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Testing the $\chi_{c1}\,p$ composite nature of the $P_c(4450)$

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ABSTRACT

Making use of a recently proposed formalism, we analyze the composite nature of the $P_c(4450)$ resonance observed by LHCb. We show that the present data suggest that this state is almost entirely made of a χ_{c1} and a proton, due to the close proximity to this threshold. This also suppresses the decay modes into other, lighter channels, in our study represented by $J/\Psi p$. We further argue that this is very similar to the case of the scalar meson $f_0(980)$ which is located closely to the $K\bar{K}$ threshold and has a suppressed decay into the lighter $\pi\pi$ channel.

1. Introduction

A clear peak has been observed in the invariant mass distribution of the $J/\Psi p$ subsystem in the three-body decay $\Lambda_b \to K^- J/\Psi p$ in Ref. [1]. This signal is interpreted as a new pentaquark resonance $P_c(4450)$. These results have spurned a plethora of theoretical investigations, see Refs. [2–11]. Clearly, it is still necessary to obtain further experimental information to confirm that it is actually a resonance and not an anomalous-threshold singularity on top of the $\chi_{c1}p$ branch-point threshold [7]. Other proposals like in Ref. [4] consider the $P_c(4450)$ as a composite of much heavier channels, with some of them having a large width. Here, we further explore the ideas first given in Ref. [7], as the extreme close proximity of the mass of the $P_c(4450)$ to the $\chi_{c1}p$ threshold cannot be accidental. Independent of its true nature, to simplify the argumentation, we employ the term resonance to indicate the peak $P_c(4450)$ unveiled in Ref. [1].

We proceed by analogy with the $J^{PC} = 0^{++}$ resonance $f_0(980)$ which couples mainly to two channels, one lighter ($\pi\pi$) and another heavier ($K\bar{K}$), with the latter threshold almost coinciding with the resonance mass. The lighter channel is the one that drives the relatively small width of the $f_0(980)$, in the sense that it is responsible for this resonance to develop a width, although it owes its origin to the nearby $K\bar{K}$ threshold [12]. In our present case for the $P_c(4450)$ resonance, the $J/\Psi p$ state is assumed to play the role of the $\pi\pi$ channel in the case of the $f_0(980)$ because, despite the resonance having plenty of phase space to decay into this channel, the $P_c(4450)$ width is rather small. This indicates that the coupling to this channel is suppressed. Following this line of reasoning, the $\chi_{c1}p$ channel is assumed to play the analogous role of the $K\bar{K}$ one for the $f_0(980)$, because it is almost on top of the mass of the $P_c(4450)$, dragging the mass of the resonance towards its threshold due to its large coupling, being furthermore the main contribution in the resonance composition.

In the following, we work out the consequences of such a scenario making use of the formalism developed in Ref. [13]. This allows to calculate the compositeness coefficients for the involved channels and also the partial decay widths. As we will show, the present data can be well described with the assumption that the $P_c(4450)$ is a $\chi_{c1}p$ composite, either a resonance or an anomalous threshold singularity. How to exclude the latter scenario was already discussed in Ref. [7]. In particular, a precise measurement of the partial decay widths would be a clear test of the $\chi_{c1}p$ composite nature for the $P_c(4450)$ as we discuss here.

2. Compositeness condition and the width of the $P_c(4450)$

As stated above, we consider a two-channel scenario, where the extreme proximity of the $P_c(4450)$ to the second channel, $\chi_{c1}p$, suggests that the resonance is in fact a composite of this latter constituent state, suppressing in this way the otherwise large width to the lighter mass channel $J/\Psi p$. To further investigate this possibility, we make use of the recent work from Ref. [13] that established a well-defined procedure to interpret in a standard probabilistic way the compositeness of a resonance from the compositeness relation [14–19]. The final result is a simple prescription

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consisting in changing the phase of every coupling separately, such that the compositeness coefficient of the corresponding channel is a positive real number. In this way, if the original couplings are denoted by γ^i, the procedure of Ref. [13] requires to transform them as

$$\gamma_i \to \gamma_i e^{i\varphi_i}. \tag{1}$$

This change results from the determination of the physically suited transformation of the S-matrix driven by a unitary matrix. Each of these unitary matrices implies a new compositeness relation, all of them being associated with the same or at most rotated resonance projection operator $\mathcal{A}$. One requires, however, that the resonance state associated with $\mathcal{A}$ couples to the different channels with the same strength as the original resonance, which fixes the remaining rotation. Then each coupling is only determined up to a phase factor, cf. Eq. (1), which in turn implies that the compositeness coefficient X_i transforms as

$$X_i = -\gamma_i^2 \frac{\partial G_i(s_P)}{\partial s} \to X_i^f = -e^{2i\varphi_i}\gamma_i^2 \frac{\partial G_i(s_P)}{\partial s} = |X_i|, \tag{2}$$

where X_i^f is the final compositeness coefficient for channel i, representing the amount of this two-body channel in the resonance state. In the previous equation, s_P is the pole position of the resonance and $G_i(s)$ is the scalar unitary loop function for channel i, which requires a subtraction constant, see e.g. Refs. [20,21] for explicit expressions of this function. But since only the derivative of G_i enters in $|X_i|$, this coefficient is independent of the subtraction constant. The derivative of $G_i(s)$ is a definite function of s and the precise values of the masses of the particles involved, and it would correspond to a convergent three-point one-loop function with unit vertices. A detailed account of the theory to arrive at Eq. (2) is developed in Ref. [13] to which we refer the interested reader for further details.

Our basic criterion is to impose that the $P_c(4450)$ is a composite mainly of $\chi_{c1}p$ with some contribution from $J/\Psi p$. Indeed, if we assumed that this last channel were the main contribution in the composition of the resonance (in the following we indicate by 1 the channel $J/\Psi p$ and by 2 the $\chi_{c1}p$, in order of increasing thresholds) then from the requirement

$$|X_1| \simeq 1 \tag{3}$$

it would follow that

$$|\gamma_1|^2 = \left|\frac{\partial G_1(s_P)}{\partial s}\right|^{-1}. \tag{4}$$

We determine s_P from the values of the mass, m_P, and width, Γ_P, of the $P_c(4450)$ obtained in Ref. [1], as

$$s_P = (m_P - \frac{i}{2}\Gamma_R)^2. \tag{5}$$

Performing this simple exercise one would then obtain a huge and completely unrealistic width

$$\Gamma_P = \frac{q_1(m_P^2)|\gamma_1|^2}{8\pi m_P^2} \simeq 1.5 \text{ GeV}, \tag{6}$$

where we denote by $q_i(s)$ the three-momentum of channel i at s (the total energy squared in the center-of-mass frame). Note that the threshold of $J/\Psi p$ is around 400 MeV lighter than the nominal mass of the resonance $P_c(4450)$ so that there is plenty of phase space to allow the previous decay.[1] This is similar to the case of the $f_0(980)$ resonance where the $\pi\pi$ channel is required to couple weakly to the $f_0(980)$, otherwise the width of the latter would be huge.

[1] Because of this, one should take s_P in the unphysical Riemann sheet for this channel when evaluating the derivative of G_1 at s_P.

Let us now proceed to reach quantitative conclusions within our working assumption, consisting in the analogy, on the one hand, between the channels $\pi\pi$, $K\bar{K}$ and $J/\Psi p$, $\chi_{c1}p$, in order, and on the other hand, between the resonances $f_0(980)$ and $P_c(4450)$. Our main equations stem from imposing saturation of the compositeness relation and the width of the $P_c(4450)$ by the channels considered, $J/\Psi p$ and $\chi_{c1}p$, respectively. These conditions imply two equations, in order,

$$1 = |\gamma_1|^2 \left|\frac{\partial G_1(s_P)}{\partial s}\right| + |\gamma_2|^2 \left|\frac{\partial G_2(s_P)}{\partial s}\right|, \tag{7}$$

$$\Gamma_P = \frac{q_1(m_P^2)|\gamma_1|^2}{8\pi m_P^2} + |\gamma_2|^2 \rho_2. \tag{8}$$

The second term on the right-hand side (rhs) of the last equation is the partial decay width of the $P_c(4450)$ to $\chi_{c1}p$ and, since the threshold of this channel is almost on top of the mass of the resonance, one should calculate it taking into account its finite width. For that we follow the procedure of Ref. [12] and introduce a Lorentzian mass distribution around the mass of the resonance due its width. In this way, the formula for the partial decay width is recast as in the last term on the rhs of Eq. (8) with

$$\rho_2 = \frac{1}{16\pi^2} \int_{m_1+m_2}^{\infty} dW \frac{q(W^2)}{W^2} \frac{\Gamma_P}{(m_P - W)^2 + \Gamma_P^2/4}. \tag{9}$$

In practical terms we restrict ourselves to the resonance region so that we integrate in this equation up to $m_P + 2\Gamma_P$ as in Ref. [12]. Otherwise, the tail of the integrand takes too long to converge. We have checked that in this way once the resonance mass is above the $\chi_{c1}p$ threshold, within the one-sigma interval according to the error in the mass provided by Ref. [1] (we add quadratically the statistical and systematic errors), the standard formula for the decay width (used in the first term on the rhs of Eq. (8) for the partial decay width to $J/\Psi p$) and the one based on Eq. (9) provide consistent results. However, by making use of this procedure we avoid having a zero decay width to $\chi_{c1}p$ once m_P is smaller than the $\chi_{c1}p$ threshold. Furthermore, this would be an unphysical result.

Eqs. (7), (8) are valid for any partial wave or combination of partial waves in which each state $J/\Psi p$ or $\chi_{c1}p$ could be involved. For the case of the partial decay widths in Eq. (8) higher powers of three-momentum are reabsorbed in the residues squared and this is why the resulting expression seems the typical one for an S wave decay. On the other hand, independently of the quantum numbers to characterize a given partial wave, e.g. the basis ℓSJ (orbital angular momentum, total spin and total angular momentum, respectively), the threshold is always the same and fixed by the particle masses. As a result the derivative of $G_i(s_P)$ in Eq. (7) and the phase space driven factors in Eq. (8) do not depend on the specific partial wave and then each $|\gamma_i|^2$ represents indeed the sum of residues squared to the involved partial waves.

Regarding the pole position s_P of the $P_c(4450)$, the fact that the threshold for $\chi_{c1}p$ is so close to its mass makes necessary the distinction on the Riemann sheet in which s_P lies. In the following, we discuss our results distinguishing between whether the pole is located in the 2nd or 3rd Riemann sheet. In the former, $\mathrm{Im}\, q_1 < 0$, $\mathrm{Im}\, q_2 > 0$, while in the latter $\mathrm{Im}\, q_1 < 0$, $\mathrm{Im}\, q_2 < 0$. These are the two Riemann sheets that connect continuously with the physical axis below and above the $\chi_{c1}p$ threshold, respectively. Notice that

Table 1
Average couplings from the solution of Eqs. (7) and (8) are shown in columns 2, 3. We also give the average partial decay widths Γ_i (columns 4, 5) and resulting compositeness coefficients X_i^f (columns 6, 7). Below each quantity we provide its corresponding error. The Riemann sheet (RS) where s_P lies is given in the first column.

RS	$\lvert\gamma_1\rvert$ (GeV)	$\lvert\gamma_2\rvert$ (GeV)	$\lvert\Gamma_1\rvert$ (MeV)	$\lvert\Gamma_2\rvert$ (MeV)	X_1^f	X_2^f
2nd	1.9	15.6	5.6	34.6	0.004	0.996
	±0.5	±1.3	±3.0	±10.9	±0.002	±0.002
3rd	2.4	14.6	9.5	30.0	0.006	0.994
	±0.6	±1.1	±4.5	±9.0	±0.003	±0.003

Table 2
Variation of the composite coefficient. The lhs of Eqs. (7) is equal to x (first column) and the pole lies in the 2nd Riemann sheet. For further notation, see Table 1.

x	$\lvert\gamma_1\rvert$ (GeV)	$\lvert\gamma_2\rvert$ (GeV)	$\lvert\Gamma_1\rvert$ (MeV)	$\lvert\Gamma_2\rvert$ (MeV)	X_1^f	X_2^f
0.8	2.7	13.8	12.0	27.0	0.008	0.792
	±0.5	±1.2	±4.8	±8.5	±0.003	±0.003
0.4	4.0	9.6	26.0	13.0	0.016	0.384
	±0.6	±0.8	±8.7	±4.0	±0.005	±0.005

$dG_2(s_P)/ds$ in Eq. (7) depends on the actual Riemann sheet taken for the pole position s_P. Nevertheless, our results are rather stable under the change of sheet because the calculation of $|\gamma_2|^2$ from Eq. (7) depends mainly on $\left|\partial G_2(s_P)/\partial s\right|^{-1}$, which is rather stable under the change of sheet because the threshold of $\chi_{c1}p$ is very close to the mass of the relatively narrow resonance $P_c(4450)$. Due to the latter reason it is also necessary to solve Eqs. (7), (8) taking into account the error bars in the mass and width of $P_c(4450)$ from Ref. [1].

We present in Table 1 the results of solving Eqs. (7), (8), where in the first column we show the Riemann sheet and then the couplings, partial decays widths and final compositeness coefficients are given. As argued above, the channel $\chi_{c1}p$ has a much stronger coupling to $P_c(4450)$ than to the lighter one, otherwise it would be an extremely wide resonance. The heavier channel is also by far the largest component in our composite assumption for the $P_c(4450)$ resonance. Regarding the partial width, we observe that the $\chi_{c1}p$ has a larger partial decay width than the $J/\Psi p$. This fact is a rather robust prediction of our model, because even if we reduced the weight of the two considered channels in the compositeness relation to 0.8 instead of 1 in the lhs of Eq. (7), still the partial decay width to $\chi_{c1}p$ would be twice the one to $J/\Psi p$. This is shown in the second and third rows of Table 2. For definiteness, we give all the results in this table in the 2nd Riemann sheet since they are rather stable if changed to the 3rd Riemann sheet, similarly to the results shown above in Table 1.

This observation could be turned around. If the partial decay width of the $P_c(4450)$ to either of the two channels were measured, but still assuming that the total width is the sum of these two partial decay widths, Eq. (8), one could find out whether the compositeness relation for the $P_c(4450)$ is saturated by the two channels $J/\Psi p$ and $\chi_{c1}p$. This is illustrated in the last two rows of Table 2 where now the lhs of Eqs. (7) is 0.8 and 0.4. As we can see, only when the $P_c(4450)$ has other large contributions in its composition beyond the two-body states $J/\Psi p$ and $\chi_{c1}p$, the partial decay width to the former channel is the largest.

3. Summary

In this work, we have analyzed the composite nature of the $P_c(4450)$ resonance measured by LHCb, in a two-channel framework. The first, lower mass channel, is $J/\Psi p$ and the heavier one is $\chi_{c1}p$, with its threshold extremely close to the mass of the resonance [7]. We employ the present data on the relatively small width and mass of the resonance to conclude that within our assumption the $P_c(4450)$ is almost entirely a $\chi_{c1}p$ resonance, coupling much more strongly to this channel than to $J/\Psi p$, so that the former has clearly the largest decay width too. As first noted here, this is very similar to the scalar meson $f_0(980)$, that sits very close to the $K\bar{K}$ threshold because of its strong coupling to this channel. However, the coupling to $\pi\pi$ is much smaller which explains its suppressed width, though this lighter channel has plenty of phase space available. We have shown that this two-channel composite nature of the $P_c(4450)$ can be tested by measuring precisely the partial widths into these two channels.

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