Abstract  The effects of Single Walled Carbon Nanotube and Copper nanoparticles on natural convection heat transfer in an open cavity are investigated numerically. The problem is studied for different volume fractions of nanoparticles (0–1%) and aspect ratio of the cavity (1–4) when Rayleigh number varies from $10^3$ to $10^5$. The volume fraction of added nanoparticles to Water is lower than 1% to make a dilute suspension. Although, results show that adding nanoparticles to the base fluid enhances the heat transfer, make a comparison between SWCNT and Cu-nanoparticles shows that the SWCNT-nanoparticle has better performance to enhance the convection rate. It is found that the aspect ratio of the cavity plays an important role on natural convection. An increase of this parameter leads to heat transfer reduction in the target problem. It is concluded that the Carbon Nanotubes can be applied as a passive way to enhance heat transfer in convection problems.

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1. Introduction

The enhancement of fluids heat transfer is an interesting topic for different kinds of industrial and engineering applications. Heat Exchangers, boilers and condensers are some examples of important industrial equipments which have a close relation with the heat transfer subject. There are varied methods to enhance the convection rate in different conditions. Generally, these methods are classified in three sections named Active, Passive and Component methods. The passive methods have...
no need for external forces for making enhancement in heat transfer rate. When these methods are adopted in heat exchangers can prove that the overall thermal performance improves significantly. Liu and Sakr presented a paper [1] which reviews experimental and numerical works taken by researchers on the passive techniques such as twisted tape, wire coil, and swirl flow generators. The authors concluded that twisted tape inserts perform better in laminar flow than turbulent flow. However, the other several passive techniques such as ribs, conical nozzle, and conical ring are generally more efficient in the turbulent flow than in the laminar flow.

A well-known passive way to enhance the rate of heat transfer of conventional fluids such as water, oil and ethylene glycol which has low thermal conductivity [2,3] is adding nanoscale conductive particles. The added particles can be metals [4], nonmetals [5] or Carbon nanotubes [6,7]. Because of the high thermal conductivity of these particles, they can improve conductivity of the suspensions systematically. Nowadays, nanoparticle added fluids are known as nanofluid that first time Choi [8] named this kind of fluid suspensions. The great potential of nanofluids for heat transfer enhancement in highly suited practical applications may create an opportunity to develop compact and effective heat transfer equipments for many industrial applications, including transportation, nuclear reactors, electronics, biomedicine, and food.

At this way, in recent years, many studies conducted to study heat transfer of nanofluids numerically and experimentally [9–15]. Khanafaer et al. [9] presented a heat transfer enhancement by adding nanoparticles to fluid in a two dimensional enclosures at natural convection regime for the different Grashof numbers. They presented an increase of $N\text{Nu}_{\text{ave}}$ about 7.5%, 12%, 15.5% and 20%, respectively, by adding a volume fraction of added Cu nanoparticles to the Water equal to 4%, 6%, 8% and 10% when $Gr = 10^4$. A useful review paper presented by Hussein et al. [10] shows that, according to the majority of experimental and numerical studies, suspensions of solid nanoparticles significantly enhance heat transfer and the heat transfer coefficient of nanofluids is found to be larger than that of its base fluid at the same physical condition.

In 1991, Iijima [16] discovered Carbon nanotubes as an allotrope of Carbon which is made of long-chained molecules of Carbon with Carbon atoms arranged in a hexagonal complex to form a tubular structure. Single Walled (SWCNT), Double Walled (DWCNT) and Multi Walled (MWCNT) are different classes of Carbon nanotubes depending on the number of tubules of Carbon in their structure.

In last decade Carbon nanotubes have been mentioned as an attractive topic by many researches which is generally due to their special properties at physical view such mechanical and thermal properties. At this point of view, the Carbon nanotubes have extraordinary thermal properties such as thermal conductivity about twice as high as diamond [17] or thermal stability up to 2800 °C in vacuum. The higher thermal conductivity of Carbon nanotubes relative to other nanoparticles led to that the nanofluids containing cylindrical Carbon nanotubes are expected to have better heat transfer properties compared with the other nanofluids with spherical nanoparticles [18,19]. Gavili et al. [20] simulated the mixed convection in a two-sided lid-driven differentially heated square cavity for nanofluids containing Carbon nanotubes for nanofluids.

Natural convection in a square cavity and its fluid flow is a classical benchmark in heat transfer problems. Open cavities are a kind of 2-D cavity which has an open side. These kinds of cavities have special physics for both flow and temperature fields in open side because of outgoing of flow from this side. Many studies have been done on analysis of buoyant flows and their heat transfer in open cavities [21–28]. Javam and Armfield [21] investigated stability of stratified natural convection flow in open cavities. Mohammad et al. [22] presented natural convection in an open ended cavity and slots they analyzed the

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>( c )</th>
<th>discrete lattice velocity</th>
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<tr>
<td>( \text{Cu} )</td>
<td>copper</td>
<td></td>
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<tr>
<td>( c_s )</td>
<td>speed of sound in Lattice scale</td>
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<tr>
<td>( c_p )</td>
<td>specific heat at constant pressure (kj kg(^{-1}) k(^{-1}))</td>
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<td>( F )</td>
<td>external force in direction of lattice velocity</td>
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<td>( f, g )</td>
<td>distribution function</td>
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<tr>
<td>( g_s )</td>
<td>acceleration due to gravity (m s(^{-2}))</td>
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<tr>
<td>( H )</td>
<td>height of cavity (m)</td>
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<tr>
<td>( k )</td>
<td>thermal conductivity (W m(^{-1}) k(^{-1}))</td>
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<tr>
<td>( Ma )</td>
<td>mach number</td>
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<td>( Nu )</td>
<td>Nusselt number</td>
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<tr>
<td>( Pr )</td>
<td>Prandtl number ((\nu/\alpha))</td>
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<td>( Ra )</td>
<td>Rayleigh number ((g\beta(T_w - T_c)L^3/\nu\alpha))</td>
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<td>( \text{SWCNT} )</td>
<td>Single Walled Carbon Nanotube</td>
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<td>( T )</td>
<td>dimensional temperature (K)</td>
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<tr>
<td>( W )</td>
<td>width of cavity (m)</td>
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<td>( X, Y )</td>
<td>dimensionless coordinate</td>
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<tr>
<th>Greek symbols</th>
<th>( \beta )</th>
<th>thermal expansion coefficient (1 k(^{-1}))</th>
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<tr>
<td>( \theta )</td>
<td>dimensionless temperature</td>
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<tr>
<td>( \Delta t )</td>
<td>lattice time step</td>
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<tr>
<td>( \rho )</td>
<td>lattice fluid density</td>
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<td>( \tau )</td>
<td>lattice relaxation time</td>
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<tr>
<td>( \varphi )</td>
<td>solid volume fraction</td>
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<tr>
<td>( \psi )</td>
<td>dimensionless stream-function ((\psi = \tilde{\psi}/W_u_0))</td>
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<td>( w )</td>
<td>weighting factor</td>
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<th>Subscripts</th>
<th>( \text{ave} )</th>
<th>average</th>
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<td>( c )</td>
<td>cold</td>
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<td>solid particle</td>
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<td>( (-) )</td>
<td>dimensional parameters</td>
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effect of aspect ratio of cavity on heat transfer rate. They also presented a good procedure for simulating open boundaries in Lattice Boltzmann Method. Their technique demonstrated the ability of the LBM to simulate Natural convection in open cavities. In a numerical study via Lattice Boltzmann Method, natural convection in an open enclosure in the presence of water/Cu nanofluid has been investigated by Kefayati et al. [23]. Calculations were performed for different Rayleigh numbers \((10^2-10^6)\) and volume fractions of nanoparticles (1–5%). Their results show that the average Nusselt number increases with augmentation of Rayleigh number and the volume fraction of nanoparticles. They reported an enhancement of natural convection heat transfer about 3.98%, 7.30%, 11.62%, 14.27% and 18.42% by adding a \(\phi = 1\%\), 2%, 3%, 4% and 5% of Cu nanoparticles at \(Ra = 10^4\). Also when Rayleigh number is equal to \(10^5\), and other conditions are same, these values are about 2.97%, 5.73%, 8.71%, 11.26%, and 13.60%.

The convection heat transfer of different nanoparticle-based nanofluids in a cavity has been mentioned in current years [29–32]. Fattahi et al. [29] applied Lattice Boltzmann Method to study natural convection heat transfer of \(Al_2O_3\) and Cu nanofluid in a cavity. Their results show that increasing the solid volume fraction led to a heat transfer enhancement at any Rayleigh number and also heat transfer increases with increase in Rayleigh number for a particular volume fraction. Nemati et al. [30] applied LBM to investigate the effect of Cu, \(Al_2O_3\), and \(CuO\) nanoparticles on mixed convection in a lid-driven cavity. Their results show that adding nanoparticles increase the rate of mixed convection heat transfer of the base fluid for all tested Reynolds numbers. Abu-Nada and Chamkha [31] conducted to study of mixed convection heat transfer of Water–\(Al_2O_3\) nanofluid in an inclined cavity. Heat transfer enhancement due to increase of nanoparticles volume fraction at different Richardson and Grashof numbers was presented in their results. Their results show that \(Nu_{ave}\) on hot wall enhances about 3.3%, 8.4%, 11.9% and 17.2% respectively for \(\phi = 2\%\), 5%, 7% and 10% at \(Ri = 0.2\) and \(Gr = 10^4\) when the inclination angle of the cavity is equal to zero and also these values are 3%, 8.5%, 12% and 17% for \(Ri = 2\).

The investigation of the heat transfer enhancement by adding the Single Walled Carbon Nanotubes (SWCNTs) to the base fluid in an open-ended cavity is the main aim of the present study. Also a comparison between SWCNTs and Cu-nanoparticles is one of the major tasks of the study. In this simulation the effective conductivity and viscosity were calculated based on the new theoretical models. The presented model by Masoumi et al. [33] is applied for effective viscosity of the Cu and SWCNT-nanofluid. To simulate thermal conductivity of SWCNT-nanofluid a new theoretical model presented by Sabbaghzadeh and Ebrahimi [34] is used. This new thermal conductivity model captures the effects of different phenomena of energy transport in nanofluids. At molecular point of view, some of the most important incidents in energy transport are: Collision among the base fluid molecules, thermal diffusion of nanoparticles, thermal diffusion in the nanolayer in the fluid, thermal interaction of dynamic complex nanoparticles with the base fluid molecules and finally collision between nanoparticles caused by Brownian motion. On the other hand, the study applies the Patel model [35] to evaluate the effective thermal conductivity of Cu-nanofluid.

By considering the extraordinary thermal conductivity and low density of Carbon nanotubes in comparison with other nanoparticles and also using a good expansion of effective thermal conductivity of nanofluids which covers molecular phenomena of energy transport of nanofluids, it is predicted that a great heat transfer enhancement can be achieved in this study. This enhancement can be assumed as a very important result for many industrial applications which are concern about the convection rate. Therefore the importance of new studies about the ways of heat transfer enhancement is still obvious.

The progress of using the Lattice Boltzmann Method (LBM) as a numerical technique to simulate the heat transfer and fluid flow has been obvious in the last decade [36–41]. The Lattice Boltzmann Method has well-known advantages such easy implementation, possibility of parallel coding and simulating of complex geometries and fluid dynamic problems such as melting, fuel cell, porous media, and nanofluids. To the best of author knowledge, the effects of Carbon nanotubes on flow and thermal fields of natural convection are an unknown perspective which is not understand heretofore. Also, a lack of using Lattice Boltzmann method (LBM) as a suitable numerical method to study of convection heat transfer of nanofluids containing Carbon nanotubes is sensible. Therefore, in this study the used numerical method is LBM with coupled double population approach for flow and temperature fields. Also the Bounce Back method is applied in LBM to model boundary condition in solid boundaries which supply second order accuracy in both flow and temperature fields. The effect of the volume fraction of the Cu and SWCNT-particles on average Nusselt number, streamlines and temperature contours is investigated for various values of Rayleigh number. Also the aspect ratio of the cavity is studied as an important geometric parameter of the 2D cavities in different conditions to exhibit the role of this parameter on both heat transfer rate and fluid flow of the base fluid and the nanofluid.

2. Lattice Boltzmann Method for flow and thermal fields

The LBM utilizes two distribution functions, for the flow and temperature fields. It uses modeling of movement of fluid particles to define macroscopic parameters of fluid flow. The basic form of LBM applies uniform Cartesian cells to discrete problem domain. Each cell of the grid has a constant number of distribution functions, which represent the number of fluid

![Figure 1](image.png)
particles movement in these separated directions. The distribution functions are obtained by solving the Lattice Boltzmann Equation (LBE), which is a special form of the Kinetic Boltzmann Equation.

Lattice Boltzmann Method can be operated on a number of different lattices, both cubic and triangular, and with or without rest particles in the discrete distribution function. A well known system of classifying the different methods by lattice is the “DnQm” scheme. Here “Dn” stands for “n dimensions” while “Qm” stands for “m speeds”. The D2Q9 model (see Fig. 1) is used in the current study which is a two-dimensional Lattice Boltzmann model on a square grid, with rest particles present. For a two dimensional model, a particle is restricted to stream in a possible of 9 directions, including the one staying at rest.

The basic form of the Lattice Boltzmann Equation with an external force by introducing BGK approximations can be written as follows for both the flow and the temperature fields [36]:

\[ f_s(x + c_s \Delta t, t + \Delta t) = f_s(x, t) + \frac{\Delta \tau_m}{\tau_m} [f_s^e(x, t) - f_s(x, t)] + \Delta c_s F_s \]  

\[ g_s(x + c_s \Delta t, t + \Delta t) = g_s(x, t) + \frac{\Delta \tau_s}{\tau_s} [g_s^e(x, t) - g_s(x, t)] \]  

where \( f_s(x, t) \) is a distribution function on the mesoscopic scale. Here \( \Delta c \) is a flowing speed. Also, \( \Delta x \) and \( \Delta t \) are the lattice cell size and the lattice time step size, respectively. At these equations (Eqs. (2) and (3)), \( c_s \) is the discrete lattice velocity in direction \( x \) and \( F_s \) is the external force term in the direction of discrete velocity. Also, \( \omega_s \) is a weighting factor and \( \rho \) is the lattice fluid density. \( \tau_m \) and \( \tau_s \) are the dimensionless collision-relaxation times for the flow and temperature fields, respectively. They defined as follows [42]:

\[ \tau_m = 0.5 + \frac{Ma \cdot W \sqrt{3Pr}}{c^2 \Delta \sqrt{Ra}} \]
\[ \tau_s = 0.5 + \frac{3v}{Pr c^2 \Delta t} \]  

Characteristic velocity for both natural \( (V_{natural} = \sqrt{\beta f \Delta \bar{H}}) \) and force \( (V_{force} = \nu Re/W) \) regimes must be small in comparison with the fluid speed of sound. By considering D2Q9 model for applied lattice scheme for both flow and temperature fields, equilibrium distribution functions for flow field \( (f_s^e) \) and temperature field \( (g_s^e) \) are calculated as follows in different \( x \) directions:

\[ f_s^e = \omega_s \rho \left[ 1 + 3(e_x \cdot \bar{u}) + \frac{9}{2}(e_x \cdot \bar{u})^2 - \frac{3}{2} \bar{u}^2 \right] \]

\[ g_s^e = -\omega_s \rho RT \left[ \frac{3}{2} \bar{u}^2 \right] \]

\[ g_s^e = \omega_s \rho RT \left[ \frac{3}{2} + \frac{9}{2}(e_x \cdot \bar{u}) + \frac{9}{2}(e_x \cdot \bar{u})^2 - \frac{3}{2} \bar{u}^2 \right] \]  for \( x = 1 - 4 \)

\[ g_s^e = \omega_s \rho RT \left[ \frac{3}{2} + \frac{9}{2}(e_x \cdot \bar{u}) + \frac{9}{2}(e_x \cdot \bar{u})^2 - \frac{3}{2} \bar{u}^2 \right] \]  for \( x = 5 - 8 \)  

The D2Q9 lattice is a Cartesian 2D lattice with nine velocity directions. For this model, the values of \( e_s \) and \( w_s \) for various \( x \) directions will be, respectively:

\[ e_s = \begin{cases} (0,0) & z = 0 \\ c(\cos \theta_s + \sin \theta_s), & z = 1,2,3,4 \\ \sqrt{2}c(\cos \theta_s + \sin \theta_s), & z = 5,6,7,8 \end{cases} \]

\[ w_s = \begin{cases} 4/9 & x = 0 \\ 1/9 & x = 1,2,3,4 \\ 1/36 & x = 5,6,7,8 \end{cases} \]  

In order to incorporate external force in collision part of Lattice Boltzmann model (Eq. (1)), radiation heat transfer and viscous dissipation are neglected at the numerical simulation. Therefore, to capture buoyancy force effects in the flow field, the Boussinesq approximation is applied. Thus, to model buoyancy force in Eq. (1), the external force needs to be assumed as below in the needed direction:

\[ F_s = 3w_s \beta_g(T - T_{ref}) \]

Finally, macroscopic variables can be calculated as follows:

Flow density : \( \rho = \sum_s f_s \), Momentum :

\[ \rho u = \sum_s f_s c_s, \]  

Temperature : \( T = \sum_s g_s \)  

3. Nanofluids modeling

In the present study, nanofluid is assumed as a single phase fluid. The added nanoparticles to the base fluid changes thermal conductivity, viscosity and other basic characteristic of the base fluid, thus innovated Carbon-based nanofluid has special characteristic as a combination of the pure fluid and Carbon nanotubes. Thermal diffusivity of nanofluid is as follows:

\[ \alpha_{nf} = \frac{\kappa}{(\rho c_p)_{nf}} \]  

The density, heat capacitance and thermal expansion of nanofluid can be defined as [43]:

\[ \rho_{nf} = (1 - \varphi)\rho_f + \varphi(\rho_{p})_{nf} \]

\[ \alpha_{nf} = (1 - \varphi)(\alpha_{f})_f + \varphi(\alpha_{p})_p, \beta_{nf} = (1 - \varphi)\beta_f + \varphi\beta_p \]  

3.1. Effective viscosity of nanofluid

The effects of nanoparticles on the viscosity of nanofluids are introduced by the so-called apparent viscosity which is presented by \( \mu_{app} \). The numerical procedure to simulate the nanofluid viscosity is same as those presented by Masoumi et al. [33]. According to this analytical model of nanofluid viscosity, the effective viscosity can be defined as follows:

\[ \mu_{nf} = \mu_{nf} + \mu_{app} = \mu_{nf} + \frac{\rho_{np} V_{Brownian} d_{np}}{72Cd} \]  

As presented, Masoumi et al. [33] considered Brownian motion as an important parameter that creates a relative velocity between the nanoparticle and the base fluid in nanofluids [33]. \( V_B \) is the Brownian velocity and the corresponding Reynolds number is defined as follows [44]:

\[ Re = \frac{V_B d_{np}}{v} \]
\[ V_{Brownian} = \frac{1}{d_{op}} \sqrt{\frac{18K_b T}{\pi \rho_d d_{op}}} \]  
\[ \text{Re}_{Brownian} = \frac{1}{\nu_{op}} \sqrt{\frac{18K_b T}{\pi \rho_d d_{op}}} \]  
(13)  
(14)

While, \( K_b \) is the Boltzmann Constant (\( \approx 1.38 \times 10^{-23} \text{ J} \cdot \text{K}^{-1} \)). Also, \( \delta \) is the approximately distance between the centers of particles and it will be obtained from [33]:

\[ \delta = \frac{1}{\sqrt{6\pi}} d_{op} \]  
(15)

Now the correction factor, \( C \), must be determined.

\[ C = \mu_{d}^{2/3} [(c_1 d_{op} + c_2)\varphi + (c_3 d_{op} + c_4)] \]  
(16)

Using the experimental data associated with the nanofluid viscosity \( c_1, c_2, c_3 \) and \( c_4 \) can be determined. For Cu-nanoparticles the values of these constants are assumed as follows [33],

\[ c_1 = -0.000001133, \quad c_2 = -0.000002771, \quad c_3 = 0.000000099, \quad c_4 = -0.000000393 \]  
(17)

Also, according to the experimental data about the viscosity of Carbon nanotube based nanofluids presented by Chen et al. [45] the values of c-constants are estimated as follows:

\[ c_1 = -0.000000896, \quad c_2 = -0.000002054, \quad c_3 = 0.000000738, \quad c_4 = -0.000000306 \]  
(18)

3.2. Thermal conductivity of nanofluid

The applied effective thermal conductivity of Carbon nanotubes-based nanofluid is same as those proposed by Sabbaghzadeh and Ebrahimi [34]. This theoretical model contains five phenomena of energy transport in nanofluids: collision among the base fluid molecules; thermal diffusion of nanoparticles, thermal diffusion in the nanolayer in the fluid; thermal interaction of dynamic complex nanoparticles (nanoparticles and nanolayer) with the base fluid molecules and finally collision between nanoparticles due to Brownian motion. Their theoretical model can be expressed as [34]:

\[ K_{eff} = K_b[(1 - \varphi M^2) + \varphi (\kappa K_{op} + K_w(M^2 - 1)) + \varphi K_b M^2] \]

\[ \times \frac{df}{D} \left( 0.35 + 0.56 \text{Re}_{eff}^{0.22} \right) P_r_{eff}^{0.3} \]  
(19)

where \( \kappa, D \) and \( M \) are respectively thermal Kapitza resistance, diameter of nanoparticles complex and ratio of radius complex nanoparticle to nanoparticle which is defined as follows:

\[ M = \frac{r_{at} + r_{up}}{r_{op}} \]  
(20)

The term of \( K_{eff} \) in Eq. (19) indicates the thermal conductivity of nanolayer which has a value equal to:

\[ K_{eff} = \frac{K_b r_{op} \left( \frac{M_{bf}}{K_b} - 1 \right) \ln(M)}{d_{op} \ln \left( \frac{M_{bf}}{K_b} \right)} \]  
(21)

The relation is valid for \( 10^{-1} < \text{Re}_{eff} < 10^{5} \), The Reynolds number, \( \text{Re}_{eff} \) is defined as follows:

\[ \text{Re}_{eff} = \frac{u^* L}{v_{bf}} \]  
(22)

\( u^* \) is the mean velocity of the complex nanoparticles and \( L \) is the specific length of the cylindrical shape that can be calculated as follows:

\[ L = \sqrt{\frac{\pi r_{np}^2 M^2}{\varphi}} \]  
(23)

where \( l \) is the cylinder length, from the molecular theory of heat, \( \bar{u} \) is determined as follows [46]:

\[ \bar{u} = \sqrt{\frac{3K_b T}{m_o}} \]  
(24)

As cited, \( K_b \) is the Boltzmann Constant, \( m_o \) is the mass of the complex nanoparticle which can be calculated as follows:

\[ m_o = \pi r_{np}^2 \rho_{np} \left( \frac{\rho_{bf}}{\rho_{np}} (M^2 - 1) + 1 \right) \]  
(25)

Sabbaghzadeh and Ebrahimi [34] presented more details of thermal conductivity calculation in the nanofluids based on Carbon nanotube existence in convective fluids in their published research paper. The effective thermal conductivity of Cu–Water nanofluid is calculated using the presented model by Patel et al. [35] which is as follows:

\[ k_{bf} = 1 + \frac{K_b A_s}{k_{bf} A_f} + c_k \frac{P_e}{k_{bf} A_f} \]  
(26)

where

\[ A_s = \frac{d_s}{d_f} \frac{\varphi}{1 - \varphi}, \quad P_e = \frac{u_s d_s}{k_{bf}}, \quad u_s = \frac{2K_b T}{\pi \mu d_s^2} \]  
(27)

where \( u_s \) is the Brownian motion term of nanoparticle velocity and \( K_b \) is the Boltzmann constant.

4. Main problem

4.1. Problem description

In the present study, the effects of Single Walled Carbon Nanotubes (SWCNT) and Cu-nanoparticles on natural convection heat transfer in a 2D deep open cavity are investigated using the Lattice Boltzmann Method based on double population approach and bounce back method. The horizontal walls of the cavity are assumed to be insulated while the left wall is...
maintained at a uniform temperature (\(T_s\)) differentially higher than the open end side temperature (\(T_e\)). The considered physical system is presented in Fig. 2. The effect of SWCNT/Cu-nanoparticles on both fluid flow and heat transfer is investigated for different values of Rayleigh number at the range of \(10^3 \leq Ra \leq 10^5\) while the Prandtl number of based fluid (Water) at reference temperature (22 °C) is 6.57.

It is assumed that the both nanotubes and nanoparticles are in constant form with uniform diameter which are at thermal equilibrium state with base fluid and also the nanotubes and nanoparticles have same velocity with the base fluid thus there is no slip velocity between added nanotubes/nanoparticles and molecules of the base fluid. Thermo-physical properties of SWCNT-particles, Cu-nanoparticles and Water at reference temperature are tabulated in Table 1. The dimensionless quantities are as follows:

\[
\begin{align*}
x &= \frac{\bar{x}}{H}, & y &= \frac{\bar{y}}{H}, & u &= \frac{\bar{u}}{u_0}, & v &= \frac{\bar{v}}{u_0}, & \theta &= \frac{\bar{\theta}}{0} \\
\text{Pr} &= \frac{\nu \rho C_p}{K}, & \text{Ra} &= \frac{g \beta \Delta \theta W^3}{\nu^2}
\end{align*}
\]

(28)

The boundary conditions on the solid walls are in the following forms:

\[
\begin{align*}
u &= \text{for } x = 0, 0 \leq y \leq 1 \\
u &= \text{for } 0 < x < W, y = 0 \\
u &= \text{for } 0 < x < W, y = 1
\end{align*}
\]

(29)

The boundary conditions on the east open side \((x = W; 0 \leq y \leq 1)\) are in the following forms:

\[
\begin{align*}
u &= \text{for } x = 0, 0 \leq y \leq 1 \\
u &= \text{if } u > 0 \\
u &= \text{if } u < 0
\end{align*}
\]

(30)

The boundary conditions in the LBM can be implemented through the distribution function. For both flow and thermal fields, the distribution functions out of the domain are known from the streaming process. The unknown distribution functions are those toward the domain. Bounce back boundary condition is used on the straight solid boundaries (no-slip velocity wall) which have adiabatic thermal conditions, the unknown distribution function in needed direction are calculated as follows:

\[f_x = \tilde{f}_x(\text{flow field})], \quad g_x = \tilde{g}_x(\text{thermal field})\]

(31)

Which \(\bar{z}\) indicates needed direction and (–) shows opposite direction. The (~) symbol presents the known distribution function which obtained after the streaming and collision steps. At the west wall, for the flow field the Bounce Back is applied \((f_x = \tilde{f}_x)\) when the thermal field uses the following boundary condition for constant temperature boundary conditions at this wall:

\[
g_x = \begin{cases} 
(\omega_x + \omega_y)\theta_x - g_x & \text{if } u > 0 \\
(\omega_x + \omega_y)\theta_x - g_x & \text{if } u < 0 \\
\end{cases}
\]

(32)

At the outlet, the used boundary condition is same as those presented by Mohamad et al. [22]. At this side, the temperature boundary condition is applied according to the axial velocity component:

\[
g_x = \begin{cases} 
\frac{1}{3}[4\theta_x(n-1) - \theta_x(n-2)] & \text{if } u > 0 \\
(\omega_x + \omega_y)\theta_x - g_x(n) & \text{if } u < 0 \\
(\omega_x + \omega_y)\theta_x - g_x(n) & \text{if } u < 0 \\
\end{cases}
\]

(33)

where \(n\) is the lattice on the boundary, \((n-1)\) and \((n-2)\) are the lattices inside the enclosure adjacent to the open-side.

The local Nusselt number is defined along the hot bottom wall as follows:

\[Nu = \left| \frac{\frac{\partial \theta}{\partial x}}{H} \right| \]

(34)

where \(H\) is the height of the cavity. By integrating the local Nusselt number along the \(Y\) direction, the average Nusselt number is calculated as follows:

\[Nu_{\text{avg}} = \frac{1}{H} \int_X^H Nudy\]

(35)

4.2. Code validation, solution convergence and mesh independent treatment

The numerical code is validated by the results presented by Mohammad et al. [22] for natural convection in an open cavity at \(Ra = 10^5\); \(Pr = 0.71\) and \(A = 1\) (Fig. 3). Also for more consistency, Table 2 shows the comparison of the values of average Nusselt number of this study and those presented by Mohamad et al. [22] which shows a good agreement. The agreement of the result with the pervious results confirms the accuracy of numerical code to simulate the flow and temperature fields of convection heat transfer.

A grid independency treatment is used to select an adequate number of nodes in the mesh grid. This treatment is presented in Fig. 4 for a special case study. It is visible that a grid with 100 nodes in each direction can satisfy the grid independency treatment when aspect ratio of the cavity is equal to unity. Therefore, in used numerical code to model various aspect ratios, the lattices inside the enclosure adjacent to the open-side.

To get a steady state result in the numerical solution, the results are compared at different time steps. This algorithm was done for all case studies to get convergent results by applying an error function as presented in the following equation in the used numerical code:

\[Err = \left( \frac{\Psi^n_{ij} - \Psi^{n-1}_{ij}}{\text{number of mesh nodes}} \right) \leq 10^{-5}\]

(36)

where \(\Psi^n_{ij}\) stands for a variable of flow or temperature fields such as temperature or velocity component in each node of the grid at \(n_{th}\) iteration. Steady state solution was declared when the relative change in the variables of flow and tempera-
ture fields between two consecutive iterations at through solution domain is satisfied by the presented criterion. The Schematic of applied numerical convergence treatment to get convergent condition is presented in Fig. 5 for an arbitrary point in problem domain in a special case study. The values of CPU-time of Calculations are tabulated in Table 3 at different geometrical and physical conditions. The presented data in this table explain how aspect ratio and Rayleigh number can change value of numerical code’s run-time.

5. Result and discussion

At pervious sections, the numerical simulation of natural convection heat transfer in an open ended enclosure has been
explained in details. The target problem is investigated for three types of convectional fluids which are named as Water, Water–Cu and Water-SWCNT nanofluids. The volume concentration of added nanoparticles to Water, as base fluid of Water/Cu and Water/SWCNT nanofluids, is one of the important parameters which must be studied to clear the role of these nanoparticles on both thermal and fluid fields of the base fluid. Besides studying of the volume fraction of nanoparticles (0 ≤ φ ≤ 1), different values of aspect ratio of the cavity (1 ≤ A ≤ 4) and Rayleigh number (10^3 ≤ Ra ≤ 10^5) are assumed as important geometrical and physical parameters at the main problem.

The enhancement of the heat transfer of the nanofluids may be caused by the suspended nanoparticles increasing the thermal conductivity of fluids, and the chaotic movement of ultrafine particles increases fluctuation and turbulence of the fluids, accelerating the energy exchange process. Therefore, current study tries to make a comparison between SWCNT and Cu nanoparticles to answer this question that “which kind of these nanoparticles can present a better heat transfer enhancement”. At following subsection, the effects of the nanoparticles on thermal and fluid fields at the problem domain are discussed. Then the values of average Nusselt number are presented to clarify the role of the nanoparticles on the convection rate.

5.1. Nanoparticles effects

The presence of nanotube particles with high thermal conductivity in suspensions has different effects on temperature and flow fields. The effect of added nanotubes and Cu nanoparticles on flow and temperature fields of the Water (the base fluid) is presented in Fig. 6 for the cavity with A = 1 at different values of Rayleigh number. In the present study to be sure that nanofluids behave completely same as a single phase fluid, the volume fraction value of added nanotubes is selected in a range less than 1% to make a dilute suspension, therefore it can be acceptable that the overall shape of the contours is same for both pure fluid and nanofluid with added nanotube particles. Also, the maximum value of stream-function in the flow field is presented in this figure for Water, Water–Cu and Water-SWCNT nanofluids at different case studies. The presented values for maximum stream-function show that adding the 1% volume fraction of SWCNT-nanoparticles increases the maximum value of stream-function about 57%, 59% and 75% respectively for Ra = 10^3, 10^4 and 10^5 when the Cu nanoparticles change the value of maximum stream function about 10%, 11% and 14% respectively for Ra = 10^3, 10^4 and 10^5. The increase of stream-function values shows that the nanoparticles have more effect on fluid flow at a high Rayleigh number. This result can be due to the importance of the fluid density at a high Rayleigh number when natural convection is completely dominant.

Although, the Cu nanoparticles have higher density, the results show that SWCNT-nanoparticles have more positive effects on fluid flow of the base fluid. To explore this phenomenon, it is important to notice that the extra ordinary thermal conductivity can play an important role on flow and thermal fields of convectional flows when the dominant regime is natural convection. In natural convection regime, there is a double-way couple between flow and thermal field. This double-way connection causes that the role of thermal conductivity be more important in comparison with the forced connection. At the forced convection the connection between flow and thermal field is a one-way connection which means that just flow fields make changes in thermal field and the thermal field has no effect on the flow field. For more exploration, the variation pattern of velocity components of SWCNT-nanofluid at horizontal mid-section of the cavity is plotted in Fig. 7 for different values of Rayleigh

| Table 3 CPU-time of calculations at different case studies. |
|---------------------|-----------------|-----------------|
| Aspect ratio       | Rayleigh number | Rayleigh number |
|                    | Ra = 10^3       | Ra = 10^4       | Ra = 10^5       |
| A = 1              | 4957            | 5177            | 5608            |
| A = 2              | 10,583          | 11,006          | 11,573          |
| A = 3              | 16,061          | 16,285          | 16,451          |
| A = 4              | 21,810          | 21,374          | 20,092          |
Figure 6  Streamlines and temperature contours of the base fluid, SWCNT and Cu-nanofluid (\(\varphi = 1\%\)) at \(A = 1\). (a) \(Ra = 10^3\) (b) \(Ra = 10^4\) (c) \(Ra = 10^5\).
number and volume fraction of the added nanotubes when the aspect ratio is equal to unity. It is visible that adding nanoparticles to the base fluid has more obvious effects at a higher Rayleigh number.

Investigation of the role of SWCNT-nanoparticles on convection rate in an open enclosure is main aim of the present study. Therefore, the values of average Nusselt number ($Nu_{ave}$) aspect to the volume fraction of SWCNT and Cu nanoparticles are presented in Fig. 8 for different values of Rayleigh number. Also, some correlations which show the relation between average Nusselt number and volume fraction of nanoparticles is presented in this figure. It is found that the presence of a low value of Carbon nanotubes obviously enhances the heat transfer rate. According to the presented results, adding the 0.6% volume fraction of Cu-particles enhances the average Nusselt number about 13.3%, 13.8% and 14% at $Ra = 10^3$, $Ra = 10^4$ and $Ra = 10^5$, respectively, and these values are about 54%, 56.6% and 56.7% for the SWCNT-particles with same volume fraction when the cavity aspect ratio is equal to unity. Improvement of thermal conductivity of the base fluid and increasing of flow momentum are some of the advantages of nanoparticles presence in the base fluid. These increases in flow momentum and thermal conductivity lead to heat transfer enhancement of the base fluid. As presented before, the volume fraction value of added single-walled Carbon nanotubes is less than 1% that means significant effects on convection heat transfer and its flow are observed by adding a low value of added nanotubes which is due to extraordinary thermal conductivity and other properties of the Carbon nanotubes. The thermo-physical properties of the added nanoparticles specify the magnitudes of heat transfer enhancement of the base fluid. The values of average Nusselt number increase about 22% and 92%, respectively, by adding the 1% volume fraction of Cu and SWCNT-nanoparticles when $A = 1$ and $Ra = 10^5$.
Figure 9  Streamlines and temperature contours of the base fluid and SWCNT-nanofluid ($\phi = 1\%$) at $Ra = 10^7$. (a) $A = 2$ (b) $A = 3$ (c) $A = 4$.  

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$Ra = 10^5$ are 24% and 98%. It is observed that SWCNT-nanoparticles make better enhancement in heat transfer rate in comparison with Cu-nanoparticles. The main reason of this matter is higher thermal conductivity of SWCNT nanoparticles. Although, higher density of added nanoparticles can make a better enhancement of flow strength, the couple of flow and thermal fields cause that the thermal conductivity has a powerful role on flow strength in the natural convection. Therefore, the SWCNT-nanoparticles have more effects on values of stream function even with lower density in comparison with Cu-nanoparticles.

5.2. Aspect ratio effects

At studying of convection problems in 2D cavities, aspect ratio of a cavity must be mentioned as characteristic geometrical

![Figure 10](image1.png)

**Figure 10** Pattern of u and v-velocity components of SWCNT-nanofluid ($\phi = 1\%$) in horizontal mid-section of the cavity with different aspect ratios when $Ra = 10^5$.

![Figure 11](image2.png)

**Figure 11** The effects of aspect ratio on average Nusselt number at different Rayleigh numbers. (a) Water (b) Cu-Water ($\phi = 0.2\%$) (c) SWCNT-Water ($\phi = 0.2\%$).
parameter of the cavity. In this section, the aspect ratio is studied in different graphs and contours. In this way, the streamlines and temperature contours of the base fluid and SWCNT-nanofluid ($\phi = 1\%$) in the cavity with different values of aspect ratio are presented in Fig. 9 when $Ra = 10^5$. The overall view of this figure shows that the increase of cavity’s aspect ratio leads to reduce of flow strength. The distance between hot and cold side in an enclosure is one of most important parameter of natural convection regime, the increase of this distance changes the natural convection regime to a conduction-like manner in the problem domain.

To observe of the pattern of velocity components in mid-section of the cavity with different aspect ratios, some graphs which show the velocity components along the cavity’s length are illustrated in Fig. 10.

The effect of aspect ratio on flow structure and heat transfer rate in the problem domain is sensible at these figures. For more clarification of the leading role of the aspect ratio on heat transfer as the target outcome, the values of $Nu_{ave}$ at different values of Rayleigh number are plotted in Fig. 11 for cavities with various aspect ratios. Also some practical correlations

Figure 12   The Nusselt number distribution along the hot wall of the cavity with different aspect ratios for the base fluid and SWCNT-nanofluid ($\phi = 1\%$). (a) $Ra = 10^3$ (b) $Ra = 10^4$ (c) $Ra = 10^5$.

Figure 13   Average Nusselt number at different aspect ratios and volume fractions of nanoparticles at $Ra = 10^5$. (a) Cu-nanoparticles (b) SWCNTs.
are added to this figure. Paying an attention to the presented results shows that by increasing of the aspect ratio, the rate of natural convection heat transfer reduces for all studied values of Rayleigh number and fluids (see Fig. 11).

To delineate the details of heat transfer reduction by increasing of the cavity’s aspect ratio, the Nusselt number distribution along the left hot wall is presented in Fig. 12 for different physical conditions. Changing of aspect ratio affects on heat transfer rate especially for a high Rayleigh number. Decreasing of heat transfer by increasing of the aspect ratio is due to increase of the distance between hot wall and open side. Increase of hot and cold walls distance leads to decrease of both convection and conduction heat transfer in the problem domain. At high aspect ratios, the heat transfer pattern is likely to be conductive that is because of the big gap existence between hot and cold walls therefore it is found that Rayleigh number loses its effects on heat transfer rate at high aspect ratio of the cavity.

Finally, for more observation about the effects of nanoparticles effects on convection rate at different physical conditions, the values of $N_u_{\text{avg}}$, aspect to the volume fraction of $\text{Cu}$ and SWCNT nanoparticles, participated with needed correlations, are plotted in Fig. 13 at different aspect ratios. Taking an overview to the plotted data and especially the presented correlations emphasize that the nanoparticles have better performance at the cavity with lower aspect ratio.

6. Conclusion

The effect of Single Walled Carbon Nanotubes and Cu nanoparticles on laminar natural convection heat transfer in an open-end enclosure was studied numerically. The double population approach in lattice Boltzmann Method was used at this study and the Bounce Back Method with second order accuracy in both flow and temperature field is applied to model solid boundaries. The problem was investigated at different aspect ratios of the cavity ($1 \leq A \leq 4$) and the volume fractions of SWCNT and Cu-nanoparticles ($0 \leq \varphi \leq 1$) when Rayleigh number varies from $10^3$ to $10^5$. Some of most important results that have been achieved in this study are as follows:

- Single Walled Carbon Nanotubes and Cu-nanoparticles can enhance both flow strength (stream-function) and convection rate (Nusselt number) in an open-end enclosure.
- Results show that adding a low value of SWCNT to the base fluid led to significant enhancement of convection heat transfer.
- The heat transfer rate is at closed relation with thermal conductivity of the suspensions therefore the use of nanoparticles with better thermal conductivity leads to better heat transfer enhancement at base fluid.
- Make a comparison between SWCNT and Cu-nanoparticles shows that the SWCNT-nanoparticle has better performance to enhance convection rate.
- The aspect ratio of the cavity plays an important role on natural convection heat transfer. An increase of this parameter leads to heat transfer reduction in a 2D deep open cavity.
- Rayleigh number loses its importance on natural convection in the high aspect ratios.

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