
<tr>
<td>Cyclic tension</td>
<td>LSA</td>
<td>100% (316) 99% (313) 99% (313) 101% (318) 99% (314) 100% (318) 97% (307)</td>
</tr>
<tr>
<td></td>
<td>WEAK</td>
<td>135% (428) 137% (428) 137% (428) 135% (423) 135% (428) 135% (428) 135% (431)</td>
</tr>
<tr>
<td>Cyclic bending</td>
<td>LSA</td>
<td>100% (316) 98% (308) 98% (309) 101% (319) 99% (313) 100% (316) 97% (305)</td>
</tr>
<tr>
<td></td>
<td>WEAK</td>
<td>113% (358) 119% (366) 118% (366) 113% (359) 113% (354) 113% (358) 113% (346)</td>
</tr>
</tbody>
</table>

LSA (Local-Stress Approach): % value is the fatigue-effective stress of a given (multiaxial) criterion normalised with respect to the fatigue-effective stress of the normal stress criterion.
WEAK (Weakest-Link Approach): % value is the WEAK fatigue-effective stress of a given (multiaxial) criterion normalised with respect to the corresponding LSA fatigue-effective stress.
The stress analyses have been performed in Abaqus 6.12-1 using C3D20R elements, the fatigue analyses in LINKpfat version 1.1.0.308.
The volume formulation [7] of the weakest-link model has been used.

Table 2. Fatigue data used for riser joint.

<table>
<thead>
<tr>
<th>Yield</th>
<th>Tensile strength</th>
<th>Push-pull fatigue limit (stress range)</th>
<th>Torsional fatigue limit (stress range)</th>
<th>Shape parameter $\Lambda_{[4, 8]}$</th>
<th>Reference volume $V_s [4]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>518 MPa</td>
<td>563 MPa</td>
<td>450 MPa @ R = -1</td>
<td>260 MPa @ R = -1</td>
<td>30</td>
<td>2356 mm³</td>
</tr>
</tbody>
</table>

Another situation presents itself for equipment with (smooth) geometric transitions giving only a small increase in stress as demonstrated by the following fatigue assessment of the riser joint of Fig. 2. This is analysed separately.
under pressure cycling, cyclic tension and cyclic bending. The riser joint is exposed to an internal pressure cycle of $\Delta p = 138$ MPa, which is equal to the rated working pressure of the joint. For simplicity, the stress analysis for pressure cycling assumes the riser to be open-ended. The cyclic tension and bending levels are chosen so that the peak stress according to the normal stress criterion is equal to that under pressure cycling, see Table 1. The estimated fatigue stresses for the local stress (LSA) and the weakest-link approaches (WEAK) are given in Table 1 for several multiaxial stress criteria using the fatigue data of Table 2. For the influence of mean stress to be minimised, symmetric load cycles ($R = -1$) only have been considered. The following observations can be made from Table 1:

- The choice of the appropriate stress criterion is found not to be important except for pressure cycling. For pressure cycling, all considered stress criteria give consistent fatigue stresses except for the normal stress criterion. The normal stress criterion is therefore not considered to be appropriate for un-welded riser joints.
- In contrast to equipment with design features giving highly localised stresses, according to the weakest-link approach, the fatigue-effective stress will become greater than the peak stress, since the highly stressed volume is larger than the stressed volume of the smooth fatigue test specimen tested to establish the design $S$-$N$ curve, cf. [10]. The effect of the highly stressed volume is largest for cyclic tension and smallest for cyclic bending. Fatigue-effective stresses according to the Sines criterion and the associated probabilities of fatigue failure of single finite elements estimated by means of the weakest-link approach are shown in Fig. 3 for pressure cycling and cyclic bending.

![Fatigue stress range in MPa](image1)

![Probability of fatigue failure single elements shown as.](image2)

Figure 3: Fatigue-effective stress according to Sines’ criterion and the local-stress approach and (right-hand figures) the corresponding probability of fatigue failure of single elements (expressed as $\ln[-\ln(1-P_f)]$) according to the weakest-link approach.
4. Conclusions

- Traditional fatigue assessment methods, including commercially available FEA post-processors focus on peak-stress cycles and give limited attention to the surrounding stress field.
- Taking the distribution of stresses into account makes it possible to perform a weakest-link analysis of the structure based on S-N data and thus to predict its probability of failure.
- The use of weight functions together with the stress field of the crack-free structure in the plane of the crack makes possible a computationally highly efficient fatigue-crack growth analysis.
- If the size distribution of crack initiating defects is known, the life distribution of a structure can be estimated by means of a Monte-Carlo analysis.
- The workflow for performing a weakest-link analysis of an offshore riser joint by means of the probabilistic post-processor LINKpfat has been outlined and compared with the standard procedure (peak stress approach).
- The capability of the weakest-link method to reveal the size effect of the joint has been demonstrated. Fatigue tests of full-scale riser joint have been foreseen to verify predictions.

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References