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Deformation and break-up of viscoelastic droplets Using Lattice Boltzmann Models

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Abstract

We investigate the break-up of Newtonian/viscoelastic droplets in a viscoelastic/Newtonian matrix under the hydrodynamic conditions of a confined shear flow. Our numerical approach is based on a combination of Lattice-Boltzmann models (LBM) and Finite Difference (FD) schemes. LBM are used to model two immiscible fluids with variable viscosity ratio (i.e. the ratio of the droplet to matrix viscosity); FD schemes are used to model viscoelasticity, and the kinetics of the polymers is introduced using constitutive equations for viscoelastic fluids with finitely extensible non-linear elastic dumbbells with Peterlin’s closure (FENE-P). We study both strongly and weakly confined cases to highlight the role of matrix and droplet viscoelasticity in changing the droplet dynamics after the startup of a shear flow. Simulations provide easy access to quantities such as droplet deformation and orientation and will be used to quantitatively predict the critical Capillary number at which the droplet breaks, the latter being strongly correlated to the formation of multiple neckings at break-up. This study complements our previous investigation on the role of droplet viscoelasticity (A. Gupta & M. Sbragaglia, Phys. Rev. E 90, 023305 (2014)), and is here further extended to the case of matrix viscoelasticity.

Keywords: Droplet Microfluidics, Deformation and Break-up, Viscoelasticity, Lattice Boltzmann Models;

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1. Introduction

Emulsion properties are largely determined by their microstructure which can be tuned and designed for a huge variety of applications\(^1\). In particular, deformation and break-up of dispersed droplets determine the emulsion rheology\(^2\). Droplet deformation and break-up in Newtonian fluids have been extensively studied in the literature\(^3,4,5\). The effect of an unconfined shear flow on droplets of one fluid suspended in another immiscible fluid was first considered long time ago by Taylor\(^3\): he estimated the largest stable droplet radius by balancing the surface stresses due to interfacial tension and viscous stresses due to shear. A dimensionless measure of this balance is provided by the Capillary number \(Ca = \frac{\eta_M \dot{y} R}{\sigma}\), where \(\eta_M\) is the dynamic viscosity of the fluid matrix, \(\dot{y}\) the shear rate, \(R\) the droplet radius at rest and \(\sigma\) the surface tension. Break-up occurs at a critical Capillary number \(Ca_{cr}\) when the viscous forces overcome the surface forces. The problem of droplet deformation and break-up under confined shear flow between two parallel plates has also been addressed in a series of theoretical and experimental papers (see\(^6,7,8,9\) and references therein). It was suggested that under confined conditions, a uniform shear flow can be exploited to generate quasi monodisperse emulsions by controlled break-up at near-critical conditions\(^7,10\). The properties of confined droplets that contain viscoelastic components are less studied\(^12,13,14\) and the critical conditions for break-up have been rarely explored so far. Recent experiments suggest that viscoelasticity changes profoundly the critical Capillary numbers in confined conditions\(^14\). Complementing these kind of results with the help of numerical simulations would be of extreme interest. Simulations provide easier access to quantities such as droplet deformation and orientation as well as the velocity flow field and pressure field inside and outside the droplet. The goal of this paper is to use numerical simulations to characterize the idealized problem of a Newtonian/viscoelastic droplet subject to simple shear in a confined viscoelastic/Newtonian matrix.

2. Theoretical Model

Our numerical approach is based on a hybrid combination of Lattice-Boltzmann models (LBM) and finite difference (FD) schemes, the former used to model two immiscible fluids with variable viscosity ratio, and the latter used to model viscoelasticity using the FENE-P constitutive equations. LBM have already been used to model droplet deformation problems\(^17,21,20,22\) and also viscoelastic flows\(^16,23\). The approach we use has already been studied and validated in a dedicated work\(^26\), where we have provided evidence that the model is able to capture quantitatively rheological properties of dilute suspensions as well as deformation and orientation of single droplets in confined shear flows. We just recall here the relevant continuum equations which are integrated in both the droplet (d) and the matrix (M) phases. In the droplet phase we integrate both the NS (Navier-Stokes) and FENE-P reference equations:

\[
\rho \left[ \partial_t u + (u \cdot \nabla) u \right] = -\nabla P + \nabla \left( \eta_M (\nabla u + (\nabla u)^T) \right) + \frac{\eta_p}{\tau_p} \nabla \cdot [f(r_p)C]; \tag{1}
\]

\[
\partial_t C + (u \cdot \nabla) C = C \cdot (\nabla u) + (\nabla u)^T \cdot C - \frac{f(r_p)C - I}{\tau_p}. \tag{2}
\]
Here, $\eta_A$ is the dynamic viscosity of the fluid, $\eta_p$ the viscosity parameter for the FENE-P solute, $\tau_p$ the polymer relaxation time, $\rho$ the solvent density, $P$ the solvent pressure, $(\nabla u)^T$ the transpose of $(\nabla u)$, $C$ the polymer-conformation tensor, $I$ the identity tensor, $f(r_p) \equiv (L^2 - 3)/(L^2 - r_p^2)$ the FENE-P potential that ensures finite extensibility, $r_p \equiv \sqrt{T\tau(C)}$ and $L$ is the maximum possible extension of the polymers$^{15}$. In the outer matrix (M) phase (indicated with a prime), the corresponding equations are

$$
\rho' [\partial_t u' + (u' \cdot \nabla)u'] = -\nabla P' + \nabla \left( \eta_B (\nabla u' + (\nabla u')^T) \right) + \frac{\eta_B'}{\tau_p'} \nabla \cdot \left[ f(r_p') C' \right];
$$

$$
\partial_t C' + (u' \cdot \nabla)C' = C' \cdot (\nabla u') + (\nabla u')^T \cdot C' - \frac{f(r_p') C' - I}{\tau_p'}.
$$

with $\eta_B$ the solvent matrix shear viscosity. In all the cases, immiscibility between the droplet phase and the matrix phase is introduced using the so-called “Shan-Chen” model$^{26}$. In all the numerical simulations presented in this paper, we work with unitary viscosity ratio, defined in terms of the total (solvent+polymer) shear viscosity. In particular, when studying matrix viscoelasticity (MV), we will choose a case with $\tau_p' = 0$ in equation (1Theoretical Modelequation.2.1) and $\lambda = \eta_d/\eta_M = \eta_A/\eta_B + \eta_p'/\tau_p = 1$ and polymer concentration $\eta_p/\eta_M = 0.4$; for the simulations with droplet viscoelasticity (DV), we will choose a case with $\tau_p = 0$ in equation (3Theoretical Modelequation.2.3) with $\lambda = \eta_d/\eta_M = (\eta_A + \eta_p)/\eta_B = 1$ and polymer concentration $\eta_p/\eta_d \approx 0.4$. The degree of viscoelasticity is computed from the Deborah number

$$
De = \frac{N_1 R}{2\sigma C a^2}
$$

where Ca is always computed in the matrix phase while the Deborah number is computed in either the matrix or the droplet phase, dependently on the case studied. In Eq. (5Theoretical Modelequation.2.5), $N_1$ is the first normal stress difference which develops in homogeneous steady shear. Solving the constitutive equations for such a hydrodynamic problem, $u_x = \dot{\gamma} z, u_y = 0, u_z = 0$, both the polymer feedback stress and the first normal stress difference $N_1$ for the FENE-P model$^{15,25}$ follow (primed variables replace non-primed variables for matrix phases)

$$
\frac{\eta_p}{\tau_p} f(r_p)(C_{xx} - C_{yy}) = \frac{2\eta_p}{\tau_p} \left( \frac{L^2}{6} \right)^{1/2} \sinh \left( \frac{1}{3} \text{arsinh} \left( \frac{\dot{\gamma} \tau_p L^2}{4} \left( \frac{L^2}{6} \right)^{-3/2} \right) \right)
$$

$$
N_1 = \frac{\eta_p}{\tau_p} f(r_p)(C_{xx} - C_{yy}) = \frac{8\eta_p}{\tau_p} \left( \frac{L^2}{6} \right) \sinh^2 \left( \frac{1}{3} \text{arsinh} \left( \frac{\dot{\gamma} \tau_p L^2}{4} \left( \frac{L^2}{6} \right)^{-3/2} \right) \right).
$$

In the Oldroyd-B limit ($L^2 \gg 1$) we can use the asymptotic expansion of the hyperbolic functions and we get

$$
De = \frac{\tau_p \eta_p}{\tau_{em} \eta_M}
$$

Equation (8Theoretical Modelequation.2.8) shows that $De$ is clearly dependent on the ratio between the polymer relaxation time $\tau_p$ and the droplet emulsion time

$$
\tau_{em} = \frac{R \eta_M}{\sigma}.
$$

As evident from Eq. (6Theoretical Modelequation.2.6), the model supports a thinning effect at large shear, although such effect will not be important in our calculations, all the numerical simulations being performed with fluid pairs with nearly constant shear viscosities. In the following sections, we report the Deborah number based on the definition (8Theoretical Modelequation.2.8), as we estimated the difference between (8Theoretical Modelequation.2.8) and (5Theoretical Modelequation.2.5) to be at maximum of a few percent for the values of $L^2$ considered. Also, we focus mainly on the droplet deformation and break-up problems, being the quantitative benchmarks against known analytical results for the rheology of dilute suspensions$^{15,24}$ present in another dedicated methodological publication$^{26}$. In a previous study$^{27}$ we investigated the role of droplet viscoelasticity: a non trivial interplay between confinement and viscoelasticity has emerged. With the use of numerical simulations we had the opportunity to change separately the viscosity ratio of the Newtonian phases, the maximum extension of the polymers, and the degree of viscoelasticity,
Thus allowing for a systematic analysis of the viscoelastic effects while keeping the shear viscosity of the droplet fixed to the reference Newtonian case. In particular, by increasing the finite extensibility of the polymers, it was observed that the resistance against elongation may be enough to prevent both droplet elongation and subsequent triple breakup, thus altering significantly the critical Capillary number for viscoelastic droplets under confinement. In this paper, we push the analysis a bit further and we propose a comparative study between matrix and droplet viscoelasticity.

To simplify matters, we will also keep the maximum elongation of the polymers fixed to $L^2 = 100$, since we have exhaustively treated the importance of $L^2$ in our previous study.27

3. Results

In all the cases discussed in this section, a spherical droplet with radius $R$ is initially placed halfway between the walls. The critical Capillary number is computed by identifying the pre-critical ($U_{w,pre}$) and the post-critical wall velocity ($U_{w,post}$), i.e. the largest (smallest) wall velocity for which the droplet is stable (breaks). All the simulations described refer to cases with polymer relaxation times ranging in the interval $0 \leq \tau_P \leq 5000$ lbu and finite extensibility parameter $L^2 = 10^2$, corresponding to Deborah numbers ranging in the interval $0 \leq De \leq 2$. The numerical simulations have been carried out in three dimensional domains $L_x \times L_y \times W$. The droplet radius $R$ and the vertical gap $W$ have been changed in the ranges $50 \leq R \leq 60$ lattice cells and $128 \leq W \leq 256$ lattice cells to achieve different confinement ratios $2R/W$. The stream-flow ($x$) direction is resolved with $1024 \leq L_x \leq 1356$ lattice cells, depending on the droplet elongation properties, while the transverse-flow ($y$) direction is resolved with 128 lattice cells. Periodic conditions are applied in the stream-flow and in the transverse-flow directions. The droplet is subjected to a linear shear flow $u_x = \dot{\gamma} z$, $u_y = u_z = 0$, with the shear introduced with two opposite velocities in the stream-flow direction ($u_x(x, y, z = W) = -u_x(x, y, z = 0) = U_w$) at the upper ($z = W$) and lower wall ($z = 0$).

In Figs. 2 Break-up after the startup of a shear flow with confinement ratio $2R/W = 0.52$. We report the time history of droplet deformation and break-up including 3 representative time frames: initial deformation (left column); deformation prior to break-up (middle column); post break-up frame (right column). We use the droplet emulsion time $\tau_{em}$ (9 Theoretical Modelequation.2.9) as a unit of time. Panels (a)-(c): Newtonian droplet in a Newtonian matrix. Panels (d)-(f): viscoelastic droplet (DV) with Deborah number $De = 2.0$ in a Newtonian matrix. Non-Newtonian properties
stabilize (very little) the droplet deformation with a slightly larger critical Capillary number, \( \text{Ca}_c = 0.397 \). Panels (g)-(i): Newtonian Droplet in a viscoelastic matrix (MV) with Deborah number \( \text{De} = 2.0 \). Matrix viscoelasticity has an almost insignificant effect. In all cases, the viscosity ratio between the droplet phase and the matrix phase is kept fixed to \( \lambda = \eta_d/\eta_M = 1 \), independently of the degree of viscoelasticity.

Fig. 2. Break-up after the startup of a shear flow with confinement ratio \( 2R/W = 0.52 \). We report the time history of droplet deformation and break-up including 3 representative time frames: initial deformation (left column); deformation prior to break-up (middle column); post break-up frame (right column). We use the droplet emulsion time \( \tau_{em} \) (9Theoretical Model) as a unit of time. Panels (a)-(c): Newtonian droplet in a Newtonian matrix. A distinctive feature of this confined case is the emergence of triple break-up\(^9\). The critical Capillary number is estimated to be \( \text{Ca}_c = 0.426 \). Panels (d)-(f): viscoelastic droplet (DV) with Deborah number \( \text{De} = 2.0 \) in a Newtonian matrix. Droplet viscoelasticity stabilizes the droplet deformation and inhibits droplet break-up. The critical Capillary number is estimated to be \( \text{Ca}_c = 0.64 \). Panels (g)-(i): Newtonian droplet in a viscoelastic matrix (MV) with Deborah number \( \text{De} = 2.0 \). Matrix viscoelasticity destabilizes the formation of long droplet shapes, and the critical Capillary number, \( \text{Ca}_c = 0.32 \), is very similar to the unbounded case (see Fig. 2). Break-up after the startup of a shear flow with confinement ratio \( 2R/W = 0.52 \). We report the time history of droplet deformation and break-up including 3 representative time frames: initial deformation (left column); deformation prior to break-up (middle column); post break-up frame (right column). We use the droplet emulsion time \( \tau_{em} \) (9Theoretical Model) as a unit of time. Panels (a)-(c): Newtonian droplet in a Newtonian matrix. A distinctive feature of this confined case is the emergence of triple break-up\(^9\). The critical Capillary number is estimated to be \( \text{Ca}_c = 0.426 \).
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The critical Capillary number is estimated to be \( Ca_{cr} \). The viscosity ratio between the droplet phase and the matrix phase is kept fixed to \( \eta_d/\eta_M = 1 \), independently of the degree of viscoelasticity.

Since the shape of highly deformed and confined droplets deviates from an ellipsoid, we estimated the droplet elongation from the projection of the droplet length \( L_p \) in the velocity direction. In Panel (a) of Fig. 4 Evolution of the dimensionless droplet length after the startup of a shear flow for various Capillary numbers and Deborah numbers for a fixed confinement ratio \( 2R/W = 0.78 \) and finite extensibility parameter \( L^2 = 10^2 \). Since the shape of highly deformed droplets may deviate from an ellipsoid, we estimated the droplet elongation from the projection of the droplet length \( L_p \) in the velocity direction. The viscosity ratio between the droplet phase and the matrix phase is kept fixed to \( \lambda = \eta_d/\eta_M = 1 \), independently of the degree of viscoelasticity. Similarly to Figs. 2 Break-up after the startup of a shear flow with confinement ratio \( 2R/W = 0.52 \). We report the time history of droplet deformation and break-up including 3 representative time frames: initial deformation (left column); deformation prior to break-up (middle column); post break-up frame (right column). We use the droplet emulsion time \( \tau_{em} \) (9 Theoretical Model equation.2.9) as a unit of time. Panels (a)-(c): Newtonian droplet in a Newtonian matrix. A distinctive feature of this confined case is the emergence of \textit{triple break-up} \( \tau_{em} \) (9 Theoretical Model equation.2.9) as a unit of time. Panels (d)-(f): viscoelastic droplet (DV) with Deborah number \( De = 2.0 \) in a Newtonian matrix. Droplet viscoelasticity stabilizes the droplet deformation and inhibits droplet break-up. The critical Capillary number is estimated to be \( Ca_{cr} = 0.64 \). Panels (g)-(i): Newtonian droplet in a viscoelastic matrix (MV) with Deborah number \( De = 2.0 \). Matrix viscoelasticity destabilizes the formation of long droplet shapes, and the critical Capillary number, \( Ca_{cr} = 0.32 \), is very similar to the unbounded case (see Fig. 2). In all cases, the viscosity ratio between the droplet phase and the matrix phase is kept fixed to \( \lambda = \eta_d/\eta_M = 1 \), independently of the degree of viscoelasticity.
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Panel (a) of Fig. 5: Panel (a): Critical Capillary number for break-up as a function of confinement ratio for systems with finite extensibility parameter $L^2 = 10^2$. The critical Capillary number has been normalized to the value of the Capillary number in the unbounded case which is essentially the same for Newtonian and viscoelastic cases. The viscosity ratio between the droplet phase and the matrix phase is kept fixed to $\lambda = \eta_d/\eta_M = 1$ in all cases. Different Deborah numbers are considered, by changing the polymer relaxation time in equations (1Theoretical Modelequation.2.1)-(4Theoretical Modelequation.2.3). We consider both the cases of droplet viscoelasticity (DV) and matrix viscoelasticity (MV). Black open circles indicate situations where multiple necking occur. Panel (b): data analyzed in Panel (a) are reported in terms of the dimensionless maximum elongation of the droplet $L_{em}^{(M)}/2R$ (figure.5) summarizes and extends the findings of Figs. 2. Break-up after the startup of a shear flow with confinement ratio $2R/W = 0.52$. We report the time history of droplet deformation and break-up including 3 representative time frames: initial deformation (left column); deformation prior to break-up (middle column); post break-up frame (right column). We use the droplet emulsion time $\tau_{em}$ (9Theoretical Modelequation.2.9) as a unit of time. Panels (a)-(c): Newtonian droplet in a Newtonian matrix. Panels (d)-(f): viscoelastic droplet (DV) with Deborah number $De = 2.0$ in a Newtonian matrix. Non-Newtonian properties stabilize (very little) the droplet deformation with a slightly larger critical Capillary num-
Fig. 4. Evolution of the dimensionless droplet length after the startup of a shear flow for various Capillary numbers and Deborah numbers for a fixed confinement ratio $2R/W = 0.78$ and finite extensibility parameter $L^2 = 10^2$. Since the shape of highly deformed droplets may deviate from an ellipsoid, we estimated the droplet elongation from the projection of the droplet length ($L_p$) in the velocity direction. The viscosity ratio between the droplet phase and the matrix phase is kept fixed to $\lambda = \eta_d/\eta_M = 1$, independently of the degree of viscoelasticity. Similarly to Figs. 2-4, we report the time history of droplet deformation and break-up including 3 representative time frames: initial deformation (left column); deformation prior to break-up (middle column); post break-up frame (right column). We use the droplet emulsion time $\tau_{em}$ (9Theoretical Modelequation.2.9) as a unit of time. Panels (a)-(c): Newtonian droplet in a Newtonian matrix. 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We use the droplet emulsion time $\tau_{em}$ (9Theoretical Modelequation.2.9) as a unit of time. Panels (a)-(c): Newtonian droplet in a Newtonian matrix. Panels (d)-(f): viscoelastic droplet (DV) with Deborah number $De = 2.0$ in a Newtonian matrix. Non-Newtonian properties stabilize (very little) the droplet deformation with a slightly larger critical Capillary number, $Ca_{cr} = 0.397$. Panels (g)-(i): Newtonian Droplet in a viscoelastic matrix (MV) with Deborah number $De = 2.0$. Matrix viscoelasticity destabilizes the formation of long droplet shapes, and the critical Capillary number, $Ca_{cr} = 0.32$, is very similar to the unbounded case (see Fig. 2-4). Evolution of the dimensionless droplet length after the startup of a shear flow with confinement ratio $2R/W = 0.52$. We report the time history of droplet deformation and break-up including 3 representative time frames: initial deformation (left column); deformation prior to break-up (middle column); post break-up frame (right column). We use the droplet emulsion time $\tau_{em}$ (9Theoretical Modelequation.2.9) as a unit of time. Panels (a)-(c): Newtonian droplet in a Newtonian matrix. A distinctive feature of this confined case is the emergence of triple break-up$^5$. The critical Capillary number is estimated to be $Ca_{cr} = 0.426$. Panels (d)-(f): viscoelastic droplet (DV) with Deborah number $De = 2.0$ in a Newtonian matrix. Droplet viscoelasticity stabilizes the droplet deformation and inhibits droplet break-up. The critical Capillary number is estimated to be $Ca_{cr} = 0.64$. Panels (g)-(i): Newtonian droplet in a viscoelastic matrix (MV) with Deborah number $De = 2.0$. Matrix viscoelasticity destabilizes the formation of long droplet shapes, and the critical Capillary number, $Ca_{cr} = 0.32$, is very similar to the unbounded case (see Fig. 2-4). Evolution of the dimensionless droplet length after the startup of a shear flow with confinement ratio $2R/W = 0.52$. We report the time history of droplet deformation and break-up including 3 representative time frames: initial deformation (left column); deformation prior to break-up (middle column); post break-up frame (right column). We use the droplet emulsion time $\tau_{em}$ (9Theoretical Modelequation.2.9) as a unit of time. Panels (a)-(c): Newtonian droplet in a Newtonian matrix. Panels (d)-(f): viscoelastic droplet (DV) with Deborah number $De = 2.0$ in a Newtonian matrix. Non-Newtonian properties stabilize (very little) the droplet deformation with a slightly larger critical Capillary number, $Ca_{cr} = 0.397$. Panels (g)-(i): Newtonian Droplet in a viscoelastic matrix (MV) with Deborah number $De = 2.0$. Matrix viscoelasticity has an almost insignificant effect. In all cases, the viscosity ratio between the droplet phase and the matrix phase is kept fixed to $\lambda = \eta_d/\eta_M = 1$, independently of the degree of viscoelasticity (see Fig. 2-4). Evolution of the dimensionless droplet length after the startup of a shear flow with confinement ratio $2R/W = 0.52$. We report the time history of droplet deformation and break-up including 3 representative time frames: initial deformation (left column); deformation prior to break-up (middle column); post break-up frame (right column). We use the droplet emulsion time $\tau_{em}$ (9Theoretical Modelequation.2.9) as a unit of time. Panels (a)-(c): Newtonian droplet in a Newtonian matrix. Panels (d)-(f): viscoelastic droplet (DV) with Deborah number $De = 2.0$ in a Newtonian matrix. Non-Newtonian properties stabilize (very little) the droplet deformation with a slightly larger critical Capillary number, $Ca_{cr} = 0.397$. Panels (g)-(i): Newtonian Droplet in a viscoelastic matrix (MV) with Deborah number $De = 2.0$. Matrix viscoelasticity has an almost insignificant effect. In all cases, the viscosity ratio between the droplet phase and the matrix phase is kept fixed to $\lambda = \eta_d/\eta_M = 1$, independently of the degree of viscoelasticity.
matrix (MV) with Deborah number $\text{De} = 2.0$. Matrix viscoelasticity has an almost insignificant effect. In all cases, the viscosity ratio between the droplet phase and the matrix phase is kept fixed to $\lambda = \eta_d/\eta_M = 1$, independently of the degree of viscoelasticity. Break-up after the startup of a shear flow with confinement ratio $2R/W = 0.7$. We report the time history of droplet deformation and break-up including 3 representative time frames: initial deformation (left column); deformation prior to break-up (middle column); post break-up frame (right column). We use the droplet emulsion time $\tau_{\text{em}}$ (9Theoretical Modelequation.2.9) as a unit of time. Panels (a)-(c): Newtonian droplet in a Newtonian matrix. A distinctive feature of this confined case is the emergence of triple break-up$^9$. The critical Capillary number is estimated to be $C_{a_c} = 0.426$. Panels (d)-(f): viscoelastic droplet (DV) with Deborah number $\text{De} = 2.0$ in a Newtonian matrix. Droplet viscoelasticity stabilizes the droplet deformation and inhibits droplet break-up. The critical Capillary number is estimated to be $C_{a_c} = 0.64$. Panels (g)-(i): Newtonian droplet in a viscoelastic matrix (MV) with Deborah number $\text{De} = 2.0$. Matrix viscoelasticity destabilizes the formation of long droplet shapes, and the critical Capillary number, $C_{a_c} = 0.32$, is very similar to the unbounded case (see Fig. 2). We report the time history of droplet deformation and break-up including 3 representative time frames: initial deformation (left column); deformation prior to break-up (middle column); post break-up frame (right column). We use the droplet emulsion time $\tau_{\text{em}}$ (9Theoretical Modelequation.2.9) as a unit of time. Panels (a)-(c): Newtonian droplet in a Newtonian matrix. Panels (d)-(f): viscoelastic droplet (DV) with Deborah number $\text{De} = 2.0$ in a Newtonian matrix. Non-Newtonian properties stabilize (very little) the droplet deformation with a slightly larger critical Capillary number, $C_{a_c} = 0.397$. Panels (g)-(i): Newtonian Droplet in a viscoelastic matrix (MV) with Deborah number $\text{De} = 2.0$. Matrix viscoelasticity has an almost insignificant effect. In all cases, the viscosity ratio between the droplet phase and the matrix phase is kept fixed to $\lambda = \eta_d/\eta_M = 1$, independently of the degree of viscoelasticity. Break-up after the startup of a shear flow with confinement ratio $2R/W = 0.52$. We report the time history of droplet deformation and break-up including 3 representative time frames: initial deformation (left column); deformation prior to break-up (middle column); post break-up frame (right column). We use the droplet emulsion time $\tau_{\text{em}}$ (9Theoretical Modelequation.2.9) as a unit of time. Panels (a)-(c): Newtonian droplet in a Newtonian matrix. Panels (d)-(f): viscoelastic droplet (DV) with Deborah number $\text{De} = 2.0$ in a Newtonian matrix. Matrix viscoelasticity has an almost insignificant effect. In all cases, the viscosity ratio between the droplet phase and the matrix phase is kept fixed to $\lambda = \eta_d/\eta_M = 1$, independently of the degree of viscoelasticity. Break-up after the startup of a shear flow with confinement ratio $2R/W = 0.625$, whereas for larger confinement ratios a monotonous increase of $C_{a_c}$ is observed. The emergence of the up-turn in $C_{a_c}$ is intimately connected to the change of the break-up mechanism. For small Deborah numbers the curve $C_{a_c}$ vs. $2R/W$ does not change much for droplet viscoelasticity, whereas some decrease in $C_{a_c}$ can be readily seen for the case of matrix viscoelasticity. The black open circles indicate situations where ternary break-up is observed. Actually, the tendency of viscoelasticity to alter and change the stability properties of confined droplets is perceptibly more pronounced in the case of matrix viscoelasticity than droplet viscoelasticity. For Deborah number just above unity ($\text{De} = 2.0$) both matrix and droplet viscoelasticity alter significantly the critical Capillary number at break-up and the changes are more pronounced and amplified at larger confinement ratios. In Panel (b) of Fig. 5 PANEL (a): Critical Capillary number for break-up as a function of confinement ratio for systems with finite extensibility parameter $L^2 = 10^2$. The critical Capillary number has been normalized to the value of the Capillary number in the unbounded case which is essentially the same for Newtonian and viscoelastic cases. The viscosity ratio between the droplet phase and the matrix phase is kept fixed to $\lambda = \eta_d/\eta_M = 1$ in all cases. Different Deborah numbers are considered, by changing the polymer relaxation time in equations (1Theoretical Modelequation.2.1)-(4Theoretical Modelequation.2.3). We consider both the cases of droplet viscoelasticity (DV) and matrix viscoelasticity (MV). Black open circles indicate situations where multiple necking occur. Panel (b): data analyzed in Panel (a) are reported in terms of the dimensionless maximum elongation of the droplet $L_p^{(M)}/2R$ figure.5, we report the maximum dimensionless elongation of the droplet, $L_p^{(M)}/2R$, as a function of the confinement ratio. It is clear that the trends for $C_{a_c}$ and $L_p^{(M)}/2R$ are quite similar.

4. Conclusions

The effects of viscoelasticity and geometrical confinement on droplet break-up in a shear flow have been studied. Our analysis strongly benefited of numerical simulations, where we could model immiscible fluids in the presence of viscoelastic behaviour in either the droplet or the matrix phase. Numerical simulations offer great flexibility and easy access to hydrodynamic quantities and droplet interface dynamics, thus revealing particularly suited for the study
Fig. 5. Panel (a): Critical Capillary number for break-up as a function of confinement ratio for systems with finite extensibility parameter $L^2 = 10^2$. The critical Capillary number has been normalized to the value of the Capillary number in the unbounded case which is essentially the same for Newtonian and viscoelastic cases. The viscosity ratio between the droplet phase and the matrix phase is kept fixed to $\lambda = \eta_d/\eta_M = 1$ in all cases. Different Deborah numbers are considered, by changing the polymer relaxation time in equations (1Theoretical Modelequation.2.1)-(4Theoretical Modelequation.2.3). We consider both the cases of droplet viscoelasticity (DV) and matrix viscoelasticity (MV). Black open circles indicate situations where multiple necking occur. Panel (b): data analyzed in Panel (a) are reported in terms of the dimensionless maximum elongation of the droplet $L_{p}^{(M)}/2R$.

at hand. We have found that the effect of viscoelasticity is rather insignificant in unbounded cases, whereas it gets amplified with confinement. In particular, viscoelasticity reduces the capability of micro-confined shear flows to generate monodisperse emulsions. This is a feature that we found in a previous study on droplet viscoelasticity27 and we confirm in presence of matrix viscoelasticity. At small Deborah number, the tendency of viscoelasticity to alter and change the stability properties of confined droplets is more pronounced in the case of matrix viscoelasticity, if compared to the case of droplet viscoelasticity.

For future research it would be extremely interesting to repeat some of the numerical simulations at changing the polymer concentration and/or at changing the geometry of the system (i.e. T-shaped channels, flow-focusing devices) to reveal other interesting features on the dynamics of viscoelastic fluids in complex geometries.

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