Available online at www.sciencedirect.com
ScienceDirect
Procedia Computer Science 45 (2015) 514 – 523
Procedia Computer Science

International Conference on Advanced Computing Technologies and Applications (ICACTA-2015)

Design of a Practical cat-righting reflex (crr) Model

Ankit Manerikar[a] , Tanvi Anandpara[b]

[a] *B.E., Electronics Engineering Dept., D.J. Sanghvi College of Engineering, Mumbai, India*
ankit.31457@gmail.com
[b] *B.E., Electronics Engineering Dept., D.J. Sanghvi College of Engineering, Mumbai, India*
tanvianandpara@yahoo.com

Abstract

The dynamic model for the Cat-Righting Reflex – the innate ability of cats to always land on their feet no matter what their initial orientation was when they were released – had been quite a debated physical problem which was solved by the mathematical solution proposed and verified by Kane and Scher and later by a more general solution by Montgomery. However, the practical implementation of this ability in a robotic structure may demand for a modified version of these models with focus on physical realization of the same. This paper proposes such a model derived from the Kane-Scher cat model and simulates the same to test its suitability. An analysis is also made of the closeness to which the practical model realizes CRR with respect to the theoretical model.

Peer-review under responsibility of scientific committee of International Conference on Advanced Computing Technologies and Applications (ICACTA-2015).

Keywords: *Cat-Righting Reflex; Kane-Scher Solution; Non-Holonomic Robots*

1. Introduction

The "Falling Cat Problem" or as it is technically known, the "Cat-Righting Reflex" - the ability of a falling cat to right itself, i.e., to land on its feet no matter what its initial position was when it was released - had been a mathematical puzzle until the twentieth century when the accurate dynamic model for the mechanics of the falling cat was finally formulated. The ability of the cat to rotate its body without twisting as it rights itself seems to violate the law of conservation of angular momentum, as there are no external forces that tend to rotate the cat's body. However, it is now known that this feline ability is nothing but a consequence of a flexible spine and a set of specific geometric trajectories followed to arrive at net zero angular momentum. This mechanical framework in its generalized form has gone further to treat the falling cat as a non-holonomic control system thus applying gauge

Peer-review under responsibility of scientific committee of International Conference on Advanced Computing Technologies and Applications (ICACTA-2015).
doi:10.1016/j.procs.2015.03.092

theory to solve its dynamics (Montgomery[7]). Today, the now well-understood cat-righting reflex sees potential in practical implementation with regards to applications such as self-orientation of artificial satellites, and human motion control in zero-gravity.

The solution to this problem was attempted in the earlier years by Ter Braak and Rademaker (1923) basing the problem on geometric foundations. Rademaker and Ter Braak put forth a model that was capable of performing motions compatible with the first two features of the Falling Cat problem as described by the Kane and Scher theory[1]. It did not trace the third criteria as it had equal backward and forward bending.

The most widely recognized and accepted solution of CRR was provided by Kane and Scher in 1969 and forms the basis of analysis for practical implementation in this paper. Like Ter Braak and Rademaker, this paper also constructed the model on a geometrical basis, however, the solution provided by Kane and Scher fulfilled all the three criteria for Cat Righting Reflex. Although it has now been replaced by a more generalized model, the Kane-Scher cat model is capable of approximating the falling cat phenomenon to a very accurate level. Hence, the model constructed in this paper will be based on Kane-Scher solution rather than the currently accepted Montgomery's model.

The more modern solution to the problem has been given by Montgomery (1993) who described the Kane–Scher model in terms of a connection in the configuration space that encapsulates the relative motions of the two parts of the cat permitted by the physics. He considered the cat to be a part of a non-holonomic model, rather a different solution for the same. Framed in this way, the dynamics of the falling cat problem becomes a prototypical example of a non-holonomic system (Batterman[5] (2003)), the study of which is among the central preoccupations of control theory. A solution of the falling cat problem is a curve in the configuration space that is horizontal with respect to the connection (that is, it is admissible by physics) with prescribed initial and final configurations. Thus, the falling cat problem, in modern context, involves finding the solution space for a non-holonomic control system by applying concepts of gauge theory[7].

Although the solution of the Cat Righting Reflex is an interesting mechanical problem in itself the practical implementation of the same in a robotic structure[3] or a vehicular system with autonomy presents a different challenge altogether. In this case, the model that needs to be developed has to be consistent from a practical point of view, thereby, revising several theoretical concepts to suit practical realization. In this paper, we have constructed such a model that can not only approximate cat-righting behaviour but can also be realized as a physical prototype. The model under proposal has been derived from the Kane-Scher model for a stricter geometrical interpretation during realization.

This paper describes the classic Kane-Scher model and discusses the modifications in it for physical implementation while still validating the model. The paper then explains the simulation of the proposed model while describing its mechanical structure in SimMechanics model and its Simulink model. The control algorithm for regulating the joint angles is also described. The results and analysis of the simulation are finally presented to show that the output is in consistency with the traditional model. A physical prototype based on this model is under development to test the validity of the model.

2. The Kane Scher Model

The solution proposed by Kane and Scher in their paper of 1969 is described briefly in the following sections. The dynamic model and the differential equation for CRR are evaluated that are then considered in the later sections for practical implementation.

The model developed by Kane and Scher is evaluated based on the fulfilment of three criteria:

- The torso of the cat bends, but does not twist.
- At the instant of release, the spine is bent forward. Subsequent to this instant, the spine is bent first to one side, then backward, then to the other side, and finally forward again, at which point the cat has turned over and the spine has the same shape as at the initial instant.
- The backward bend that occurs during the maneuver is far less pronounced than the initial and terminal forward bend.

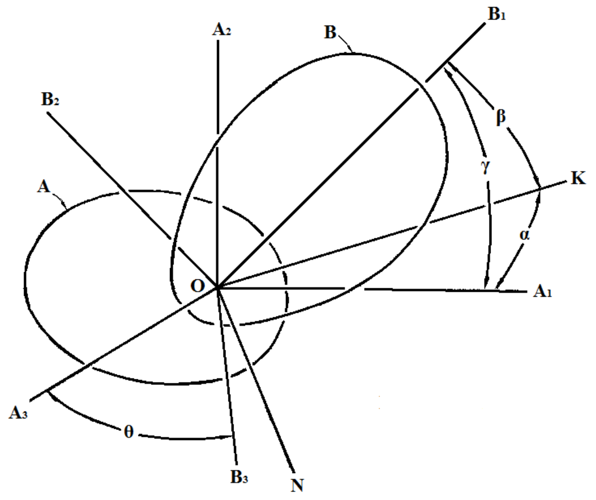

Fig. (1): Model for Cat-Righting Reflex by Kane and Scher – Adapted from [1]

In order to fulfil these criteria the model for the cat is considered to consist of two rigid body cylinders connected at a point O by a special no-twist joint. This structure is described in Fig. (1). The various labels in the figure are as follows:

Legend for Kane-Scher Model:

$\mathbf{A}_i$	Mutually perpendicular rays fixed in **A** and emanating from origin (i = 1,2,3)
K	A ray lying in the plane determined by $\mathbf{A}_1$ and $\mathbf{A}_2$
$\mathbf{B}_1$	A ray fixed in Body **B**
$\mathbf{B}_2$	A ray perpendicular to $\mathbf{B}_1$ and lying in the plane determined by B1and K (and not fixed in body B)
$\mathbf{B}_3$	A ray perpendicular to $\mathbf{B}_1$ and $\mathbf{B}_2$
N	A ray perpendicular to $\mathbf{A}_1$ and $\mathbf{B}_1$
A	The angle between $\mathbf{A}_1$ and **K**
B	The angle between $\mathbf{B}_1$ and **K**
Γ	The angle between $\mathbf{A}_1$ and $\mathbf{B}_1$
Θ	The angle between $\mathbf{A}_3$ and $\mathbf{B}_3$
$\hat{a}$	A unit vector parallel to **A**
$\hat{b}$	A unit vector parallel to **B**
$\hat{k}$	A unit vector parallel to **K**
$\hat{n}$	A unit vector parallel to **N**

The above illustration shows the two body halves **A** and **B** of the cat connected at the point O which is taken as the reference point. $\mathbf{A}_1$ and $\mathbf{B}_1$ form the principal axes of rotation for the respective body half. A net zero angular momentum is formulated as shown:

Considering a fixed reference frame formed by **N** and the angle bisector of $\mathbf{A}_1$ and $\mathbf{B}_1$, the angular velocities are given by:

$$Q_{\omega}{}^{A} = u\hat{a}_1 - (\frac{\dot{\gamma}}{2})\hat{n} \quad (2.1)$$

$$Q_{\omega}{}^{B} = v\hat{b}_1 + (\frac{\dot{\gamma}}{2})\hat{n} \tag{2.2}$$

Zero-twisting between A and B can be obtained by setting

$$u=v \tag{2.3}$$

This satisfies the first criteria mentioned. For the other criteria to be fulfilled, the equation for conservation of angular momentum is taken into consideration.

Total angular momentum = 0

$$J\omega_1^A\hat{a}_1 + \mathrm{I}\omega_2^A\hat{a}_2 + \mathrm{I}\omega_3^A\hat{a}_3 + J\omega_1^B\hat{b}_1 + \mathrm{I}\omega_2^B\hat{b}_2 + \mathrm{I}\omega_3^B\hat{b}_3 = 0 \tag{2.4}$$

Using the relations obtained by trigonometric substitution as follows:

$$\omega_1{}^A = \psi M\ (1 + T_{11}) + u \tag{2.5}$$

$$\omega_2^A = \Psi MT_{21} - (\frac{\dot{\gamma}}{2})\hat{n}.\hat{a}_2 \tag{2.6}$$

$$\omega_3^A = \Psi MT_{31} - (\frac{\dot{\gamma}}{2})\hat{n}.\hat{a}_3 \tag{2.7}$$

$$\omega_1^B = \psi M(1+T_{11}) + u \tag{2.8}$$

$$\omega_2{}^B = \Psi\ MT_{12} - (\frac{\dot{\gamma}}{2})\hat{n}.\hat{b}_2 \tag{2.9}$$

$$\omega_3^B = \Psi MT_{13} - (\frac{\dot{\gamma}}{2})\hat{n}.\hat{b}_3 \tag{2.10}$$

$$u = \theta\sin\beta T_{12}[\frac{1}{1-T_{11}^2}] \tag{2.11}$$

And substituting them in the Eqn. (2.5), we get the final expression as:

$$\frac{\partial\Psi}{\partial\theta} = \frac{(\frac{J}{I}).S}{(1+T)[1-T+(\frac{J}{I})(1+T)](1+T)^{\frac{1}{2}}} \tag{2.12}$$

Where,

$$S = -\sqrt{2}(\cos\alpha\sin\beta + \sin\alpha\cos\beta\cos\theta)\sin\beta \quad (2.13)$$

$$T = \cos\alpha\cos\beta - \sin\alpha\sin\beta\cos\theta \quad (2.14)$$

Here, the above equation introduces an angle ψ that corresponds to the relative orientation between the two body halves. The angle is chosen such that the following expression is valid:

$$\psi = 0, when \quad \theta = 0 \quad (2.15)$$

$$\psi = \pm\pi, when \ \theta = 2\pi \quad (2.16)$$

Eqn. (2.12) serves as the basic differential equation for the Cat-Righting Reflex model. This equation is a function of three variables α, β, θ and ψ that can be selectively controlled to bring about its implementation. A detailed derivation of the equation can be studied from the original solution of Kane and Scher[1]. In the next section, this equation is modified with certain assumptions to design the physical prototype.

The Kane-Scher equation explained above presents a first-order, non-linear differential equation (Eqn. (2.12)) for describing CRR as a function of the four angles α, β, θ, ψ. However given the non-linear nature of the equation, some assumptions as to the relations between the angles can be made so as to implement a simpler expression. These assumptions and considerations result in a convergent fifth-degree polynomial that can then be solved quite accurately by an appropriate numerical technique.

The first assumption involves the relation between the angular velocities of θ and ψ. Re-writing the expression for ψ, we have,

$$\psi = 0, when \theta = 0$$
$$\psi = \pm\pi, when \ \theta = 2\pi$$

Now, if the angular velocities for θ and ψ are taken to be uniform the above relation gives us,

$$\frac{d\psi}{d\theta} = \frac{1}{2} \quad (2.17)$$

This assumption can be valid only if the following conditions are satisfied:

- The velocities for θ and ψ necessarily need to be constant for the expression to be correct. This requires the actuators for the respective angles to maintain a constant rpm – such a requirement can be aptly fulfilled if the angular torque does not tend to slow the actuator speed to a considerable extent. (In which case a control loop would be necessary, thereby making the system more complicated.)

- From a more general view, the above expression can be observed to account for only a particular case, i.e., the worst case scenario in which the initial values are $\psi = 0$ and $\theta = 0$.
  When this is not the case, for example, when the angles hold non-zero values, $\psi = \psi'$ and $\theta = \theta'$, the expression now becomes

$$\frac{d\psi}{d\theta} = \frac{\pi - \psi'}{2\pi - \theta} = K \quad (2.18)$$

However, the change in value of $(d\psi/d\theta)$ has only a minimal effect in the eventual modification of the Kane-Scher equation as will be shown ahead. What more, the modified Kane-Scher equation that will now be arrived at, can thus be considered as an exemplifying case where we have $\psi' = 0$, $\theta' = 0$.

2.1. The New Kane-Scher Equation:

The above expression when substituted in Eq. (2.12) causes the Kane-Scher equation to become a zero order differential equation given by:

$$\frac{1}{2} = \frac{(\frac{J}{I}).S}{(1+T)[1-T+(\frac{J}{I})(1+T)](1+T)^{\frac{1}{2}}} \tag{2.19}$$

Further simplification of the equation is obtained by substituting the value for *J/I*. To maintain consistency with the Kane-Scher solution [1], the value is taken to be 0.25. The final equation, upon rearranging now becomes,

$$(1+T)[3T-5)](1+T)^{\frac{1}{2}} - 4S^2 = 0 \tag{2.20}$$

Here, the values for S and T are given by Eqs. (2.13) and (2.14); thus Equation is a highly non-linear, fifth order, trigonometric equation with three variables α, β and θ. Even while considering only one variable at a time, it is not possible to formulate the roots of an equation of this degree (By Abel-Ruffini theorem, it is not possible to do so for an equation of degree greater than 4). A solution can therefore be more easily found using numerical techniques. As discussed ahead, the Newton-Raphson method is utilized for the real-time solution of the problem – a numerical technique is more than consistent for practically solving the Kane-Scher equation dynamically in real time when it will be implemented in a physical prototype.

3. Practical Implementation

3.1. Mechanical Structure:

A SimMechanics model has been designed and constructed for the purpose of implementing the Kane-Scher model and testing the consistency of the modified expression to be able to realize the cat-righting reflex efficiently. The mechanical structure for this model is shown in Fig. (2) and Fig. (3).

The model is structured on the basis of the dynamical model described by the Kane-Scher model as shown in Fig. (1). Thus, the modelled cat developed consists of two symmetrical, cylindrical body halves connected to each other by a special "no-twist" joint. The model has been modified with regards to practical implementation to give the new model in Fig. (2). The different blocks of this new structure are described as follows:

Ground – This is the reference point necessary for simulating a model in SimMechanics. In this case, the Ground point is the point of origin O of Kane-Scher Model denoting the point of joining of the two body halves. All motion is considered w.r.t. to this point.

For the front body-half, the various components are given as follows:

(Alpha + Psi) Joint – This is a 2-DOF joint connected at O to generate the angular motions. As the name suggests, this joint is used to actuate the angles α and ψ as described in the Kane-Scher model.

Angle α is realized along the DOF of the revolute joint of the 2-DOF Joint Block while ψ is realized by the rotating joint. The angles α and ψ are basically described between the two body halves, however, the motion in the SimMechanics model takes place w.r.t. ground which is symmetrical to both the halves. Hence, the angle value actually fed to the joint is α/2.

Conn_Body – This is a dummy link that connects the two joints so as to integrally form the 3-DOF as described in the theoretical mode.

(Theta + Beta) Joint - This 2-DOF joint is used to actuate the angles β and θ as described in the Kane-Scher model. Angle β is realized along the DOF of the revolute joint of the 2-DOF Joint Block while θ is realized by the rotating joint. This joint is connected to the body cylinder at the end.

Front_Cyl – This is the body cylinder that constitutes the front body half of the cat. The inertia tensor for building the model is evaluated to approximate the physical prototype that will be implemented.

Leg1 and Leg2 - These are stationary legs attached to the body cylinder to serve as visual markers for the motion of the cylinder during the simulation.

Since the model is symmetric about O, the back body components are the same as the front body components. The changes in the mechanical structure and the relevant reasons are enumerated ahead:

- The joint at point O in the classic model would need to be a 3-DOF joint in order to implement all the three angles – however, simulation in SimMechanics does not allow the actuation of a 3-DOF joint. It is therefore more feasible to represent the same motion by a combination of two 2-DOF joints. Both in the simulation as well as practically. The 3-DOF joint therefore is broken down into two 2-DOF joints for the purpose of physical realization. As a consequence of this, an additional connector is required to be added between the two joints – this connector is of a small but finite mass to support the relative motion of the two joints.
- In theory the angles described are essentially formed between the two body halves for the free falling body. The simulation, however, requires a fixed reference point as ground in order to initiate simulation. Hence all the angular motions are made with regards to the ground point which is taken at point O.

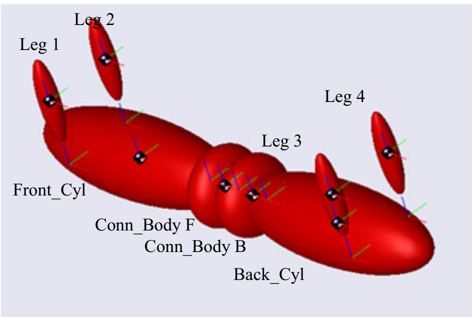

Fig.2. SimMechanics Model for CRR (Ellipsoid)

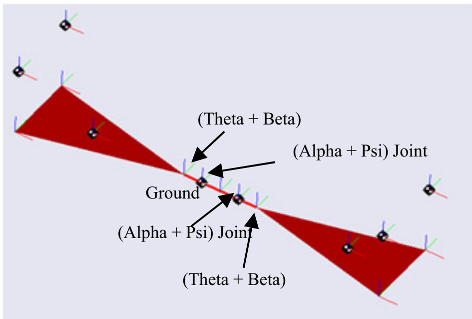

Fig.3. SimMechanics Model for CRR (Convex Hull)

3.2. Simulink Model

The Simulink model for the above simulation constitutes all the actuators, sensors and function blocks implementing the control of the various angles of the structure. The features of the model are described below:

- *Alpha Actuation* – The value for α is taken randomly to account to check for computational flexibility. The limiting value for α is taken as between 0 to 25 degrees with regards to the physical limitation of the bending of the cat spine[1].
- *Theta Actuation* – The angle θ is gradually varied to give a constant velocity signal. This serves as the primary input to the control loop that calculates β for the given set of parameters.
- *Psi Actuation* – As mentioned in the earlier section ψ is taken as a signal with half the frequency of θ. This allows for the application of Eqn. (2.20) to the model.
- *Beta Actuation* – β is calculated for each instant of θ by solving Eqn. (2.20) through numerical techniques. The β-solving loop is explained in the next section.

3.3. Solution of the Kane-Scher Equation

The operation of the model essentially involves a pseudo-control loop that takes in the angles α and θ as inputs and produces the signal value for β. Upon expansion Eqn. (2.20) is found to be a fifth degree equation in T where again T is itself a trigonometric function. The high degree of the polynomial does not allow a formulated expression for β. What more, the non-linear nature also makes it difficult to compute the values easily. Therefore, the Newton-Raphson method is adopted to calculate β from the equation.

The msfcn_newton method block accounts for the calculation of β. Adoption of this technique of control has a considerable impact on the working principle of CRR. The original model put forth by Kane and Scher suggested control of the angle ψ from the inputs α, β and θ from Eqn. (2.12). This would involve solving the differential equation by Runge-Kutta method to give the current value for β. However the current model deviates from this technique in that the relation between ψ and θ is kept fixed whereas it is the angle β that becomes the controlled parameter.

The function block thus calculates the angle value by implementing the following algorithm.

Algorithm: msfcn_newtonmethod Algorithm

01. Initialize default value of beta, *bdef.*
02. Take the input values for alpha, *a* and theta, *d*
03. Take *bold* = *bdef*
04. While (|error| <= desired value) do
05. Calculate $T = \cos\alpha\cos\beta - \sin\alpha\sin\beta\cos\theta$
06. Calculate $S = -\sqrt{2}(\cos\alpha\cos\beta + \sin\alpha\cos\beta\cos\theta)\sin\beta$
07. Calculate $fcheck = (1+T)[3T-5)](1+T)\frac{1}{2} - 4S^2$
08. Calculate derivatives *S', T', fcheck'*
09. Get *bnew* = *bdef – (fcheck'/fcheck)*
10. Get *error* = *bnew – bold*
11. Take *bold* = *bnew*
12. End While
13. Final value of beta = *bnew*

4. Practical Implementation

The simulation output sequence for the times 0.0 s, 1.1 s, 2.1 s and 2.5 s is shown for α = 0.261 radians both for the classic Kane-Scher model (Fig.(4)) as well as the revised model implementing the above algorithm (Fig. (5)). In both cases, the signal for θ is a sawtooth with a frequency of 0.4 Hz. The lower and upper bounds on the signal are 0 and 2π (6.28) radians respectively. This allows θ to vary with a constant velocity for an entire revolution. The signal is illustrated in Fig. (6).

As required, the angle ψ is taken to be of half the frequency as that of θ, i.e., 0.2 Hz, which is illustrated in Fig. (7), thus satisfying Eqn. (2.17).

The variation in β w.r.t. θ is shown in Fig. (8) – this curve has been verified for different values of α to verify the correctness of the final value.

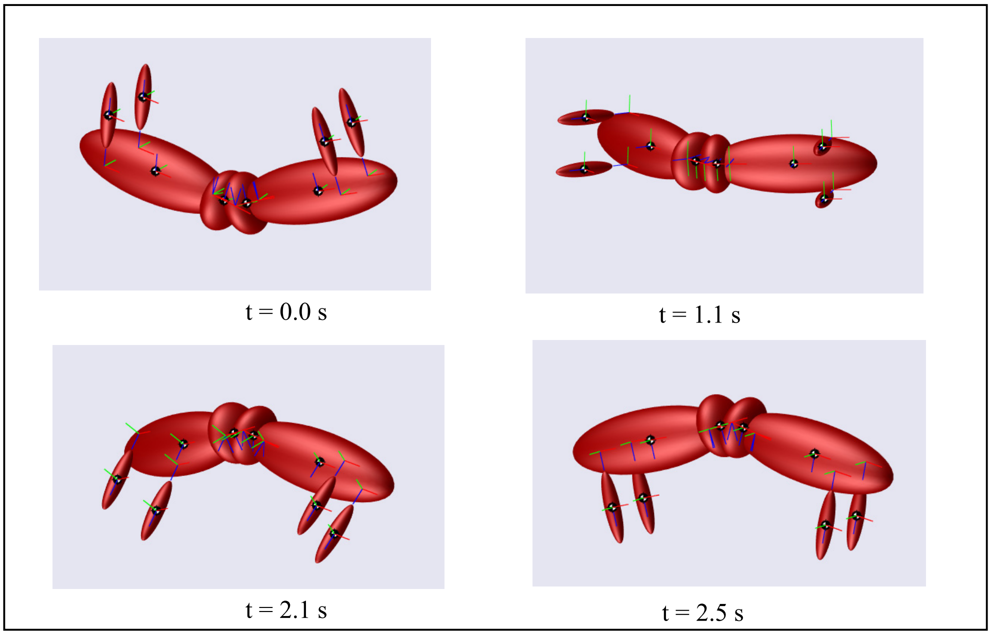

Fig. 4: Simulation sequence for CRR model in MATLAB – based on classic Kane-Scher model

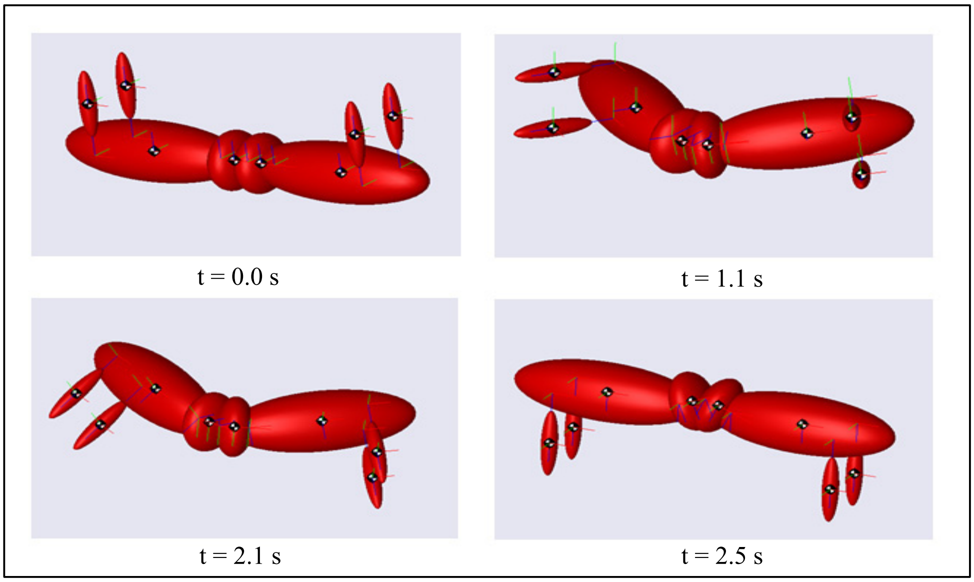

Fig.5. Simulation sequence for CRR model in MATLAB – based on revised model

Analysis:

Considering the simulation outputs from Figs. (5) – (8), we can infer the following:

- The revised model is capable of implementing CRR as described by the theoretical model to a considerable degree of accuracy

- The righting though slightly altered by the modified control loop is nevertheless observed to take place.
- A considerable computation time is required at the initial calculation of β. This computation time needs to be accounted for while programming the physical prototype.

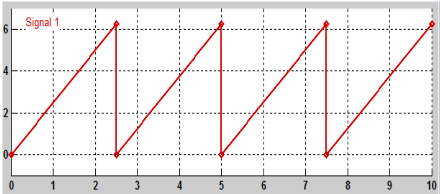

Fig.6. Signal for Theta

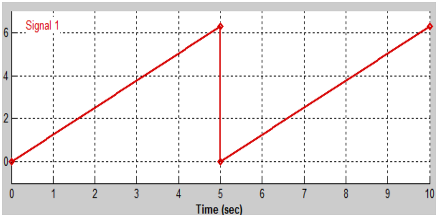

Fig.7. Signal for Psi

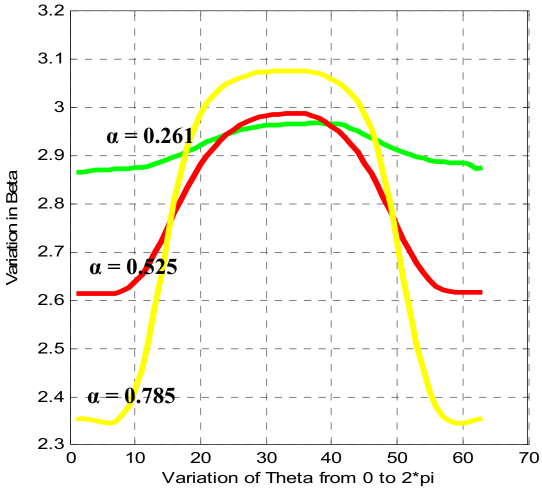

Fig.8. Variation of Beta w.r.t. Theta

5. Conclusions

The model for Cat Righting Reflex that has been simulated and analysed in the paper is found to be able to realize the required motion to a considerable accuracy. The assumptions made and modifications carried out in the model allow a simpler technique for implementation that can be utilized for the physical prototype under development.

References

1. Kane, Thomas; Scher, M. P. (1969). "*A dynamical explanation of the falling cat phenomenon*". International Journal of Solids and Structures 5(7), 663–670.
2. Asa Hopkins *"How does a falling cat turn over?"* June 6, 2003
3. Ben Shields, William S. P. Robertson, Natalie Redmond, Ross Jobson, Rian Visser, Zebb Prime, Ben Cazzolato "*Falling Cat robot lands on its feet*" Proceedings of Australasian Conference on Robotics and Automation, 2-4 Dec 2013, University of New South Wales, Sydney, Australia .
4. Yoshihisa Sakurai "*Falling cat problem*" June 9, 2009
5. Robert W. Batterman "*Falling Cats, Polarized light and Parallel Parking*" Ohio State University
6. Arabyan, A.; Tsai, D. (1998). "A distributed control model for the air-righting reflex of a cat". *Biol. Cybern.* 79 (5): 393–401
7. R. Montgomery, "Gauge Theory of the Falling Cat", field Inst. Comm., vol. 1., 193-218,1993
8. Xin-Sheng Ge, Wei-Jia Zhao, Yan-Zhu Liu "Nonholonomic Motion Planning for a Free-falling Cat Using Quasi-Newton Method" *Technische Mechanik, 31, 1, (2011), 42 – 49*