Collective behavior in Cascade and Schelling model

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Abstract

There are growing interests for studying collective behavior including the dynamics of markets, the emergence of social norms and conventions, and collective phenomena in daily life such as traffic congestion. In this paper, we focus on scale-free network and investigate the effect of number of interaction on collective behavior. In this paper, we found that collective behavior is affected in the structure of the social network and theta and the collective behavior was stochastic. Moreover, collective behavior is almost same as Schelling model, though the decision is not interactive and simultaneously. Then, we found that the collective behavior in Schelling model is similar to cascade model. That is, our results with heterogeneous rules or heterogeneous networks are possible to apply for cascade model.

Keywords: Collective behavior; Agent; Social network; Degree

1. Introduction

There are growing interests for studying collective behavior including the dynamics of markets, the emergence of social norms and conventions, and collective phenomena in daily life such as traffic congestion. Many researchers have pointed out that an equilibrium analysis does not resolve the question of how individuals behave in a particular interdependent decision situation. It is often argued "it is hard to see what can advance the discussion short of assembling a collection of individual, putting them in the situation of interest, and observing what they do"\textsuperscript{1}.

In examining collective behavior, we shall draw heavily on the interactions of individuals. We also need to work on two different levels: the microscopic level, where the decisions of the individual agents occur, and the

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macroscopic level where collective behavior can be observed\textsuperscript{2}. The greatest promise lies in analysis of linking microscopic behavior to macroscopic behavior\textsuperscript{3}. What makes collective behavior interesting and difficult is that the entire aggregate outcome is what has to be evaluated, not merely how each person does within the constraints of her own environment. The performance of the collective system depends crucially on the type of interaction as well as the heterogeneity in preference of agents\textsuperscript{4}.

Feng et al.\textsuperscript{5} brings together agent-based models and stochastic models of complex systems in financial markets and show how individual decisions give rise to macroscopic actions. Additionally, the heterogeneity in agents’ investment horizons gives rise to long-term memory in volatility. Using market data, Kenett et al.\textsuperscript{6} provides new information about the uniformity preset in the world’s economies. From their analysis, it becomes evident that this uniformity does not only stem from an increase of correlation between markets, but that there has also been an ongoing simultaneous shift towards uniformity in each single market.

There are many situations where interacting agents can benefit from coordinating their behavior. Coordination usually implies that increased effort by some agents leads the remaining agents to follow suit, which gives rise to multiplier effects. Examples where coordination is important include trade alliance, the choice of compatible technologies or conventions such as the choice of a software or language. These situations can be modeled as coordination games in which agents are expected to select the strategy the majority does\textsuperscript{7}. The traditional game theory, however, is silent on how agents know which equilibrium should be realized if a coordination game has multiple equally plausible equilibria, where these can be Pareto ranked\textsuperscript{8}. This silence is all the more surprising in games with common interest since one expects that agents will coordinate on the Pareto dominant equilibrium\textsuperscript{9}. The game theory has been also unsuccessful in explaining how agents should behave in order to improve an equilibrium situation\textsuperscript{10}.

Often an individual’s decision depends on the decisions of others because they have limited information about the problem or limited ability to process the information\textsuperscript{2,11}. An individual’s payoff is a function of the actions of others\textsuperscript{3,10}. In particular, in the diffusion of a new technology\textsuperscript{12}, early adopters impose externalities on later ones by rationally choosing technologies to suit only themselves. Then, individual has an incentive to pay attention to the decisions of others. This is known as binary decisions with externalities\textsuperscript{7}.

Threshold model\textsuperscript{7} has been postulated as one explanation for the contagion. Contagion is said to occur if one behavior can spread from a finite set of agents to the whole population. When can behavior that is initially adopted by only an infinite set of agents spread to the whole population? Morris\textsuperscript{13} shows that maximal contagion occurs when local interaction is sufficiently uniform and there is low neighbor growth, i.e., the number of agents who can be reached in $k$ steps does not grow exponentially in $k$. Lopez-Pintado\textsuperscript{14} showed that there exists a threshold for the degree of risk dominance of an action such that below the threshold, contagion of the action occurs. He also showed that networks with intermediate variance (where the connectivity of the lowest connectivity nodes are not so low) are best for diffusion purposes. Meanwhile, Watts\textsuperscript{15} showed that when the network of interpersonal influences is sufficiently sparse, the propagation of cascades is limited by the global connectivity of the network; and when it is sufficiently dense, cascade propagation is limited by the stability of the individual nodes. Therefore, the rate at which a social innovation spread depends on three factors: the topology of network, the payoff gain of the innovation and the amount of noise in the best response processes\textsuperscript{16}. Montanari\textsuperscript{17} shows that innovation spreads much more slowly on well-connected network structure dominated by long-range links than in low-dimensional ones dominated and Komatsu\textsuperscript{18} obtains the optimal network for good cascade using genetic algorithm and they show the network have a sufficient number of vulnerable nodes and hub node of medium size.

To illustrate how important spatial structure is to the emergence of cooperation in society, Nowak\textsuperscript{19} and Axelrod\textsuperscript{20} have investigated lattice models of agents confronted with a social dilemma. At the other extreme, most of human social networks were regarded as random networks whose nodes are connected randomly because of its large scale and complexity. In reality, Barabási et al. found that many complex networks have a scale-free structure\textsuperscript{21}. Moreover, another kind of network structure small-world has been defined and researched\textsuperscript{22}. Of course, the number of individual is large and the relationship is assumed to be complex. However, the world is much smaller than we think. We deal with population with scale-free network.

In our previous work\textsuperscript{23}, we showed that collective behavior is affected in the structure of the social network, the initial collective behavior and diversity of theta. In this paper, we focus on scale-free network and investigate the effect of number of interaction on collective behavior.
In this paper, we compare Schelling model with Cascade model. And we show that collective behavior is affected in the structure of the social network and theta and the collective behavior was stochastic. Moreover, collective behavior is almost same as Schelling model, though the decision is not interactive and simultaneously. Then, we show that the collective behavior in Schelling model is similar to cascade model.

2. Model

We consider the following dynamics to describe the evolution of agents’ choices through time. At time $t$, each agent plays a $2 \times 2$ game with each neighbor and chooses an action from the space $S = \{S_1, S_2\}$\textsuperscript{14,15}. The assumption that an agent cannot make his action contingent on his neighbor’s action is natural in this context. Otherwise, the behavior of an agent would be independent of the network structure. Payoffs from each interaction in each period are given by a function $\pi(s, s')$, where $s, s' \in S$ and they are summarized in the following symmetric matrix as shown in Table 1, where $0 \leq \theta_i \leq 1$. This implies that the game is a coordination game, whose strict Nash equilibria are $(S_1, S_1)$ and $(S_2, S_2)$. Agent $A_i$’s payoff from playing $s_i \in \{S_1, S_2\}$ when the strategy profile of the remaining agents $s_{-i}$ is given by $\Pi(s_i, s_{-i}) = \sum_{j \in N_i} \pi(s_i, s_j)$. Thus, an agent’s payoff is simply the sum of the payoffs obtained across all the bilateral games in which he is involved.

Table 1. Payoff matrix of agent $A_i$.

<table>
<thead>
<tr>
<th>Choice of agent $A_i$</th>
<th>S1</th>
<th>S2</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>$1 - \theta_i$</td>
<td>0</td>
</tr>
<tr>
<td>S2</td>
<td>0</td>
<td>$\theta_i$</td>
</tr>
</tbody>
</table>

Agents select the action that maximizes his benefits given the action of others in the previous period (a myopic best response). Therefore, if at time $t$ the proportion of agent $A_i$ neighbors choosing $S_1$, which we define $p_i(t)$, is higher than or equal to $\theta_i$, then agent’s best response is to choose $S_1$. Otherwise, $A_i$ chooses $S_2$. The value $\theta_i$, namely the degree of risk dominance of action $S_1$, specifies a lower bound for the fraction of individuals that must be choosing $S_1$ in order to make action $S_1$ preferred to action $S_2$. If $\theta_i \leq 1/2$ action $S_1$ is risk dominant. Also, the more risk dominant action $S_1$ is the lower the value of $\theta_i$. This rule is given by this function.

\[
p_i \geq \theta_i : \text{Agent } A_i \text{ chooses } S_1 \\
p_i < \theta_i : \text{Agent } A_i \text{ chooses } S_2
\]

(1)

The model differs from cascade models which Watts or Lopez-Pintado deals with in some respects. All these features; simultaneity, heterogeneous rule, interactive interaction and network heterogeneity are essential to collective behavior.

- Interactive interaction: Each agent can revise his behavior both of two alternatives, that is the decision is two ways. But, in cascade model, once a agent has switched on one alternative$\$$S_1$, it remains on $S_1$ for the duration of the dynamics.
- Simultaneity: Each agent decides his behavior depend on the neighbors’ behavior in previous time step for each time step. While in cascade model, a certain probability agents are chosen each period to revise their strategy
- Heterogeneous rules: Each agent has an idiosyncratic threshold. According to the threshold, agent's behavior is different from each other. Whereas, the decision rule is homogeneous in cascade model.
- Heterogeneous networks: typically modeled on regular lattices, here we are concerned with heterogeneous networks; networks in which individuals have different numbers of neighbors.

3. Simulation1: Schelling model

3.1. Settings
At first, we set that all agents have same payoff matrix, that is, population is homogeneous. And thetas of all agents are set as 0.1. Then, considering pair of agents, Nash equilibrium is for both of them to choose S1 or for both of them to choose S2. So, for population, Nash equilibrium is for all agents to choose S1 or to choose S2. And, we set a model of social network for population as regular network with 1000 agents with 10 links.

3.2. Results

We denote the collective behavior \( p(t) \) that the proportion of agents having chosen S1 in whole population at time \( t \). Here, we set the initial collective behavior as 0.01. And we assume that all agents choose at random at first time step. Then, each agent makes decision depend on best response rule each time step, and then collective behavior turns.

Figure 1 shows the simulation results in regular network. When initial collective behavior is 0.01, which means low proportion, final collective behaviors depends on trials. Final collective behaviors sometimes converges to 0.0 and no agents choose S1 at last, but other times converges to 1.0 and all agents choose S1 at last. In latter case, there occur contagion and triggered by small proportion of S1 spread to the whole population, which is rare case.

Next, we set theta \( \theta \) at intervals of 0.1 from 0.0 to 1.0. We simulate until 100 time step, we call this as a trial. Then, we set initial collective behavior as 0.01, and simulate 100 trials per theta and investigate the final collective behavior \( p^* \).

Figure 2(a) shows the simulation results. The x-axis represents theta and the y-axis represents the final proportion of agents who choose S1, which is final collective behavior \( p^* \). And 100 trials are plotted for each theta. We found that collective behavior depends on theta and becomes 0.0 or 1.0, stochastically. And if theta is more than or equal to 0.2, collective behavior converges to 0.0 and all agents choose S2 at last in any trials. Otherwise theta is 0.0 collective behavior converges to 1.0 and all agents choose S1 at last. But, if theta is 0.1, collective behaviors become 0.0 or 1.0 depending on the trials.

Other viewpoints of results are shown in Figure 2(b). In this figure, the x-axis represents theta and the y-axis represents the histogram of the final collective behavior. And blue area means final collective behavior become 1.0 and red area means final collective behavior become 0.0.

As for the regular network, the final collective behavior always become 0.0 or 1.0 depends on theta and trials. When theta was more than 0.2, the histogram of the red area \( (p^*=0.0) \) is 100 and the final collective behavior always becomes 0.0. That is, if theta is narrow large, it is benefit to choose S2 for each agent and all agents come to choose S2 at last. When theta was 0.0, the histogram of 1.0 is 100 and the final collective behavior always becomes 1.0. That is, if theta is small, it is beneficial for each agent and all agents come to choose S1 at last. Then, collective behavior sometimes becomes 0.0 or 1.0 when theta was 0.1, which depends on trials.
Fig. 2. (a) final collective behavior; (b) the histogram of final collective behavior (blue means $p^*$ become 1.0 and red means $p^*$ become 0.0.).

Next, we set a model of social network for population as scale-free network. We arrange the regular network for scale-free network using Kawachi algorism. Then, their average degrees are 10. Kawachi proposed generation algorithm from regular network to various networks by each agent's with a link of the same number changing a link. That is, a node whose number of links is large must be much larger and a node whose number of links is small must be much smaller. When all links of each node have been considered once, the procedure is repeated several times. For scale-free network, we set probability as 1 and times as 20. The scale-free network is organized as shown in Figure 3. And it is shown by log-log graph.
The simulation result in scale-free network is shown in Figure 4(a). This result is similar to the former results. But, the range that collective behavior becomes 0.0 or 1.0, which depends on the trials is wider than regular network and the range is between 0.1 and 0.5. We found that when theta is between 0.1 and 0.5; it is more possible that all agents come to choose S1 in scale-free network than regular network. Although, as shown in Figure 4(b), when theta is between 0.1 and 0.5, it is also rare case that all agents come to choose S1 in scale-free network.

### 3.3. Consideration

With comparing two network topologies, we found that collective behavior is affected in the structure of the social network and the theta. And we found that the boundary of red and blue area is keen in regular network is similar to Schelling model. But, the boundary is not keen in scale-free network. And the collective behavior was stochastic. And in regular network, the collective behavior is similar to that of Schelling model. But, in scale free network, the collective behavior is far from that of Schelling model. It is depend on trials and it is difficult to anticipate when theta is between 0.1 and 0.5.

### 4. Simulation 2 One way interaction

#### 4.1. Settings

In previous simulation, we set agents’ decision is interactive and simultaneous as denoted in Section 2. In this section, we defuse the interactive interaction and focus on the effect of two way interaction. That is, in this simulation, agent can only change the behavior for S1.

\[
p_i \geq \theta_i : \text{Agent } A_i \text{ chooses S1} \tag{2}
\]

#### 4.2. Results

We set theta at intervals of 0.1 from 0.0 to 1.0. We simulate until 100 time step, we call this as a trial. Then, we simulate 100 trials per theta and investigate the final collective behavior \(p^*\).

Figure 5 shows the simulation results in regular network and scale free network. The \(x\)-axis represents theta and the \(y\)-axis represents the final proportion of agents who choose S1, which is final collective behavior. And 100 trials are plotted for each theta. Collective behavior depends on theta and trial. And the collective behavior depends on...
theta and becomes 0.01 or 1.0, stochastically. In this model, because agent cannot change for S2, there few agents who choose S1 at first time step remain at last time step.

And in regular network, if theta is more than equal 0.2, collective behavior converges to 0.01 and almost agents choose S2 at last in any trials. Otherwise theta is 0.0 collective behavior converges to 1.0 and all agents choose S1 at last. But, if theta is 0.1, collective behaviors become 0.01 or 1.0 depending on the trials. In scale free network, the simulation result is similar to that of the regular network. But, the range that collective behavior becomes 0.01 or 1.0 depending on the trials is wider than regular network and the range is between 0.1 and 0.5. We found that when theta is between 0.1 and 0.5; it is more possible that all agents come to choose S1 in scale-free network than regular network.

Fig. 5. (a) final collective behavior in regular; (b) final collective behavior in scale free network.

Other viewpoints of results are shown in Figure 6. In this figure, the x-axis represents theta and the y-axis represents the histogram of the final proportion of agents. And blue area means final collective behavior become 1.0 and red area means final collective behavior become 0.01. We found that when theta is between 0.2 and 0.5; there are chances that all agents come to choose S1 in scale-free network than regular network, though, it is rare case.

Fig. 6. (a) the histogram of final collective behavior in regular network; (b) the histogram of final collective behavior in scale free network.

4.3. Consideration

Similarly to the previous section, we showed that collective behavior is affected in the structure of the social network and theta. And the collective behavior was stochastic. Moreover, collective behavior is almost same as Schelling model, though the decision is not interactive.
5. Simulation3: Cascade model

5.1. Settings

In previous simulation, we defuse the interactive interaction and focus on the effect of two way interaction. In this section, we defuse the simultaneity and focus on the effect of simultaneity. That is, in this simulation, all agents cannot decide at the same time, randomly chosen agent can only decide. This is cascade model.

5.2. Results

We denote the collective behavior $p(t)$ that the proportion of agents having chosen S1 in whole population at time $t$. Here, we set the initial collective behavior as 0.01. And we assume that all agents choose at random at first time step. Then, randomly chosen agent makes decision depend on best response rule each time step, and then collective behavior turns. Figure 7 shows the simulation results in regular network. When initial proportion is 0.01, which means low proportion, final collective behaviors depends on trials. Final collective behaviors sometimes converges to 0.01 and few agents choose S1 at last, but other times converges to 1.0 and all agents choose S1 at last. In latter case, there occur contagion and triggered by small proportion of S1 spread to the whole population.

![Fig. 7. The transition of collective behavior in regular network.](image)

We set theta at intervals of 0.1 from 0.0 to 1.0. We simulate until 10000 time step, we call this as a trial. Then, we simulate 100 trials per theta and investigate the final collective behavior. Figure 8 shows the simulation results in regular network and scale free network. The $x$-axis represents theta and the $y$-axis represents the final proportion of agents who choose S1, which is final collective behavior. And 100 trials are plotted for each theta. Collective behavior depends on theta and trial. And the collective behavior depends on theta and becomes 0.01 or 1.0, stochastically. In this model, because agent cannot change for S2, there few agents who choose S1 at first time step remain at last time step.

And in regular network, if theta is more than equal 0.3, collective behavior converges to 0.01 and almost agents choose S2 at last in any trials. Otherwise theta is 0.0 collective behavior converges to 1.0 and all agents choose S1 at last. But, if theta is 0.1 and 0.2, collective behaviors become 0.01 or 1.0 depending on the trials. In scale free network, the simulation result is similar to the regular network. But, the range that collective behavior becomes 0.0 or 1.0 depending on the trials is wider than regular network and the range is between 0.1 and 0.5.
for our simulation results. Moreover, that contagion threshold suit for 0.1 in regular network and theta. In this figure, the x-axis represents theta and the y-axis represents the histogram of the final proportion of agents. And blue area means final collective behavior become 1.0 and red area means final collective behavior become 0.01. We found that when theta between 0.3 and 0.5; there are chances that all agents come to choose S1 in scale-free network than regular network, though, it is rare case.

5.3. Consideration

Similarly to the previous section, we showed that collective behavior is affected in the structure of the social network and theta. In this section, it takes 10000 time step for a trial, but, in former section it takes only 100 time steps. And the collective behavior was stochastic. Moreover, collective behavior is almost same as Schelling model, though the decision is not interactive and simultaneously. Then, we found that the collective behavior in Schelling model is similar to cascade model. That is, our former results\(^\text{23}\) with heterogeneous rules or heterogeneous networks are possible to apply for cascade model. In these populations, each agent has an idiosyncratic threshold or networks in which individuals have different numbers of neighbors.

Here, Morris\(^\text{13}\) deals with m-dimension lattice and shows that contagion of the action occurs below contagion threshold. And in a homogeneous network, where all nodes have the same connectivity, the contagion threshold equals the inverse of the connectivity \(k\). In this paper, \(k\) is 10, then contagion threshold suit for 0.1 in regular network, it is very approximate for our simulation results. Moreover, López-Pintado by mean-field approach showed that contagion threshold of homogeneous network is low than that of scale free network. And when the degree is 10, contagion threshold in regular network is about 0.08 and that in scale free network is 0.2. In our results, the
contagion threshold in regular network is about 0.2 and that in scale free network is 0.5. They are very approximate for our simulation results.

6. Conclusion

We showed that collective behavior in cooperative relationships is affected in the structure of the social network, the initial collective behavior and diversity of theta.

In our previous work, we showed that collective behavior is affected in the structure of the social network, the initial collective behavior and diversity of theta. In this paper, we focus on scale-free network and investigate the effect of number of interaction on collective behavior.

In this paper, we found that collective behavior is affected in the structure of the social network and theta and the collective behavior was stochastic. Moreover, collective behavior is almost same as Schelling model, though the decision is not interactive and simultaneously. Then, we found that the collective behavior in Schelling model is similar to cascade model. That is, our results with heterogeneous rules or heterogeneous networks are possible to apply for cascade model.

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