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Masses of the $\eta_c(nS)$ and $\eta_b(nS)$ mesons

A.M. Badalian [a], B.L.G. Bakker [b,*]

[a] Institute of Theoretical and Experimental Physics, Moscow, Russia
[b] Department of Physics and Astronomy, Vrije Universiteit, Amsterdam, The Netherlands

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Abstract

The hyperfine splittings in heavy quarkonia are studied using new experimental data on the di-electron widths. The smearing of the spin–spin interaction is taken into account, while the radius of smearing is fixed by the known $J/\psi - \eta_c(1S)$ and $\psi(2S) - \eta_c'(2S)$ splittings and appears to be small, $r_{ss} \approx 0.06$ fm. Nevertheless, even with such a small radius an essential suppression of the hyperfine splittings ($\sim 50\%$) is observed in bottomonium. For the nS $b\bar{b}$ states ($n = 1, 2, \ldots, 6$) the values we predict (in MeV) are 28, 12, 10, 6, 6, and 3, respectively. In single-channel approximation for the $3S$ and $4S$ charmonium states the splittings 16(2) and 12(4) MeV are obtained.

1. Introduction

At present two spin-singlet S-wave states $\eta_c(1S)$ and $\eta_c(2S)$ are discovered [1–3]; still, no spin-singlet $\eta_b(nS)$ levels have been seen [4]. Though the masses of the $\eta_b(nS)$ were predicted in many papers [5–12], the calculated hyperfine (HF) splittings, $\Delta_{\rm HF}(nS) = M(n^3S_1) - M(n^1S_0)$, vary in a wide range: from 35 up to 100 MeV for the $b\bar{b}$ $1S$ state and for the $2S$ state between 19 and 44 MeV [11]. However, at the modern level of the theory and experiment there exist well-established limits on the factors which determine the spin–spin potential $V_{\rm HF}(r)$ in heavy quarkonia. First of all, the wave function (w.f.) at the origin for a given n^3S_1 ($c\bar{c}$ or $b\bar{b}$) state can be extracted from di-electron width which are now measured with high accuracy [13,14]. Concerning the quark masses, the pole (current) mass, present in a correct relativistic approach, and the constituent mass, used in nonrelativistic or in more refined approximations, are also known with good accuracy [15,16]. Therefore the only uncertainties comes from two sources.

First, in perturbative QCD there is no strict prescription how to choose the renormalization scale μ in the strong coupling $\alpha_{\rm HF}$, entering $V_{\rm HF}(r)$.

Secondly, the role of smearing of the spin–spin interaction is not fully understood and the true size of the smearing radius r_{ss} is still not fixed.

Moreover, the masses of higher triplet and singlet states can be strongly affected by open channel(s), thus modifying the HF splittings.

In our calculations the smearing radius r_{ss} is taken to fit the $J/\psi - \eta_c(1S)$ and $\psi(2S) - \eta_c(2S)$ splittings. To reach agreement with experiment it is shown to be small, $r_{ss} \leqslant 0.06$ fm. Our value $r_{ss} = 0.057$ fm practically coincides with the number used in Ref. [10]. However, in spite of this coincidence the splitting $\Delta_1 = \Upsilon(1S) - \eta_b(1S) = 28$ MeV in our calculations appears to be two times smaller than that in Ref. [10], where $\Delta_1 = 60$ MeV.

From our point of view the use of the w.f. at the origin $|\tilde{R}_n(0)|^2_{\rm exp}$, extracted from di-electron widths, is the most promising one, because these w.f. take implicitly into account the relativistic corrections as well as the influence of open channel(s), in this way drastically simplifying the theoretical analysis. A comparison of these w.f. with those calculated in different models puts serious restrictions on the static potential used and also on many-channel models.

* Corresponding author.
E-mail address: blg.bakker@few.vu.nl (B.L.G. Bakker).

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The HF interaction is considered here in two cases. First one corresponds to the standard perturbative (P) spin–spin interaction with a δ-function:

$$\hat{V}_{ss}^{\mathrm{P}}(r) = \mathbf{s}_1 \cdot \mathbf{s}_2 \frac{32\pi}{9\omega_Q^2} \alpha_s(\tilde{\mu}) \left(1 + \frac{\alpha_s}{\pi}\rho\right) \delta(\mathbf{r}) \equiv \mathbf{s}_1 \cdot \mathbf{s}_2 V_{\mathrm{HF}}(r), \quad (1)$$

which in one-loop approximation gives the following HF splitting [6]:

$$\Delta_{\mathrm{HF}}^{\mathrm{P}}(nS) = \frac{8}{9} \frac{\alpha_s(\tilde{\mu})}{\omega_Q^2} \left|R_n(0)\right|^2 \left[1 + \frac{\alpha_s(\tilde{\mu})}{\pi}\rho\right], \quad (2)$$

where $\rho = \frac{5}{12}\beta_0 - \frac{8}{3} - \frac{3}{4}\ln 2$ and the second term in brackets is small: $\lesssim 0.5\%$ in bottomonium ($n_f = 5$) and $\lesssim 3\%$ in charmonium ($n_f = 4$).

It is very probable that $\delta(\mathbf{r})$ may be considered as a limiting case and the "physical" spin–spin interaction is smeared with a still unknown "smearing" radius. For the Gaussian smearing function

$$\delta(\mathbf{r}) \to \frac{4\beta^3}{\sqrt{\pi}} \int r^2\, dr \exp\left(-\beta^2 r^2\right) \quad (3)$$

the splitting can be rewritten as

$$\Delta_{\mathrm{HF}}^{\mathrm{P}}(nS) = \frac{8}{9} \frac{\alpha_s(\tilde{\mu})}{\omega_Q^2} \xi_n(\beta) \left|R_n(0)\right|^2 \left(1 + \frac{\alpha_s}{\pi}\rho\right), \quad (4)$$

where by definition the "smearing factor" $\xi_n(\beta)$ is

$$\xi_n(\beta) = \frac{4}{\sqrt{\pi}} \frac{\beta^3}{|R_n(0)|^2} \int \left|R_n(r)\right|^2 \exp\left(-\beta^2 r^2\right) r^2\, dr. \quad (5)$$

The general expression (4) is evidently kept for any other smearing prescription which may differ from Eq. (3).

2. Wave function at the origin

The w.f. at the origin is very sensitive to the form and parameters of the gluon-exchange interaction and also to the value of the quark mass used. Therefore we make the following remarks:

(a) To minimize the uncertainties in the w.f. at the origin, $R_n(0)$, we shall use the w.f. extracted from the experimental data on leptonic widths and denote them as $|\tilde{R}_n(0)|^2_{\mathrm{exp}}$. In this way the relativistic corrections to the w.f. and the influence of open channel(s) are implicitly taken into account.

(b) In Eqs. (2) and (4) the constituent mass ω_q enters (this fact can be rigorously deduced from relativistic calculations [15,16]): $\omega_q(nS) = \langle\sqrt{\mathbf{p}^2 + m_Q^2}\rangle_{nS}$, where under the square-root the pole mass $m_Q \equiv m_Q$ (pole) is present. This mass is known with good accuracy and we take here $m_b(\text{pole}) = 4.8 \pm 0.1$ GeV and $m_c(\text{pole}) = 1.42 \pm 0.03$ GeV, which correspond to the well-established current masses $\bar{m}_b(\bar{m}_b) = 4.3(1)$ GeV, $\bar{m}_c(\bar{m}_c) = 1.2(1)$ GeV [3], while the constituent masses lie also in a rather narrow range for all nS states, both in charmonium and bottomonium: $\omega_b(nS) = 5.05 \pm 0.15$ GeV, $\omega_c(nS) = 1.71 \pm 0.03$ GeV ($n \geqslant 2$), and $\omega_c(1S) = 1.62 \pm 0.04$ GeV ($n = 1$). Note, that just these mass values are mostly used in nonrelativistic calculations, thus implicitly taking into account relativistic corrections.

(c) The leptonic width of the n^3S_1 states in heavy quarkonia are defined by the Van Royen–Weisskopf formula with QCD correction γ_Q,

$$\Gamma_{ee}\left(n^3S_1\right)\big|_{\mathrm{exp}} = \frac{4e_Q^2\alpha^2}{M_n^2} \left|\tilde{R}_n(0)\right|^2_{\mathrm{exp}} \gamma_Q, \quad (6)$$

where $e_Q = \frac{1}{3}(\frac{2}{3})$ for a $b(c)$ quark, $\alpha = (137)^{-1}$, $M_n \equiv M(n^3S_1)$, and $\gamma_Q(nS) = 1 - \frac{16}{3\pi}\alpha_s(2m_Q)$, with the renormalization scale μ in α_s equal to $2m_Q$ (pole), as in Refs. [9,10] and also in $\eta_b \to \gamma\gamma$ decay [17]. In some cases $\mu = M_n$ is also taken, but with accuracy $\lesssim 1\%$ both choices coincide (here $2m_b = 9.6$ GeV and $2m_c = 2.9$ GeV are taken).

Since for $n_f = 5$ the QCD constant $\Lambda_{\overline{\mathrm{MS}}}^{(5)}$ is well known from high energy experiments [3], the factor γ_b is also defined with a good accuracy. For $\Lambda_{\overline{\mathrm{MS}}}^{(5)}(\text{3-loop}) = 210(10)$ MeV, which corresponds to $\alpha_{\overline{\mathrm{MS}}}(M_Z) = 0.1185$, one has $\gamma_b = \gamma_{bn} = 0.700(5)$ and $\alpha_s(2m_b) = 0.177(3)$. In charmonium ($n_f = 4$) for $\Lambda_{\overline{\mathrm{MS}}}^{(4)} = 0.260(10)$ MeV the coupling $\alpha_s(2m_c = 2.9\ \text{GeV}) = 0.237(5)$, $\gamma_c = 0.60(2)$. Then the w.f. at the origin, extracted from the di-electron width (6),

$$\left|\tilde{R}_n(0)\right|^2_{\mathrm{exp}} = \frac{M_n^2 \Gamma_{ee}(n^3S_1)}{4e_Q^2\alpha^2\gamma_Q}, \quad (7)$$

implicitly takes into account the relativistic corrections as well as the influence of open channels, which gives rise to smaller values for $|R_n(0)|$ as well as for the HF splitting. The extracted values of $|\tilde{R}_n(0)|^2_{\mathrm{exp}}$ in the $b\bar{b}$ and $c\bar{c}$ systems are presented in Table 1.

The extracted $|\tilde{R}_n(0)|^2_{\mathrm{exp}}$ can be compared to the predicted values, which chiefly depend on the strong coupling used in the gluon-exchange term. In particular, if the asymptotic-freedom

Table 1
The w.f. $|\tilde{R}_n(0)|^2_{\mathrm{exp}}$ (in GeV^3) and the leptonic widths $\Gamma_{ee}(\Upsilon(nS))$ and $\Gamma_{ee}(\psi(nS))$ (in keV)[a,b] ($\gamma_b = 0.70$, $\gamma_c = 0.60$)

	$b\bar{b}$		$c\bar{c}$	
	$\Gamma_{ee}(nS)_{\mathrm{exp}}$	$\lvert\tilde{R}_n(0)\rvert^2_{\mathrm{exp}}$	$\Gamma_{ee}(nS)_{\mathrm{exp}}$	$\lvert\tilde{R}_n(0)\rvert^2_{\mathrm{exp}}$
1S	1.314(29)[a]	7.094(16)	5.40(22)	0.911(37)
	1.336(28)[b]	7.213(15)	5.68(24)	0.959(40)
2S	0.576(24)[a]	3.49(15)	2.12(12)	0.51(3)
	0.616(19)[b]	3.73(12)	2.54(14)	0.61(3)
3S	0.413(10)[b]	2.67(7)	0.75(1)	0.22(1)
			0.89(8)	0.26(2)
4S	0.25(3)[a]	1.69(20)	0.47(15)	0.16(5)
			0.71(10)	0.24(4)
5S	0.31(7)[a]	2.21(49)		
6S	0.13(3)[a]	0.95(22)		

[a] The upper values of the leptonic widths in bottomonium are taken from PDG [3] and the lower values of $\Gamma_{ee}(nS)$ are taken from the CLEO data [13].
[b] The upper entries in charmonium are taken from [3] and the lower ones from [14].

Table 2
The factor $S_n(9)$ for the potential $V_B(r)(8)$ in charmonium and bottomonium

	$1S$	$2S$	$3S$	$4S$	$5S$	$6S$
$b\bar{b}$	1.08(4)	1.02(4)	1.02(4)	0.72(9)	1.03(22)	0.47(10)
$c\bar{c}$	1.01(4)	0.82(5)	0.41(2)	0.32(10)		

behavior of $\alpha_{\text{static}}(r)$ is neglected, then theoretical numbers can be 2–1.5 times larger than $|\tilde{R}_n(0)|^2_{\text{exp}}$, even for the $\Upsilon(nS)$ ($n = 1, 2, 3$) states, which lie far below the $B\bar{B}$ threshold [8].

Here, as well as in our analysis of the spectra and fine structure splittings in heavy quarkonia [7,15,16], we use the static potential $V_B(r)$ in which the strong coupling $\alpha_B(r)$ is defined as in background perturbation theory:

$$V_B(r) = \sigma r - \frac{4}{3}\frac{\alpha_B(r)}{r},$$
$$\alpha_B(r) = \frac{8}{\beta_0}\int dq \frac{\sin qr}{q}\frac{1}{t_B(q)}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\ln t_B}{t_B}\right], \tag{8}$$

where $t_B(q) = \ln\frac{q^2+M_B^2}{\Lambda_B^2(n_f)}$. Here $M_B = 0.95(5)$ GeV is the background mass, $\Lambda_B(n_f)$ is expressed through $\Lambda_{\overline{\text{MS}}}(n_f)$ and in 2-loop approximation $\Lambda_B(n_f = 4) = 360(10)$ MeV and $\Lambda_B(n_f = 5) = 335(5)$ MeV [15]; the string tension $\sigma = 0.18$ GeV^2. Our calculations show that in bottomonium (in single-channel approximation) the potential $V_B(r)$ gives values of $|R_n(0)|^2_{\text{theory}}$ very close to the values $|\tilde{R}_n(0)|^2_{\text{exp}}$. For illustration in Table 2 the ratios

$$S_n = \frac{|\tilde{R}_n(0)|^2_{\text{exp}}}{|\tilde{R}_n(0)|^2_{\text{theory}}} \tag{9}$$

are given for all known nS levels in charmonium and bottomonium.

As seen from Table 2, using potential $V_B(r)$ the influence of open channels in bottomonium appears to be important only for the $4S$ and $6S$ levels, while for the other states single-channel calculations are in good agreement with experiment. This is not so for many other potentials [8] and it means that any conclusions about the role of open channels cannot be separated from the $Q\bar{Q}$ interaction used in a given theoretical approach.

In charmonium the effect from open channels is much stronger and already reaches $\sim 60\%$ for the $3S$ and the $4S$ states ($S_n \approx 0.4$) and about 20% for the $\psi(2S)$ meson.

3. Hyperfine splitting

Now we discuss the HF splitting for both bottomonium and charmonium.

3.1. Bottomonium

In bottomonium the HF splittings are considered in two cases:

(A) No smearing effect, i.e., in Eq. (4) the smearing parameter $\xi_{bn} = 1.0$ ($\forall n$).

(B) The smearing parameter ξ_{bn} (5) is calculated with $\beta = \sqrt{12}$ GeV, or a smearing radius $r_{ss} = \beta^{-1} = 0.057$ fm.

Table 3
$\Delta^{\text{P}}_{\text{HF}}(nS)$ (in MeV) in bottomonium for $\alpha_{\overline{\text{MS}}}(\tilde{\mu}) = 0.21$, $\omega_b = 5.10$ GeV and $|\tilde{R}_n(0)|^2$ from Table 2

	$\xi_b = 1.0$ (no smearing)	ξ_{bn} for the smeared HF interaction with $\beta = \sqrt{12}$ GeV, $r_{ss} = 0.057$ fm
$1S$	51(4) (4)	28(2) (3)
$2S$	25(3) (2)	12(2) (1)
$3S$	22(5) (2)	10(2) (1)
$4S$	12(3) (1)	5.1(2) (1)
$5S$	16(2) (1)	6.4(1) (1)
$6S$	7(2) (1)	2.7(1) (1)

Table 4
Theoretical predictions of the HF splittings $\Delta M = M(n^3S_1) - M(n^1S_0)$ in MeV

ΔM	$n = 1$	$n = 2$	$n = 3$
GI [10]	63	27	18
PTH [6]	35	19	15
LNR [12]	79	44	35
EFG [9]	60	30	27
case (A)	51	25	22
case (B)	28	12	10

Unfortunately, at present there is no a precise prescription how to choose the renormalization scale in the HF splitting (2): in $\alpha_{\overline{\text{MS}}}(\tilde{\mu})$ the scale $\tilde{\mu} = m_b(\text{pole}) \approx 4.80 \pm 0.01$ GeV is often used. With $\Lambda_{\overline{\text{MS}}}(n_f = 5) = 210(10)$ MeV (just the same as in our calculations of γ_b (6)) one finds

$$\alpha_s(b\bar{b}, \tilde{\mu}) = \alpha_{\overline{\text{MS}}}(4.8\text{ GeV}) = 0.21(1). \tag{10}$$

With this $\alpha_s(\tilde{\mu})$ and $|\tilde{R}_n(0)|^2_{\text{exp}}$ from Table 1, one obtains the HF splittings in bottomonium presented in Table 3, second column. (The numbers in Table 3 contain experimental errors coming from $\Gamma_{ee}(nS)$ [3] (first number) and theoretical errors (second number).) For the $\Upsilon(nS)$ states ($n = 1, 2, 3$) the calculated HF splittings ($\xi_n = 1.0$) appear to be very close to the splittings from Refs. [9].

If smearing of the HF interaction (3) is taken into account (the smearing radius, $r_{ss} = \beta^{-1} = 0.057$ fm for $\beta = \sqrt{12}$ GeV, is taken to fit the experimental values of the $J/\psi - \eta_c(1S)$ and $\psi(2S) - \eta_c(2S)$ splittings), then even for such a small radius $\Delta_{\text{HF}}(nS)$ turn out to be 50% ($n = 1, \dots, 4$), 60% ($n = 5, 6$) smaller as compared to the "nonsmearing" case. In particular, the $\Upsilon(1S) - \eta_b(1S)$ splitting turns out to be 28 MeV instead of 51(4) MeV for $\xi_{bn} = 1.0$. For higher excitations very small splittings, $\Delta_{\text{HF}} \approx 6$ MeV and 3 MeV for the $5S$ and $6S$ states, are obtained, see Table 3.

Note that our value of $r_{ss} = 0.057$ fm is very close to that from Ref. [10] where $r_{ss} = 0.060$ fm is taken. However, in spite of this coincidence our numbers are about two times smaller than in [10], where $\Upsilon(1S) - \eta_b(1S) = 60$ MeV is obtained. For the $2S$ state our value of the splitting is 12 MeV, still smaller than 20 MeV in [10]. From this analysis it is clear that the observation of an $\eta_b(nS)$ meson could clarify the role of smearing

Table 5
The splittings $\Delta^{\rm P}_{\rm HF}(nS)$ and $\Delta^{\rm NP}_{\rm HF}(nS)$ (in MeV) in charmonium[a]

	$\Delta^{\rm P}_{\rm HF}(nS)$ (no smearing: $\xi_c = 1.0$) $\alpha_s(\mu_1) = 0.36$; $\alpha_s(\mu_n) = 0.30$ $(n = 2, 3, 4)$	$\Delta^{\rm P}_{\rm HF}(nS)$ $r_{ss} = 0.29$ GeV^{-1} $\alpha_s(\mu_n) = 0.36$ $(n = 1, \ldots, 4)$	$\Delta^{\rm NP}_{\rm HF}(nS)$[b] $G_2 = 0.043$ GeV^4
$1S$	117(5)	102(6)[a] 108(7)[c]	9 ± 2
experiment $J/\psi - \eta_c(1S)$	117(2)	117(2)	
$2S$	51(5) 61(5)[c]	46(3) 55(4)	3.5 ± 1.5
experiment $\psi(2S) - \eta_c(2S)$	48(4)	48(4)	
$3S$	21(2)	16(2)	2 ± 1
$4S$	15(4)	12(4)	1.5 ± 0.5

[a] The w.f. $|\tilde{R}_n(0)|^2$, taken from Table 1, correspond to $\Gamma_{ee}(nS)$ from PDG [3].
[b] The NP splittings are calculated in [18].
[c] Here $|\tilde{R}_1(0)|^2_{\rm exp} = 0.959$ GeV^3 and $|\tilde{R}_2(0)|^2 = 0.61$ GeV^3 from the CLEO data [14] are taken.

in the spin–spin interaction between a heavy quark and antiquark. In Table 4 we compare our predictions with those which were done in Refs. [6,9,10,12].

Cases (A) [B] correspond to the unsmeared [smeared] HF interactions, see Table 3.

3.2. Charmonium

Also in charmonium the splitting (4) in fact depends on the product $\alpha_s(\tilde{\mu}) \cdot \xi_n$, therefore it is convenient to discuss an *effective* HF coupling: $\alpha_{\rm HF}(nS) = \alpha_s(\tilde{\mu}_n)\xi_{cn}$, which is the only unknown factor. (The masses $\omega_c(nS)$ may be specified for different nS states [15,18].)

As discussed in [7], the experimental splittings $J/\psi - \eta_c(1S)$ and $\psi(2S) - \eta_c(2S)$ can be fitted if different values of $\alpha_{\rm HF}$ for the $1S$ and $2S$ states are taken namely, $\alpha_{\rm HF}(1S) \approx 0.36$ and $\alpha_{\rm HF}(2S) \approx 0.30$. Such a choice implies two possibilities. The first one, case (A), is

$$\text{(A)} \quad \alpha_s(\mu_1) = 0.36, \qquad \alpha_s(\mu_2) = 0.30,$$
$$\alpha_s(\mu_3) = \alpha_s(\mu_4) \leqslant 0.30, \qquad \xi_{cn} = 1.0 \quad (\forall n), \tag{11}$$

i.e., the renormalization scale is supposed to grow for larger excitations. In particular, for $\Lambda^{(4)}_{\overline{\rm MS}}(\text{2-loop}) = 270$ MeV one finds $\mu_1 = 1.25$ GeV $\approx \bar{m}_c(\bar{m}_c)$ while the scale $\mu_2 = 1.60$ GeV is essentially larger. For this choice of $\alpha_{\rm HF}$ the perturbative HF splittings are given in Table 5, second column.

Besides, we have also calculated the contributions coming from the nonperturbative (NP) spin–spin interaction. In bottomonium their values are small, $\Delta^{\rm NP}_{\rm HF}(nS) < 1$ MeV, and can be neglected. In charmonium, as well as in light mesons, the situation is different, e.g. due to the NP spin–spin interaction in the $1Pc\bar{c}$ state a cancellation of perturbative and NP terms takes place [19]. As a result, the mass difference $M_{\rm cog}(\chi_{cJ}) - M(h_c) = (+1 \pm 1)$ MeV turns out to be close to zero or even positive, in accord with experiment [20]. The values of $\Delta^{\rm NP}_{\rm HF}(nS)$ are given in Table 5, fourth column.

Thus one can conclude that in case (A) with different renormalization scales μ_n, the splittings $J/\psi - \eta_c(1S)$ and $\psi(2S) - \eta_c(2S)$ can be obtained easily in agreement with experiment.

If the renormalization scales μ_n are supposed to be (almost) equal for all nS states:

$$\text{(B)} \quad \alpha_s(\mu_n \approx \bar{m}_c = 1.25 \text{ GeV}) = 0.36, \tag{12}$$

then to explain the relatively small $\psi(2S) - \eta_c(2S)$ splitting, a smearing effect needs to be introduced. Then for the potential used, the values $\xi_n(c\bar{c}) = 0.84, 0.80, 0.78$, and 0.76 for the $1S$, $2S$, $3S$, and $4S$ states, respectively, are calculated. In this case the $\Delta^{\rm P}_{\rm HF}(c\bar{c}, nS)$ are also given in Table 5. For the higher $3S$ $(4S)$ levels our predicted numbers are about 21(15) MeV (no smearing) and 16(12) MeV (with smearing), i.e., the difference between cases (A) and (B) is only $\sim 20\%$. Notice that in case (B) the NP contribution improves the agreement with experiment for $J/\psi - \eta_c(1S)$. As a whole, in charmonium the smearing effect appears to be less prominent than in bottomonium.

4. Conclusions

Thus we come to the following conclusions:

(1) In bottomonium $\Delta^{\rm P}_{\rm HF}(nS)$ appears to be very sensitive to the smearing of the spin–spin interaction. Due to this effect the splitting decreases from 51 MeV to 28 MeV for the $1S$ state and from 25 MeV to 12 MeV for the $2S$ state; very small values are obtained for higher states.

(2) In charmonium there are two possibilities to describe $\Delta_{\rm HF}(1S)$ and $\Delta_{\rm HF}(2S)$, which are known from experiment. First one refers to a different choice of the renormalization scale: $\mu_1 = 1.25$ GeV and $\mu_2 \approx 1.60$ GeV for the $1S$ and $2S$ states, if the smearing effect is absent. The second possibility implies equal renormalization scales μ_n $(n = 1, \ldots, 4)$ for all nS states. Then to explain the $\psi(2S) - \eta_c(2S)$ splitting the smearing of the spin–spin interaction needs be taken into account. We also expect that for the $1S$ level a small contribution (~ 8 MeV) comes from the NP spin–spin interaction.

(3) The $\psi(3S) - \eta_c(3S)$ splitting in single-channel approximation is predicted to be around 16(2) MeV, without and 12(4) MeV with smearing effect. However, in charmonium a strong effect of the open channels is probable.

From the theoretical point of view, the cleanest cases refer to the η_b mesons, which lie below the $B\bar{B}$ threshold. Therefore, in order to understand the true role of the smearing effect in the spin–spin interaction the observation of an $\eta_b(nS)$ is crucially important.

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