Dynamic probability evaluation of safety levels of earth-rockfill dams using Bayesian approach

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Abstract: In order to accurately predict and control the aging process of dams, new information should be collected continuously to renew the quantitative evaluation of dam safety levels. Owing to the complex structural characteristics of dams, it is quite difficult to predict the time-varying factors affecting their safety levels. It is not feasible to employ dynamic reliability indices to evaluate the actual safety levels of dams. Based on the relevant regulations for dam safety classification in China, a dynamic probability description of dam safety levels was developed. Using the Bayesian approach and effective information mining, as well as real-time information, this study achieved more rational evaluation and prediction of dam safety levels. With the Bayesian expression of discrete stochastic variables, the a priori probabilities of the dam safety levels determined by experts were combined with the likelihood probability of the real-time check information, and the probability information for the evaluation of dam safety levels was renewed. The probability index was then applied to dam rehabilitation decision-making. This method helps reduce the difficulty and uncertainty of the evaluation of dam safety levels and complies with the current safe decision-making regulations for dams in China. It also enhances the application of current risk analysis methods for dam safety levels.

Key words: dynamic probability evaluation; dam safety levels; Bayesian approach; sorting decision-making; dam rehabilitation and reinforcement

1 Introduction

Evaluating and predicting the safety levels of dams dynamically over their entire life cycles, in order to control the aging process, is an important task. It is always very difficult to predict the future development of a specific dam. We do not have a perfect understanding of the influence of the aging process on the dam safety levels. In order to accurately predict and control the aging process, new information should be collected continuously. This aids the quantitative evaluation of the safety levels by renewing the probabilistic description of the uncertainty of dam safety. In recent years, the idea of dynamic renewal of information has gained favor in construction fields. A calculation formula for dynamic reliability based on analysis of the time-varying rules of the structural resistance and the load effect has been...
introduced in order to predict the optimal remaining life of an in-service concrete structure, providing a reference for project rehabilitation and reinforcement decision-making (Mori and Ellingwood 1993; Enright and Frangopol 1998; Flourentzou et al. 2000; Hong 2000; Zhao et al. 2002; Wang et al. 2004; Liu and Zuo 2006). However, owing to the complex structural characteristics of dams, it is difficult to predict the time-varying factors affecting dam safety levels. It is not feasible to employ dynamic reliability indices to evaluate the actual safety levels of dams by generalizing their aging processes on the basis of statistical methods. In China, at present, the analysis of dam safety levels is still at the stage of static testing evaluation and data accumulation.

The process of dynamic renewal of information is theoretically based on the Bayesian rules, which can provide a framework of support (Enright and Frangopol 1998, 1999; Igusa et al. 2002; Liu and Zuo 2006). In accordance with related regulations for the safety classification of earth-rockfill dams in China (MWRPRC 2001), a dynamic probability description of the dam safety levels has been developed. Based on the Bayesian method, the probability information for evaluating earth-rockfill dam safety levels has been renewed by utilizing the prior probability evaluations of safety levels determined by experts and the likelihood evaluation of real-time sampling information affecting earth-rockfill dam safety levels. The probability information has been further used in sorting decision-making for earth-rockfill dam rehabilitation and reinforcement.

This research aids in the dynamic prediction and control of the safety level of an aging dam and makes the existing risk analysis methods for dam safety more rational and practical.

2 Probability description of dam safety levels

From the perspective of reliability analysis, the evaluation of dam safety levels generally divides dams into two complementary discrete sets: reliable or unreliable. From the perspective of dam safety management, however, it is difficult to deal with these two sets because they are too broad. Therefore, in the current dam safety appraisal methods, according to some items of engineering performance including flood-resistance capacity, structural stability, and seepage stability, dam safety levels are divided into three levels: Level A, Level B and Level C, where Level A is safe and reliable, Level B is basically safe but with defects, and Level C is unsafe. According to the characteristics of the dams, these safety levels are used to determine three dam safety classes: Class I is safe and reliable, Class II is basically safe, and Class III is unsafe. In the Guidelines on Dam Safety Evaluation (MWRPRC 2001), the boundary definitions of dam safety levels are clearly described and can be used to divide the three discrete states. The dam safety classification and division originate from engineering practice and are widely used.

There is obvious uncertainty in division of dam safety levels. Probability estimates are thus required to quantify the uncertainty so that the dam safety evaluation can be transformed from a traditional deterministic perspective to a probabilistic one. As shown by the equation
below, probabilities of the three discrete sets can be used to describe the dam safety performance intuitively, and the sum of probabilities of the three levels is 1:

\[ P_A + P_B + P_C = 1 \]  

(1)

where \( P_A \), \( P_B \) and \( P_C \) are, respectively, the probabilities of an item at Level A, Level B, and Level C.

The probabilities of the three levels vary with time over the service life of a dam. If the development trends of \( P_A \), \( P_B \) and \( P_C \) can be evaluated and predicted, the probability distributions of the three time-varying safety levels can be obtained, as shown in Fig. 1. In the figure, the area below line “a” corresponds to Level A, the area between line “a” and line “b” corresponds to Level B, and the area above line “b” corresponds to Level C. The vertical line at \( t = 50 \) years shows the probabilities of safety levels of a dam in its 50th service year.

![Fig. 1 Probability distributions of dam safety levels](image)

Essentially, the idea of using probability to evaluate dam safety levels corresponds to the traditional concept of dam risk rate evaluation, as the results are all described by probabilities and the difference is only in the formulation for limit states (Salmon and Hartford 1995; Lemperiere 1999; Beacher 2000). Generally, the limit state of the dam risk rate is represented by dam breakage, which is distinctly different from the representation of the limit state of the dam safety level. The correlation between the dam safety level and the risk rate has been discussed, the threshold values of the risk rate for the different safety levels have been verified and calibrated, and the breakage probabilities of dams with different safety levels have been determined (Jiang and Fan 2008), but the randomness of dam safety level classification and the probability evaluation of the safety levels have not been discussed.

The probability evaluation method for the dam safety levels mainly has the following two advantages:

1. The dam risk rate is used for low-probability dam breakage events, and accurate probability evaluation depends on sufficient random information. Dam breakage occurs under rigorous conditions, usually during extreme events such as severe flooding, but there is no record or there is an inaccurate record of such infrequent events, and people lack experience in dealing with them. Especially as the dam ages, the lack of information often diminishes the accuracy of predictions (Wang et al. 2004). The conditions for dam safety level classification are flexible, and the probability of each safety level is relatively large. Therefore, the
probability evaluation method helps compensate for lack of experience and data and reduce the
difficulty and uncertainty of evaluating the probability of dam safety levels.

(2) The ultimate purpose of evaluating dam safety levels is to provide a scientific basis for
making decisions and taking measures to protect the dam, such as dam reinforcement,
degradation and rejection. Current methods for evaluating dam safety levels (MWRPRC 2001)
consider Class III the critical threshold for safety control, and hold that a Class III dam is
unsafe and unsound, and must be reinforced, degraded or rejected. The control criteria for dam
safety are based on past engineering experience, and are within the macroscopic public
acceptance range. As there is no accordant evaluation system, the target reliability of dam
safety needs to be determined by calibration methods when the risk rate method is used in
decision-making. The steps of the decision-making process increase as a consequence. Using
the probability evaluation of dam safety levels to determine the properties of dam classification
and division according to the principle of maximum probability complies with current
principles of safety decision-making.

3 Dynamic information mining and utilization for dam safety
level evaluation

The purpose of information renewal is to reduce the uncertainty arising from the lack of
recognition and information. By mining and utilizing new information, the safety performance
of dams can be more accurately evaluated and predicted. It is worthwhile to determine how to
effectively mine and utilize the dynamic information. The Bayesian approach provides a
framework that supports the renewal of dynamic information and is used as a theoretical basis
for mining and utilizing dynamic information.

3.1 Application of Bayesian approach

The Bayesian process integrates and renews knowledge that reduces uncertainty.
Bayesian theory holds that a random variable can be described through the distribution of its a
priori and a posteriori probabilities (West and Harrison 1997). The a posteriori distribution is
obtained from the integration of the a priori distribution and the likelihood function, and can
be used as a reliable basis for deducing the probability distribution of random variables.
Bayesian statistics take into account the utilization of all information, including objective
sample information and subjective estimation based on experience. The uncertainty of
dynamic information can be reduced during the continuous process of information renewal.

The probability evaluation and prediction of dam safety levels can be expressed by the
Bayesian expression of discrete random variables as follows:

\[
P_{HC} = \frac{P_{EA}P_{XA} + P_{EB}P_{XB} + P_{EC}P_{XC}}{P_{EC}P_{XC}}
\]

where \( P_{XA} \), \( P_{XB} \) and \( P_{XC} \) are, respectively, the a priori probabilities of an item at Level A,
Level B, and Level C, and they represent the a priori information; $P_{EA}$, $P_{EB}$, and $P_{EC}$ are, respectively, the likelihood probabilities of Level A, Level B, and Level C, and they represent the test results; and $P_{EC}$ is the a posteriori probability of an item at Level C, which is obtained by integrating the sample information and the a priori information and represents the a posteriori information. The calculation formulas of the a posteriori probabilities $P_{HA}$ and $P_{HB}$ of an item at Level A and Level B are similar to Eq. (2).

In the Bayesian deduction described above, there are two inputs: the a priori probability and the likelihood probability. The evaluation of the a posteriori probability can be carried out with a simple arithmetic operation only if these two inputs are determined.

### 3.2 Evaluation of a priori probability

According to the Bayesian theory, an estimation of the safety performance of a dam can be carried out before any new testing, and a suitable method can be used to estimate the probability of each item at Level A, Level B and Level C. In general, this estimation occurs when experts evaluate dam safety. Based on the field survey and the analysis of related monitoring data, the safety performance of the dam can be evaluated by experts according to their experience, intuition or subjective estimation. The experts’ evaluation contains a lot of a priori random information, which can be mined with the probability assignment and the weighted mean methods they use. It reflects the experts’ estimation of the dam class and level, and embodies the concept of subjective interpretation of probability (Bayesian interpretation). Previous evaluations of dam safety neglected the collection and arrangement of random information and directed attention to the qualitative estimation of the dam safety level made by the experts. It is very difficult for experts to evaluate the failure risk rate of a dam based only on their own experience, as such evaluations usually lack references; under the same conditions, it is more accurate to estimate the probability of the safety level of a dam.

### 3.3 Evaluation of likelihood probability

According to the Bayesian theory, new sample information can be continuously acquired through real-time testing, so that the uncertainty of dynamic information about an aging dam can be reduced.

As the likelihood probability is directly proportional to the conditional probability (West and Harrison 1997), a discrete random variable can be directly evaluated with the event tree method. It is convenient to describe the conditional probability with the event tree, and the event tree can help decompose a complicated problem and describe each link and corresponding probability of an event process. Generally, a chart of the event tree whose purpose is to evaluate the risk rate of a dam must begin from the initial conditions of an incoming flood. The tree branches extend to the occurrence of dam breakage according to the logical sequence of the deductive method (Jiang 1999). This requires consideration of all incoming flood conditions, flood overtopping (piping) failure, intervention to deal with
emergencies, and other factors. To evaluate the probabilities of dam safety levels, it is only necessary to provide a simplified chart of the event tree based on the related specification of the level classification for each item. For example, only the occurrence of a design flood (check flood) needs to be taken into consideration for the initial conditions; the final event only needs to be extended to the critical conditions of each item at Level B and Level C, including the actual water level exceeding the design flood level, the insufficient free board of the dam height, and the seepage gradient ratio that is larger than the allowable gradient ratio (Jiang and Fan 2008). The probability of each item at Level B and Level C can be obtained according to the real-time prototype monitoring data and the prediction of development trends, such as the sediment accumulation of the dam (Jiang and Fan 2006a), the rate of gate accidents during flood discharge (Pohl 2000; Jiang and Fan 2006b), and the change of seepage water head (Jiang and Fan 2007). As a result, it is possible to evaluate the likelihood probability directly.

As mentioned above, the Bayesian approach provides a way of deducing the time-varying random variable. It not only utilizes subjective experience information but also the sampled information to continuously correct and improve the existing probability distribution and further create conditions for evaluating and estimating the probability distributions of dam safety levels. Meanwhile, the introduction of the idea of probability evaluation for the safety levels also reduces the difficulty of practical application of the Bayesian approach.

4 Discussion on dynamic control strategy for dam safety

The evaluation and estimation of dam safety levels can aid in the selection of safety control strategies throughout the life of the dam. Safety management during the life cycle of the dam is different from the design process. It is a process during which the control strategy for dam safety is continuously adjusted and renewed, and in which the managers make dynamic estimations and decisions.

4.1 Sorting strategy for dam rehabilitation and reinforcement

Rehabilitating and reinforcing a dam is an effective way to prolong the dam’s life. In China, a lot of unsound and dangerous dams need to be rehabilitated and reinforced, but the funds are limited. It is worthwhile to discuss how to reinforce these dams in a reasonable sequence (Li et al. 2006).

Under the current regulations for dam safety management in China, Class III is used as the control standard for rehabilitation and reinforcement measures. This requires a transition from safety division to safety classification of each item of dam engineering performance. Dam safety classification should be carried out through the integration of the safety levels for each item (MWRPRC 2001). A dam for which one or more of the safety levels is Level C will be classified as Class III. Based on the concept of fuzzy membership grade, the membership grade of a Class III dam can be expressed as
\[ \mu_{III} = \sum_{i=1}^{n} K_i P_i \]  

(3)

where \( \mu_{III} \) is the membership grade of a Class III dam, \( P_i \) is the probability of the \( i \)th item at Level C, \( K_i \) is the weight coefficient of the \( i \)th item, and \( n \) is the number of items. The method for determining \( K_i \) has been discussed in several referenced papers (Li et al. 2006), and will therefore not be discussed further in this paper.

If a Class III dam has a higher value of \( \mu_{III} \) during the sorting process of rehabilitation and reinforcement, then there is more dynamic information supporting this dam’s Class III classification, and measures for rehabilitation and reinforcement should be taken immediately to ensure its safety.

Generally, a sequence is determined by the risk value \( R (R = P_t L) \), where \( P_t \) is the risk rate of dam breakage, and \( L \) is the risk loss) in the risk decision-making method. This method is scientific and reasonable. Nevertheless, it is difficult and complicated to calculate \( P_t \) and \( L \) (including personnel death, economic loss, social loss and environmental loss). This difficulty influences the generalized application of this method.

4.2 Strategy for selecting safety control measures

Rehabilitation and reinforcement are not the sole choices for dam safety control. Dam degradation and rejection, which are not engineering measures, are also suitable choices (Jiang et al. 2004).

Generally, the rehabilitation and reinforcement of a dam should be carried out according to a double-control principle of safety and economy. In other words, the economic benefit of the prolonged dam service life should be larger than the cost of the rehabilitation and reinforcement plus the annual maintenance, i.e., it should satisfy the following equation:

\[ \sum_{t=1}^{T} B(1 + r)^t - D \geq 0 \]  

(4)

where \( B \) is the designed annual economic benefit of the rehabilitated and reinforced dam, \( D \) is the cost of the rehabilitation and reinforcement as well as subsequent maintenance, \( r \) is the annual discount rate of the funding, and \( T \) is the prolonged service life after such rehabilitation and reinforcement.

If Eq. (4) cannot be satisfied, the rehabilitation and reinforcement plan is not economically reasonable, and dam degradation or rejection should be regarded as a suitable choice. The key in applying Eq. (4) is to determine the prolonged service life. The aforementioned method for predicting the probabilities of dam safety levels is a suitable way to solve the problem.

5 Engineering case

In this section, a practical application of the Bayesian approach is described based on an evaluation of the probabilities of safety levels of flood resistance capability for a reservoir dam.
The reservoir dam is a homogeneous earth dam with an original designed dam height of 35.5 m, and a storage capacity of 1.09 km³. The design flood control standard of the dam was a 1000-year flood recurrence. After this reservoir had been put into service for a period, a safety evaluation for each item of engineering performance was made. Experts independently quantified probabilities of safety levels of the flood resistance capacity of this dam: \( P_{XA} = 10\% \), \( P_{XB} = 55\% \), and \( P_{XC} = 35\% \). A preliminary survey showed that the original flood control capacity was not high and the sediment accumulation became more and more serious. This group of data was provided by experts based on preliminary survey, analysis and experience. The experts believed that the flood resistance capacity of this dam was at Level B. This kind of a priori information is very important to accurate evaluation and estimation of the probabilities of safety levels.

Due to the changing hydrological conditions, the original designed and proposed operation mode for discharging muddy water and storing clear water was not realized. As a result, more and more sediment accumulated in the reservoir, and the flood storage capacity significantly decreased. After 20 years of operation, the effective storage capacity decreased by \( 18 \times 10^6 \) m³. According to this test result, and using storage routing on the basis of a stochastic mathematical model, the probability density distribution \( f(h,t) \) of the reservoir water level, the corresponding maximum dam water level \( \mu_{h_{\text{max}}} \), and the maximum dam water level’s variance \( \sigma_{h_{\text{max}}} \) under design flood conditions were calculated (Jiang and Fan 2006a). This calculation did not take into account random conditions that influenced the reservoir water level, such as the gate accident rate during flood discharge and the accuracy of flood forecasting. Thus, the calculated results can be regarded as a result of sampling tests. According to the calculated results, the probability that the event tree cannot satisfy the safety superelevation is 70%; that is, the likelihood probability \( P_{EC} \) of the flood resistance capacity at Level C is 70%. Correspondingly, \( P_{EA} \) is 5% and \( P_{EB} \) is 25%.

According to Eq. (2), the a posteriori probability \( P_{HC} \) of the flood resistance capacity at Level C is 63%. Correspondingly, \( P_{HA} \) is 2% and \( P_{HB} \) is 35%. This probability is obtained based on the a priori information from experts’ experience and the likelihood information of sample testing, showing that there is a significant possibility that the flood resistance capacity of this dam is at Level C, and there is nearly no possibility that it is at Level A. Urgent measures are needed for rehabilitation and reinforcement.

Afterwards, this dam was redesigned according to the standard for resisting a 2000-year flood. The dam had a heightened dam height of 39.5 m and an enlarged storage capacity of 1.14 km³. The flood resistance and regulating capacity of the reservoir was largely enhanced. At this time, the probability valuations of the dam flood resistance capacity at Level A, Level B, and Level C provided by experts were \( P_{XA} = 65\% \), \( P_{XB} = 25\% \), and \( P_{XC} = 10\% \), respectively. \( f(h,t) \), \( \mu_{h_{\text{max}}} \), and \( \sigma_{h_{\text{max}}} \) under the conditions of the new designed flood process were calculated (Jiang and Fan 2006b) and used as the basis for evaluating the likelihood
probability. The results show that the probability that the event tree cannot satisfy the safety superelevation is only 5%. This means that the likelihood probability $P_{EC}$ is 5%, and, correspondingly, $P_{EB}$ is 15% and $P_{EA}$ is 80%. In this case, by using Eq. (2), the a posteriori probability $P_{HA}$ of this item at Level A is 92%. The flood resistance capacity of this dam was largely enhanced by rehabilitation and reinforcement, and afterwards the dam satisfied the safety criteria of Level A.

6 Conclusions

In order to accurately predict and control the aging process of dams, new information should be collected continuously to renew the quantitative evaluation of dam safety levels. Owing to the lack of time-varying information affecting dam safety, it is quite difficult to evaluate the actual risk level of a dam with dynamic reliability indices.

This paper describes the evaluation of the probabilities of dam safety levels, which is essentially coincident with the traditional risk rate evaluation. This method helps reduce difficulty and uncertainty in the quantitative evaluation of dam safety levels and complies with current safe decision-making principles. The application of the Bayesian approach creates conditions for effectively mining and utilizing new information, and the safety performance of the dam can also be evaluated and predicted more reasonably. It is practical to utilize the Bayesian expression of discrete random variables to renew the evaluation of the probabilities of dam safety levels based on the a priori probabilities provided by experts’ quantifications and the likelihood probability of real-time test information. The achievement of such dynamic probability evaluation also provides decision-making support for safety management throughout the life of a dam and aids in decisions about sorting for dam rehabilitation and reinforcement.

References


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