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Nonexistence of Positive Solutions to a Quasilinear Elliptic System and Blow-Up Estimates for a Non-Newtonian Filtration System

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Abstract—The prior estimate and decay property of positive solutions are derived for a system of quasilinear elliptic differential equations first. Then the result of nonexistence for a differential equation system of radially nonincreasing positive solutions is implied. By using this nonexistence result, blow-up estimates for a class of quasilinear reaction-diffusion systems (non-Newtonian filtration systems) are established to extend the result of semilinear reaction-diffusion (Fujita type) systems.

Keywords—Blow-up, Quasilinear parabolic system, Nonexistence.

1. INTRODUCTION

The aim of this paper is to derive some estimates near the blow-up point for positive solutions of a class of quasilinear reaction-diffusion systems (non-Newtonian filtration systems)

$$\begin{aligned} u_t &= \operatorname{div}\left(|Du|^{p-2}\, Du\right) + v^\alpha, \\ v_t &= \operatorname{div}\left(|Dv|^{q-2}\, Dv\right) + w^\beta, \\ w_t &= \operatorname{div}\left(|Dw|^{m-2}\, Dw\right) + u^\gamma, \qquad (x,t) \in \Omega \times (0,T), \end{aligned} \tag{1.1}$$

as well as the nonexistence of positive solutions of the related elliptic systems

$$\begin{aligned} -\operatorname{div}\left(|Du|^{p-2}\, Du\right) &= v^\alpha, \\ -\operatorname{div}\left(|Dv|^{q-2}\, Dv\right) &= w^\beta, \\ -\operatorname{div}\left(|Dw|^{m-2}\, Dw\right) &= u^\gamma, \qquad x \in \Omega, \end{aligned} \tag{1.2}$$

where $\Omega \subset \mathbf{R}^N$, $p, q, m > 1$, $\alpha\beta\gamma > (p-1)(q-1)(m-1)$. For $p = q = m = 2$, (1.1) is the classical reaction-diffusion system of Fujita type. If $p \neq 2, q \neq 2, m \neq 2$, (1.1) appears

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in the theory of non-Newtonian fluids [1,2] and in nonlinear filtration theory [3]. In the non-Newtonian fluids theory, the pair (p,q,m) is a characteristic quantity of the medium. Media with $(p,q,m) > (2,2,2)$ are called dilatant fluids and those with $(p,q,m) < (2,2,2)$ are called pseudoplastics. If $(p,q,m) = (2,2,2)$, they are Newtonian fluids.

The main result of the present paper is a natural extension of the results given by Weissler and Caristi [4,5], which concern the single equation

$$u_t(x,t) = \Delta u + u^m(x,t), \qquad (x,t) \in \Omega \times (0,T)$$

and the semilinear reaction-diffusion systems (Newtonian filtration systems)

$$u_t(x,t) = \Delta u(x,t) + v^m(x,t), \qquad v_t(x,t) = \Delta v(x,t) + u^n(x,t).$$

Throughout this paper, let $\Omega = B_R = \{x \in \mathbf{R}^N : |x| < R\}$ $(R > 0)$. In Section 2, we give sufficient conditions under which the nonexistence of positive solutions of the elliptic system (1.2) holds in $\mathbf{R}^N$ for $N \geq \max\{p,q,m\}$. Then in Section 3, by using the nonexistence result, we get the desired blow-up estimates for the reaction-diffusion system (1.1) with some additional assumptions.

2. NONEXISTENCE FOR SYSTEM (1.2)

Consider radially symmetric solutions of the elliptic system (1.2), that is, suppose that $u = u(r), v = v(r), w = w(r)$ with $r = |x|$.

Let

$$z_1 = \frac{p(q-1)(m-1) + \alpha q(m-1) + \alpha\beta m}{\alpha\beta\gamma - (p-1)(q-1)(m-1)} - \frac{N-p}{p-1},$$
$$z_2 = \frac{q(p-1)(m-1) + \beta m(p-1) + p\beta\gamma}{\alpha\beta\gamma - (p-1)(q-1)(m-1)} - \frac{N-q}{q-1},$$
$$z_3 = \frac{m(p-1)(q-1) + \gamma p(q-1) + q\alpha\gamma}{\alpha\beta\gamma - (p-1)(q-1)(m-1)} - \frac{N-m}{m-1}.$$

We have the following theorems.

THEOREM 2.1. *Assume that*

(i) $N > \max\{p,q,m\}$, $\alpha\beta\gamma > (p-1)(q-1)(m-1)$ *with* $p,q,m > 1$;
(ii) $z_1 \geq 0$ *or* $z_2 \geq 0$ *or* $z_3 \geq 0$.

Then system (1.2) has no positive radially symmetric solution.

To prove Theorem 2.1, system (1.2) can be written in radial coordinates as

$$(\Phi_p(u'))' + \frac{N-1}{r}\Phi_p(u') + v^\alpha = 0, \tag{2.1}$$
$$(\Phi_q(v'))' + \frac{N-1}{r}\Phi_q(v') + w^\beta = 0, \tag{2.2}$$
$$(\Phi_m(w'))' + \frac{N-1}{r}\Phi_m(w') + u^\gamma = 0, \tag{2.3}$$
$$u(0) > 0, \quad v(0) > 0, \quad w(0) > 0, \quad u'(0) = v'(0) = w'(0) = 0, \tag{2.4}$$

in $\mathbf{R}^N$ with $N \geq \max\{p,q,m\}$, where $\Phi_p(u) = |u|^{p-2}u$, $\Phi_q(v) = |v|^{q-2}v$, $\Phi_m(w) = |w|^{m-2}w$.

By the similar argument of Lemma 2 in [6], we can prove the following.

LEMMA 2.1. *Let (u, v, w) be a positive and radially symmetric solution of equations (2.1)–(2.4). Then for $r > 0$,*

$$\left(\frac{r^p}{N}\right)^{1/(p-1)} v^{\alpha/(p-1)} \le -ru' \le \frac{N-p}{p-1} u(r),$$
$$\left(\frac{r^q}{N}\right)^{1/(q-1)} w^{\beta/(q-1)} \le -rv' \le \frac{N-q}{q-1} v(r), \tag{2.5}$$

$$\left(\frac{r^m}{N}\right)^{1/(m-1)} u^{\gamma/(m-1)} \le -rw' \le \frac{N-m}{m-1} w(r). \tag{2.6}$$

From (2.5),(2.6), we see the following lemma.

LEMMA 2.2. *Suppose that the conditions in Theorem 2.1 are satisfied. Let (u, v, w) be a positive and radially symmetric solution of equations (2.1)–(2.4). Then*

$$u(r) \le Cr^{-(p(q-1)(m-1)+\alpha(m-1)q+\alpha\beta m)/(\alpha\beta\gamma-(p-1)(q-1)(m-1))},$$
$$v(r) \le Cr^{-(q(p-1)(m-1)+\beta m(p-1)+p\gamma\beta)/(\alpha\beta\gamma-(p-1)(q-1)(m-1))},$$
$$w(r) \le Cr^{-(m(p-1)(q-1)+\gamma p(q-1)+q\gamma\alpha)/(\alpha\beta\gamma-(p-1)(q-1)(m-1))}.$$

PROOF OF THEOREM 2.1. Let (u, v, w) be a nontrivial positive and radially symmetric solution of equations (2.1)–(2.4). We consider first the case $z_1 > 0$ or $z_2 > 0$ or $z_3 > 0$.

By Lemma 2.1,

$$\left(r^{N-p}u^{p-1}(r)\right)' = r^{N-p-1}u^{p-2}\left[(p-1)ru'(r) + (N-p)u(r)\right] \ge 0,$$

we have $u(r) \ge cr^{-(N-p)/(p-1)}$ and $(ur^{(N-p)/(p-1)})$, $(vr^{(N-q)/(q-1)})$, $(wr^{(N-m)/(m-1)})$ are nondecreasing on $(0, +\infty)$. From Lemma 2.2 and for $r > r_0 > 0$, we obtain that

$$r^{z_1} \le C \qquad \text{or} \qquad r^{z_2} \le C \qquad \text{or} \qquad r^{z_3} \le C.$$

Since $z_1 > 0$ or $z_2 > 0$ or $z_3 > 0$, this leads to a contradiction for r sufficiently large.

Suppose next that $z_1 = 0$ (the case $z_2 = 0$ or $z_3 = 0$ being similar). From (2.1), it follows that for $r \ge r_0 > 0$,

$$r^{N-1}\left|u'(r)\right|^{p-1} - r_0^{N-1}\left|u'(r_0)\right|^{p-1} = \int_{r_0}^{r} s^{N-1}v^{\alpha}(s)\,ds.$$

By Lemma 2.1, we have

$$v^{\alpha}(s) \ge Cs^{(q\alpha)/(q-1)}w^{(\alpha\beta)/(q-1)} \ge Cs^{(q\alpha(m-1)+\alpha\beta m)/((m-1)(q-1))}u^{(\alpha\beta\gamma)/((m-1)(q-1))},$$

and hence,

$$r^{N-1}\left|u'(r)\right|^{p-1} \ge C\int_{r_0}^{r} s^{N-1+(q\alpha)/(q-1)+(\alpha\beta m)/((q-1)(m-1))}u^{(\alpha\beta\gamma)/((q-1)(m-1))}(s)\,ds.$$

Now taking into account that $u(s) \ge Cs^{(p-N)/(p-1)}$, we obtain

$$\begin{aligned} r^{N-1}\left|u'(r)\right|^{p-1} &\ge C\int_{r_0}^{r} s^{N-1+(q\alpha)/(q-1)+(\alpha\beta m)/((q-1)(m-1))+(\alpha\beta\gamma(p-N))/((p-1)(q-1)(m-1))}\,ds \\ &= C\int_{r_0}^{r} s^{-1}\,ds, \end{aligned}$$

where we have used the assumption $z_1 = 0$.

Then

$$r^{N-1}\left|u'(r)\right|^{p-1} \geq C \ln\left(\frac{r}{r_0}\right). \tag{2.7}$$

On the other hand, from

$$ru'(r) + \frac{N-p}{p-1}u(r) \geq 0, \qquad \text{for } r > 0,$$

we find that

$$\left(\frac{N-p}{p-1}\right)^{p-1} u^{p-1}(r) \geq \left|u'(r)\right|^{p-1} r^{p-1}. \tag{2.8}$$

Together with (2.7), this implies that

$$r^{(N-p)/(p-1)}u(r) \geq C\left(\ln\left(\frac{r}{r_0}\right)\right)^{1/(p-1)}.$$

This is impossible, however, since from Lemma 2.2 estimate implies that

$$r^{(N-p)/(p-1)}u(r) \leq Cr^{z_1} = C.$$

This contradiction concludes the proof of the theorem.

2. BLOW-UP ESTIMATES FOR SYSTEM (1.1)

Motivated by [4,5], we impose the following initial and boundary value conditions to equations (1.1):

$$u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \quad w(x,0) = w_0(x), \qquad x \in \Omega = B_R \subset \mathbf{R}^N, \tag{3.1}$$

$$u = v = w = 0, \qquad (x,t) \in \partial\Omega \times (0,T). \tag{3.2}$$

THEOREM 3.1. *Let (u,v,w) be a solution of equations (1.1), (3.1), and (3.2). Assume that*

(i) *$u(\cdot,t)$, $v(\cdot,t)$, and $w(\cdot,t)$ are nonnegative, radially, decreasing, and symmetric functions of $r = |x|$,*
(ii) *$u_t(x,t)$, $v_t(x,t)$, and $w_t(x,t)$ attain the maxima at $x = 0$ for every $t \in (0,T)$,*
(iii) *$u_t(x,t) \geq 0$, $v_t(x,t) \geq 0$, $w_t(x,t) \geq 0$ for $(x,t) \in Q_T = B_R \times (0,T)$,*
(iv) *u, v, w have a blow-up time $T < +\infty$,*
(v) *integer $N \geq \max\{p,q,m\}$, $\alpha\beta\gamma > (p-1)(q-1)(m-1)$, and $p,q,m \geq 2$ with*

$$z_1 \geq 0 \qquad \text{or} \qquad z_2 \geq 0 \qquad \text{or} \qquad z_3 \geq 0,$$

(vi) *there are positive constants k_1, k_2, k_3, k_4 and $\eta < T$ such that*

$$k_2(u(0,t))^{\delta_2/\delta_1} \leq v(0,t) \leq k_1(u(0,t))^{\delta_2/\delta_1},$$
$$k_4(u(0,t))^{\delta_3/\delta_1} \leq w(0,t) \leq k_3(u(0,t))^{\delta_3/\delta_1}, \qquad \text{for } t \in (\eta, T).$$

Then there are positive constants c_1, c_2, c_3, and $t_1 \in (0,T)$ such that

$$u(x,t) \leq u(0,t) \leq c_1(T-t)^{-\delta_1}, \qquad v(x,t) \leq v(0,t) \leq c_2(T-t)^{-\delta_2},$$
$$w(x,t) \leq w(0,t) \leq c_3(T-t)^{-\delta_3},$$

for $(x,t) \in Q_T \times Q_{t_1}$, where

$$\delta_1 = \frac{p(q-1)(m-1) + \alpha q(m-1) + \alpha\beta m}{p\alpha\beta\gamma + (p-2)\alpha(\beta m + q(m-1)) - p(q-1)(m-1)},$$
$$\delta_2 = \frac{q(p-1)(m-1) + \beta m(p-1) + \beta\gamma p}{p\alpha\beta\gamma + (p-2)\alpha(\beta m + q(m-1)) - p(q-1)(m-1)},$$
$$\delta_3 = \frac{m(q-1)(p-1) + \gamma p(q-1) + \alpha\gamma q}{p\alpha\beta\gamma + (p-2)\alpha(\beta m + q(m-1)) - p(q-1)(m-1)}.$$

Proof of Theorem 3.1. Define

$$\mu(t) = u(0,t)^{1/\tau_1}, \qquad \theta(t) = v(0,t)^{1/\tau_2}, \qquad \delta(t) = w(0,t)^{1/\tau_3},$$

for $t \in (0,T)$, where

$$\tau_1 = \frac{p(q-1)(m-1) + \alpha q(m-1) + \alpha\beta m}{\alpha\beta\gamma - (p-1)(q-1)(m-1)},$$
$$\tau_2 = \frac{q(p-1)(m-1) + \beta m(p-1) + p\beta\gamma}{\alpha\beta\gamma - (p-1)(q-1)(m-1)},$$

and

$$\tau_3 = \frac{m(p-1)(q-1) + \gamma p(q-1) + q\alpha\gamma}{\alpha\beta\gamma - (p-1)(q-1)(m-1)}.$$

By putting $\rho(t) = \mu(t) + \theta(t) + \delta(t)$, $r = |x|$

$$h_1(r,t) = \frac{u(r/\rho(t),t)}{\rho(t)^{\tau_1}}, \qquad h_2(r,t) = \frac{v(r/\rho(t),t)}{\rho(t)^{\tau_2}}, \qquad h_3(r,t) = \frac{w(r/\rho(t),t)}{\rho(t)^{\tau_3}}.$$

Using the symmetry and Assumptions (ii)–(iii) in this theorem, it follows that:

$$0 \le (\Phi_p(h_1'))' + \frac{N-1}{r}\Phi_p(h_1') + h_2^\alpha \le \frac{u_t(0,t)}{\rho(t)^{p+(p-1)\tau_1}} + \frac{v_t(0,t)}{\rho(t)^{q+(q-1)\tau_2}} + \frac{w_t(0,t)}{\rho(t)^{m+(m-1)\tau_3}}, \tag{3.3}$$

$$0 \le (\Phi_q(h_2'))' + \frac{N-1}{r}\Phi_q(h_2') + h_3^\beta \le \frac{u_t(0,t)}{\rho(t)^{p+(p-1)\tau_1}} + \frac{v_t(0,t)}{\rho(t)^{q+(q-1)\tau_2}} + \frac{w_t(0,t)}{\rho(t)^{m+(m-1)\tau_3}}, \tag{3.4}$$

$$0 \le (\Phi_m(h_3'))' + \frac{N-1}{r}\Phi_m(h_3') + h_1^\gamma \le \frac{u_t(0,t)}{\rho(t)^{p+(p-1)\tau_1}} + \frac{v_t(0,t)}{\rho(t)^{q+(q-1)\tau_2}} + \frac{w_t(0,t)}{\rho(t)^{m+(m-1)\tau_3}}, \tag{3.5}$$

for any $t \in (0,T)$ and $r \in [0, R\rho(t))$.

Since $u(x,t)$, $v(x,t)$, and $w(x,t)$ achieve their maxima at $x=0$, we easily see that h_1, h_2, h_3 are bounded. Indeed,

$$0 \le h_1(r,t) \le \frac{u(0,t)}{\rho(t)^{\tau_1}} \le 1, \qquad 0 \le h_2(r,t) \le \frac{v(0,t)}{\rho(t)^{\tau_2}} \le 1, \qquad 0 \le h_3(r,t) \le \frac{w(0,t)}{\rho(t)^{\tau_3}} \le 1. \tag{3.6}$$

Multiplying (3.3) by $h_{1,r}$ (where $h_{1,r}$ express partial derivation of h_1 for r), and then integrating with respect to r on $(0,r)$, we have

$$\frac{p-1}{p}|h_{1,r}|^p + h_2^\alpha(r,t)h_1(r,t) - h_2^\alpha(0,t)h_1(0,t) - \alpha\int_0^r h_2^{\alpha-1}h_{2,r}h_1\,dr \le 0. \tag{3.7}$$

From (3.7) and $h_{2,r}(r,t) \le 0$, it follows that

$$|h_{1,r}| \le \left(\frac{2p}{p-1}\right)^{1/p}, \tag{3.8}$$

for $t \in (0,T)$ and $r \in [0, R\rho(t))$. Similarly, we get

$$|h_{2,r}| \le \left(\frac{2q}{q-1}\right)^{1/q}, \qquad |h_{3,r}| \le \left(\frac{2m}{m-1}\right)^{1/m}. \tag{3.9}$$

Now we proceed by contradiction to claim that

$$\liminf_{t\to T}\left(\frac{u_t(0,t)}{\rho(t)^{p+(p-1)\tau_1}} + \frac{v_t(0,t)}{\rho(t)^{q+(q-1)\tau_2}} + \frac{w_t(0,t)}{\rho(t)^{m+(m-1)\tau_3}}\right) = c > 0. \tag{3.10}$$

Otherwise, there exists a sequence $\{t_n\} \subseteq (0,T)$ with $t_n \to T$ such that

$$\liminf_{t_n\to T}\left(\frac{u_t(0,t_n)}{\rho(t_n)^{p+(p-1)\tau_1}} + \frac{v_t(0,t_n)}{\rho(t_n)^{q+(q-1)\tau_2}} + \frac{w_t(0,t_n)}{\rho(t_n)^{m+(m-1)\tau_3}}\right) = 0.$$

By using the Ascoli-Arzela theorem, there exists a sequence (still denoted by $\{t_n\}$) such that

$$h_1(\cdot,t_n) \to \bar{h}_2(\cdot), \quad h_2(\cdot,t_n) \to \bar{h}_2(\cdot), \quad \text{and} \quad h_3(\cdot,t_n) \to \bar{h}_3(\cdot), \qquad \text{as } n \to +\infty \tag{3.11}$$

hold uniformly on a compact subset of $[0,+\infty)$. Now in the sense of distributions

$$\left(\Phi_p\left(\bar{h}_1'\right)\right)' + \frac{N-1}{r}\Phi_p\left(\bar{h}_1'\right) + \bar{h}_2^{\alpha} = 0, \tag{3.12}$$

$$\left(\Phi_q\left(\bar{h}_2'\right)\right)' + \frac{N-1}{r}\Phi_q\left(\bar{h}_2'\right) + \bar{h}_3^{\beta} = 0, \tag{3.13}$$

$$\left(\Phi_m\left(\bar{h}_3'\right)\right)' + \frac{N-1}{r}\Phi_m\left(\bar{h}_3'\right) + \bar{h}_1^{\gamma} = 0. \tag{3.14}$$

The absolute continuity of h_1, h_2, h_3 implies that $\bar{h}_1, \bar{h}_2, \bar{h}_3$ are $\mathbf{C}^1(0,+\infty)$. By the local existence and uniqueness of initial value problem for (3.12)–(3.14) and using the arguments in [4,5], we conclude that $\bar{h}_1, \bar{h}_2 > 0, \bar{h}_3 > 0$ on $(0,+\infty)$ with $\bar{h}_1'(0) = \bar{h}_2'(0) = \bar{h}_3'(0) = 0$.

If $N = 2$, $p > 2$, we proceed as follows. From equations (3.12)–(3.14), we infer that $r\Phi_p(\bar{h}_1')$, $r\Phi_q(\bar{h}_2')$, and $r\Phi_m(\bar{h}_3')$ are decreasing, and that there exist $M < 0$ and $r_0 > 0$ such that

$$r\Phi_p\left(\bar{h}_1'\right) < M, \qquad \text{for } r \in (r_0,+\infty).$$

The last inequality implies that

$$\begin{aligned}\bar{h}_1(s) > \bar{h}_1(s) - \bar{h}_1(t) &= (-M)^{1/(p-1)}\int_s^t r^{-1/(p-1)}\,dr \\ &= (-M)^{1/(p-1)}\left(t^{(p-2)/(p-1)} - s^{(p-2)/(p-1)}\right),\end{aligned} \tag{3.15}$$

for $r_0 \le s \le t$. Letting $t \to +\infty$ in (3.15), we obtain a contradiction.

If $N = 2$, $p = 2$, proceeding similarly as above implies that

$$\bar{h}_1(s) > \bar{h}_1(s) - \bar{h}_1(t) > (-M)[\ln(t) - \ln(s)]$$

for $r_0 \le s \le t$. Letting $t \to +\infty$ in the last inequality, we obtain a contradiction.

Finally, if $N > \max\{p,q\} \geq 2$ holds, we know from Theorem 2.1 that system (3.12)–(3.14) has no positive solution. We conclude that (3.10) is true. It follows from (3.10) that there exists $t_1 \in (0,T)$ such that for any $t \in (t_1,T)$ we have

$$\begin{aligned} c &\leq \frac{u_t(0,t)}{\rho(t)^{p+(p-1)\tau_1}} + \frac{v_t(0,t)}{\rho(t)^{q+(q-1)\tau_2}} + \frac{w_t(0,t)}{\rho(t)^{m+(m-1)\tau_3}} \\ &\leq \frac{u_t(0,t)}{u(0,t)^{(1+\delta_1)/\delta_1}} + \frac{v_t(0,t)}{v(0,t)^{(1+\delta_2)/\delta_2}} + \frac{w_t(0,t)}{w(0,t)^{(1+\delta_3)/\delta_3}}. \end{aligned} \tag{3.16}$$

Integrating (3.16) on $(t,s) \subseteq (t_1,T)$ and then letting $s \to T$, we obtain

$$c(T-t) \leq \delta_1 u(0,t)^{-1/\delta_1} + \delta_2 v(0,t)^{-1/\delta_2} + \delta_3 w(0,t)^{-1/\delta_3}. \tag{3.17}$$

By using Condition (vi) in (3.17), we have

$$u(x,t) \leq u(0,t) \leq c_1(T-t)^{-\delta_1},$$

for any $(x,t) \in Q_T \setminus Q_{t_1}$.

We have the blow-up estimates for v and w in the same way:

$$v(x,t) \leq v(0,t) \leq c_2(T-t)^{-\delta_2}, \qquad w(x,t) \leq w(0,t) \leq c_3(T-t)^{-\delta_3}.$$

The proof is completed.

REMARK 1. For the special variational parabolic system

$$u_t = \Delta u + v^{\mu}, \qquad v_t = \Delta v + u^{\delta},$$

with $\mu, \delta > 1$, Caristi and Mitidieri [5] obtained the following blow-up estimates:

$$u(x,t) \leq u(x,0) \leq c(T-t)^{-(\mu+1)/(\mu\delta-1)}, \qquad v(x,t) \leq v(x,0) \leq c(T-t)^{-(\delta+1)/(\mu\delta-1)}. \tag{3.18}$$

The single equation case was treated by Weissler [4] with

$$u(x,t) \leq u(x,0) \leq c(T-t)^{-1/(\theta-1)}. \tag{3.19}$$

Clearly, inequalities (3.18),(3.19) agree with Theorem 3.1 if one takes $p = q = m = 2$, $\alpha = \mu$, $\beta = \gamma = \delta$, or $p = q = m = 2$, $\alpha = \beta = \gamma = \theta$, respectively. Therefore, the result in this paper extends their results essentially.

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