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REVIEWS

Edited by CATHERINE GOLDSTEIN AND PAUL R. WOLFSON

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Fleeting Footsteps: Tracing the Conception of Arithmetic and Algebra in Ancient China. By Lam Lay Yong and Ang Tian Se. Singapore (World Scientific). 1992. xvi + 199 pp. 15£.

REVIEWED BY JEAN-CLAUDE MARTZLOFF

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As stated in the introductory remarks (pp. vii–x), the authors have chosen the *Sun Zi suan jing* [Sun Zi's canonical manual of arithmetics]—one of the 'Ten mathematical manuals' (*Suan jing shi shu*) of the Sui (589–618) and Tang dynasties (618–907)—as the basis of a study which aims to reconstruct the development of arithmetic and the initial stages of algebra in China so as to prove that "the Hindu–Arabic number system has its origin in the Chinese rod numeral system" (p. ix). Given that the *Sun Zi suan jing* (*SZSJ*) is neither the oldest known Chinese mathematical manual nor the most elaborated or representative one, one may ask why such a broad and general purpose should be based merely on what the authors themselves consider a "rudimentary" text (p. 127) of very uncertain date written by an unknown author. In fact, as explained on p. vi and elsewhere in the book, this limitation becomes understandable once it is noted that, according to the authors:

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(1) after 1200, the development of mathematics was conditioned by the availability of a system of numeration conceptually identical to the Hindu–Arabic decimal and positional number system (p. vii), and

(2) the mathematical content of the *SZSJ* bears testimony of the Chinese discovery and computational mastery of a fully developed decimal and positional system of numeration many centuries before the appearance of such a system anywhere else in the world. In particular, the *SZSJ* is the oldest source which contains concrete instructions for the practice of elementary arithmetical operations.

From this perspective, it appears that Chinese mathematical sources older than the *SZSJ* (for example the *Jiuzhang suanshu* [Computational processes in nine chapters]) could not have been used for the same purpose (even if some of them would certainly have been mathematically more interesting) because prior to the *SZSJ*, known Chinese mathematical treatises do not expound the most elementary aspects of mathematics. Consequently, the authors have devoted a large part of their study to the fundamental operations of arithmetic in the *SZSJ* (pp. 29–82).

After a general presentation of the two main Chinese systems of numeration from antiquity, that is, the written number system and the rod numeral system (pp. 11–27), the authors translate the general instructions of the *SZSJ* relative to mutiplication and division. Then, they reconstruct addition and subtraction (because these are not explained in the *SZSJ*). Finally, operations on fractions and square root extraction are also carefully studied, and Chinese techniques of multiplication and division are shown to be similar to certain computational techniques found in certain early Islamic sources (p. 36 and 42 ff.).

On the whole, these reconstructions are rather convincing even though the terse instructions of the *SZSJ* would allow, in some cases, slightly different interpretations (see [13, 63]). The comparisons with Islamic techniques are no less convincing. Still, should we conclude that China influenced Islam? Not necessarily. If the Islamic techniques presented here are incontestably posterior to those of the *SZSJ*, another kind of comparison with the much earlier Roman computational techniques with counters (*calculi*) on the Roman counting-board (see [2], [3], [4], [7], [11, 315 ff.], [15]), which were developed well before the *SZSJ*, would have indicated a possible influence in the opposite direction. Indeed, the Chinese rod numerals and the Roman counters share operationally the quinary grouping for individual digits. In both cases, 6, 7, 8, and 9 are always decomposed into 5 + 1, 5 + 2, 5+ 3, and 5 + 4, respectively, and, on the Chinese and Roman counting-boards, numbers are represented decimally and according to the laws of place-value. Moreover, multiplication begins systematically by the digits of highest rank. The "bead numerals" of the Roman abacus (as well as those of the much later Chinese abacus, which is generally believed to date back to not much sooner than the fourteenth century [17]), also show the same quinary grouping. This represents a clear contrast between them and the undecomposable digits of Islamic (and Indian) sources whose individual written structure does not incorporate any separate representation of the number "5."

If the hypothesis of a Chinese origin of our number system is to be maintained, the very important problem of the zero raises other thorny issues. On page 8 of *Fleeting Footsteps*, we read that "when a number had no digit of a particular rank, the position representing that rank was left vacant." This assertion seems rather plausible. However, the existence of blank spaces on the counting-board is not in itself a special property of the Chinese counting-rod system but rather of counting-boards and abacuses in general. In that case, though, another problem presents itself since the Chinese counting-board was "a flat surface such as a table top, a mat or the floor" (p. 20) with apparently no lines, grooves, or material indications of any kind to distinguish between the different orders of unities of numbers. Under these circumstances, one may ask how the Chinese managed to distinguish numbers with only one median blank from those with two or more blanks, or to delineate numbers with initial or terminal blanks? According to *Fleeting Footsteps*, the Chinese used the character *kong* (empty) to describe vacant places in written texts (pp. 25–26). Should this character be considered as an ancestor of zeroes: Indian, Islamic, European, and even Chinese? If so, the reader would have appreciated the presentation of at least one historical example of this written *kong* used as a median zero, singly or, better yet, doubly. For example, did the Chinese write "*kong kong*" when two consecutive blank spaces appeared within a number, in the same way that, much later (from the Ming onwards), they would write "*ling ling*" for the same purpose (see [18, 575])? We have not the slightest evidence that the answer to this question could be positive. But, of course, the absence of testimony is not equivalent to the testimony of absence. Nonetheless, examples of Chinese written numerals from sources anterior to the tenth century do exist, and the structure of these militates against the idea of the Chinese origin of the Hindu–Arabic numerals.

According to the MS. Stein 930 from the Dunhuang caves (9th or 10th century AD, one of the oldest known original Chinese mathematical manuscripts), the presence of blank spaces was not automatically preserved in the written version of the rod numeral system (see [1] and [10, 191]). Incidentally, other similar examples provide evidence of irregular forms for rod numbers representing 20, 30, and 40. These were conceived as global, additive concatenations of the symbol for 10, read *nian*, *sa*, and *xi*, respectively (see the MS. Pelliot 2667 of the Bibliothèque Nationale, Paris) and were rather common in mathematical texts. These various remarks tend to prove that, in practice, the counting-rod system was not as perfectly decimal and positional as the descriptions in *Fleeting Footsteps* would imply. Still, while we cannot be certain that the Dunhuang manuscripts are representative of the Chinese mathematical practice of their time, some of these manuscripts are doubtless quite relevant for the study of the *SZSJ*: the content of the MS Stein 930 is identical to a part of the text of the first chapter of the *SZSJ*; a complete comparison between this manuscript and known editions of the *SZSJ* would perhaps bring to light new elements on the Chinese written zero.

Whatever the interest of such a study, another very important fact concerning the zero seems quite relevant to the argument presented in *Fleeting Footsteps*:

well before the compilation of the *SZSJ*, the Babylonians of the Seleucid period (312–64 B.C.) already possessed a well-developed written form of the zero. Their double slanted-wedge was inserted not only in median positions but also in initial and terminal positions of numbers as well, and was repeated as many times as necessary (see [5, 398 ff.], which draws its examples from previous publications of Babylonian mathematical and astronomical texts by O. Neugebauer and E. M. Bruins). Thus, reasoning as in *Fleeting Footsteps* (p. 143), but in an opposite direction, we might suggest that the extremely long span of time (more than 1000 years) which separates the first known appearance of the zero in antiquity and in medieval China (7th century in a Chinese adaptation of an astronomical text of Indian origin [13, 12]) renders quite plausible the idea of the Babylonian origin of the Chinese zero.

Apart from the question of the zero, I would also like to examine a very different kind of question which is of consequence for the thesis developed in *Fleeting Footsteps*: the dating of the *SZSJ*. In the past, this baffling question has been patiently studied by historians of Chinese mathematics such as Li Yan, Yan Dunjie, and Wang Ling, who based their datings mainly on internal evidence from the text of the *SZSJ*. In *Fleeting Footsteps*, the most important of these studies are carefully listed. Hence, it would seem that the *SZSJ* was not written later than 473 AD.

However, the most ancient Chinese bibliography which mentions the *SZSJ* is that of the *Sui shu* [Sui history], one of the 24 dynastic histories completed in 636 by Wei Cheng, the Confucian counselor of Emperor Taizong of the Tang dynasty. According to this source (Chap. 34, 1025), the *SZSJ* was originally composed of two chapters and not three, as stated by later sources. In the present version of the *SZSJ*, Chaps. 2 and 3 have a certain unity which differentiate them from Chap. 1; they are both composed of a series of problems, whereas the first chapter contains mainly concrete details on metrology and arithmetical operations. Consequently, the first chapter of the *SZSJ* is perhaps a later addition; its content is consistent with what we know of arithmetical usage from the Tang dynasty. Naturally, scribal error cannot be ruled out since the Chinese characters for '2' and '3' are very similar, composed as they are of 2 (resp. 3) superposed strokes. Another possibility would be a later division into three chapters of a text initially in two chapters. Regardless, the biography of Li Chunfeng (602–670) in the *Jiu Tang shu* [Old Tang History] states that Li Chunfeng and others reedited the *SZSJ* because "the text was very erroneous (or contradictory) from the point of view of the principles [*li duo chuanbo*]" (Chap. 79, 2719). The version of the *SZSJ* presented in *Fleeting Footsteps*, however, is remarkably devoid of contradictions of any kind! The idea that the original of the *SZSJ* contained contradictions may perhaps have been nothing more than a clever assertion made by a scholar–bureaucrat anxious to justify his professional activities. Yet it is well-known [9, 92] that Li Chunfeng reedited the *SZSJ* not for historical purposes but because he had been ordered to produce manuals for the teaching of mathematics at the Imperial Academy. Thus, it would not be absurd to believe that this famous scholar would have updated the text to some extent, especially from the point of view of the practice of arithmetical operations. If this conclusion were correct, it would follow that the present version of the *SZSJ* reflects the arithmetical practice of the beginning of the Tang dynasty, precisely at a period of intense contacts between China and India (where the concept of zero in its written form was already developed). As is often noted, Chinese translations of Indian mathematical and astronomical texts were made at this time and one of these, dated 712 AD, mentions precisely an Indian written zero in the form of a small dot [13, 12]. This aspect of the question is well documented [16], and certain of these translations have even survived. Still more significantly, the representation of numbers in Chinese Buddhist literature is often borrowed from Indian culture, especially in the form of phonetical transliterations of Sanskrit words into Chinese (see [8, 183ff] and [12]). Conversely, as far as I know, Chinese mathematical terms have never been detected in Indian or Islamic technical literature. Unfortunately, these aspects of the problem are passed over in silence in *Fleeting Footsteps*.

These omissions aside, *Fleeting Footsteps* contains the first translation of the *SZSJ* into an Occidental language (pp. 151–182). This aspect of the present work will be all the more appreciated by historians of medieval mathematics since the authors have meticulously shed light on many facets of the Chinese historical context: Chinese metrology (pp. 83–88); the socioeconomic context of individual problems (pp. 127–131); the mathematical presentation of problems, in particular the famous "Chinese remainder problem" (simultaneous congruences of the first degree); and other such topics.

APPENDIX: CHINESE CHARACTERS REFERRED TO IN THE TEXT

Chen Liangzuo	陳良佐
Dalu zazhi	大路雜誌
Jiu Tang shu	舊唐書
Jiuzhang suanshu	九章算術
kong	空
Li Chunfeng	李淳風
Li duo chuanbo	理多踳駁
Li Yan	李儼
ling	零
Mineshima	峰島

Suan jing shi shu	算經十書
Sûgakushi kenkyû	數學史研究
Suishu	隋書
Sun Zi suanjing	孫子算經
Yabuuchi (Kiyoshi)	藪内清
Yan Dunjie	嚴敦傑
Zôtei Zui Tô rekihôshi no kenkyû	増訂隋唐曆法史の研究

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Briefwechsel zwischen Karl Weierstraß und Sofja Kowalewskaja. Edited with an introduction and commentary by Reinhard Bölling. Berlin (Akademie-Verlag). 1993. 504 pp.

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An important edifice in the community of late 19-century mathematics was built by Weierstrass and his circle, including Gösta Mittag-Leffler, Paul du Bois-Reymond, Hermann Amandus Schwarz, Sofya Kovalevskaya, and others. The structural plan and much of the framework of this edifice were created by Weierstrass himself, but his students showed great creativity in fitting their own ideas into the general design. The unique features of the Weierstrassian school, its striving for clarity of basic principles and purity of method on the basis of algebraic ideas, and its relation to other approaches to the same subject matter in the works of Riemann and Kronecker provide a vast field for historical investigation. Just how vast cannot be gauged merely from the published works of the principals involved, for there are large amounts of unpublished material scattered all around the world, much of which the historian is likely to encounter only by accident. To give just one example, Weierstrass' 1881 course in analytic function theory was attended by one Archibald Daniels of Hudson, Michigan. Upon his return to America in 1885 Daniels found a position at Princeton University, replacing Henry Burchard Fine, who had gone on leave. From Princeton, Daniels was hired by the University of Vermont, where he taught until his retirement in 1914. His transcript of Weierstrass' course is now in the possession of his grandson, R. V. Daniels, who recently retired as Professor of History from the University of Vermont.

The number of people like Archibald Daniels, who once attended a Weierstrass course and preserved and passed on their notes, must be considerable, but the probability of discovering the present whereabouts of an individual set of notes is discouragingly small. Fortunately there are "mother lodes," such as the Institut Mittag-Leffler, whose archival treasures were first publicized by Grattan-Guinness [3]. The work of correlating, analyzing, and (with luck) publishing this material remains to be undertaken so that the definitive history of the Weierstrass school can be written. The large amount already published ([7], the samples of correspondence published by Mittag-Leffler [6], and the biographical works by P. Ya. Kochina [4] and others) tells what kind of information to look for, but the real key to