

Particle breakage and the critical state of sand: By Ghafghazi, M., Shuttle, D.A., DeJong, J.T., 2014. Soils and Foundations 54 (3), 451–461

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The authors have carried out an experimental study on the effect of particle breakage on the critical state of sand. The results show that particle breakage caused a downward shift in the critical state line (CSL) on the e-p' plane without changing its slope. The authors also assumed that detectable particle breakage does not occur unless two conditions are satisfied: (1) the contraction potential of the material induced by sliding and rolling is exhausted; and (2) a soil-specific stress threshold is surpassed.

The discussers applaud the authors' efforts and their findings on the relationship between CSL and particle breakage. The experimental results provided further support for the unified modeling of crushable soils in the framework of critical state soil mechanics. The results also assist in the understanding of many geotechnical mechanisms related to particle breakage. However, the authors assume that particle breakage only occurs when the soil stress is large, which does not seem to be accurate. Previous experimental studies have shown that particle breakage is not only determined by confining pressure and the level of shear stress, but it also increases with an increase in shear strain at a constant stress level (Miura and Ohara, 1979; Indraratna et al., 1998; Ueng and Chen, 2000;

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Coop et al., 2004). In particular, particle breakage occurs in cyclic loading, in which the stress level is moderate, and it increases with an increase in the number of loading cycles (Indraratna et al., 2005; Donohue et al., 2009).

In lieu of soil stress, consumed plastic work per unit volume by the soil, $W_p = \int \sigma'_{ij} d\varepsilon^p_{ij}$, is a better parameter to correlate with particle breakage (Miura and Ohara, 1979; Lade et al., 1996). Here, σ'_{ij} and ε^p_{ij} are the effective stress and the plastic strain components, respectively. It is understandable that plastic work is not solely consumed by particle breakage, as particle rearrangement also contributes to it. Nonetheless, particle breakage has been remarkably well correlated with the input plastic work for sandy soils (Miura and Ohara, 1979; Lade et al., 1996), soil-structure interfaces (Zeghal and Edil, 2002), and gravelly soils (Liu et al., 2014). In addition, it is convenient to calculate W_p in the constitutive model, making it suitable for practical applications.

With plastic work per unit volume W_p , the translating critical state line with particle breakage can be expressed as

$$e_c = e_\tau - \Delta e_c - \lambda \log_{10}(p'/p_a) \tag{1}$$

Here, e_{τ} and λ are the intercept and the slope of the virgin CSL without particle breakage, respectively, p' is the mean effective stress, p_a is the atmospheric pressure for stress normalization, and Δe_c is the change in critical-state void ratio. To make a comparison with the discussed experimental results, the linear

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relationship between Δe_c and particle breakage index B_r (Hardin, 1985) in Liu and Zou (2013) and Liu et al. (2014) was replaced by the following relationships:

$$\Delta e_c = c \Delta F C \tag{2a}$$

$$\Delta FC = FC - FC_{0.8} \tag{2b}$$

$$\Delta FC = \frac{(W_p - W_{th})}{a + b(W_p - W_{th})} \tag{2c}$$

Here, $FC_{0.8} = 0.8(\%)$ is the initial fines content of Fraser River Sand. FC(%) is the current fines content of Fraser River Sand. Eq. (2a) was approximated from the relationship between the critical-state void ratio and the fines content in the discussed paper. In Eq. (2c), the plastic work per unit volume W_p consists of the effects of isotropic compression and the following triaxial shearing until the end of the test, which can be calculated as

$$W_p = \int \left[(\sigma'_1 - \sigma'_3) d\varepsilon_1^p + \sigma'_3 d\varepsilon_\nu^p \right]$$
(3)

Here, σ'_1 and σ'_3 are the axial and confining effective stresses, respectively, while ε_1^p and ε_{ν}^p are the axial and volumetric plastic strains, respectively. In this discussion, due to the lack of test data, the elastic work was not subtracted from the total work to obtain W_p , but it has been shown by previous studies that elastic work is negligible compared to the magnitude of plastic work (Daouadji et al., 2001; Daouadji and Hicher, 2010; Liu et al., 2014).

For the tests on Fraser River Sand, the input plastic work per unit volume by isotropic compression was interpreted based on the initial states at the start of shearing (see Fig. 1) due to the lack of isotropic compression test data. That is to say, it was calculated by the two log-linear equations in Fig. 1 for loose and dense sands, respectively. W_{th} is the threshold value of plastic work per unit volume that would result in detectable particle breakage (Liu and Ling, 2008). Particle breakage does not occur unless W_p is larger than W_{th} , which is similar to the assumption proposed in the discussed paper.



Fig. 1. e-p' by isotropic compression tests.



Fig. 2. Relationship between plastic work and fines contents.



Fig. 3. Relationship between increment in fines content and increment in critical void ratio.

The relationship between W_p and ΔFC for Fraser River Sand was calculated; it is plotted in Fig. 2. The test data (CID-L 1400, CID-L 1400-2, CID-L 1000, CID-L 1000-peak, CID-D 600, and CID-D 600-peak) were obtained from the discussed paper as well as from Ghafghazi (2011). It can be seen that Eq. (2c) very well describes the relationship between the increase in fines content and the plastic work per unit volume. For Fraser River Sand, parameters *a* and b were 168 kPa and 0.04, respectively. And the threshold plastic work per unit volume was 138 kPa.

The hyperbolic relationship can be further validated by Test CID-M #3 600, which had an initial FC=6.3 at the beginning of triaxial shearing. Using Eq. (2c) and the aforementioned parameters, FC=6.3 corresponds to an initial $W_p=1415$ kPa. The input plastic work W_p during triaxial shearing was 456 kPa, and FC=7.7, based on Eq. (2c), is very close to the value of 7.8 measured after the test.

Fig. 3 shows the relationship between Δe_c and ΔFC . Apparently, a linear relationship can be approximated, and parameter $c = \Delta e_c / \Delta FC$ in Eq. (2c) was approximately equal to 0.02.

Table 1 Plastic work and change in void ratio for some CID tests in discussed paper.

Test name	L 100	L #6 100	L 190 UR	L #3 150	M #3 200	M #3 600
$W_p \\ \Delta e_c$	99 0	74 0	122 0	158 0.0024	141 0.0005	429 0.032
Test name	L #4 100	L #4 300	L #7 100	L #7 300	L #8 100	L #8 300
$W_p \\ \Delta e_c$	96 0	242 0.012	83 0	228 0.01	110 0	276 0.016

Therefore, the translating critical state line of Fraser River Sand can be expressed as

$$e_c = e_\tau - 0.02 \frac{(W_p - 138)}{168 + 0.04(W_p - 138)} - \lambda \log_{10}(p'/p_a)$$
(4)

where e_{τ} is equal to 0.944 and λ is equal to 0.138. Eq. (4) is similar to the CSL equations for crushable soils seen in previous studies (Daouadji et al., 2001; Daouadji and Hicher, 2010; Liu and Zou, 2013; Liu et al., 2014).

It is also interesting to analyze the plastic work for the tests that achieved critical states at the end of shearing. For the tests that were employed to obtain the virgin CSL (CID-L 100, CID-L #6 100, and CID-L 190 UR, Table 1 and Fig. 5 of the discussed paper), the values were all smaller than 138 kPa, as can be seen in Table 1. The values were smaller or somewhat larger than 138 kPa for the tests on a remolded specimen, and except for CID-M #3 600, the calculated Δe_c by Eq. (4), was small, as can be seen in Table 1. CID-M #3 600 resulted in additional particle breakage, as can be seen in Table 2 of the discussed paper, and strictly speaking, it should not have been employed to obtain the CSL after particle breakage together with CID-L #3 200 (Fig. 8b of the discussed paper).

In summary, the framework of translating CSL based on a hyperbolic function of the input plastic work per unit volume can describe the behavior of crushable soils well. By the threshold value, W_{th} , the first segment of the CSL in Fig. 1 of the discussed paper can be properly modeled. And, with the hyperbolic function, the limiting state of the soil, when plastic work is very large and particle breakage has basically been completed, can also be described (the elastic-like behavior in Fig. 1 of the discussed paper). More importantly, with the translating state line, depending on the input plastic work, the salient features of crushable soils under cyclic loading, the critical state of which is not necessarily on the second segment in Fig. 1 of the discussed paper, can be properly simulated.

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