Multi-objective optimization of hard turning of AISI 6150 using PCA-based desirability index for correlated objectives

K. Wonggasem\textsuperscript{a,\*}, T. Wagner\textsuperscript{b}, H. Trautmann\textsuperscript{a}, D. Biermann\textsuperscript{b}, C. Weihs\textsuperscript{a}

\textsuperscript{a}Faculty of Statistics, Technische Universität Dortmund, Vogelpothsweg 87, 44227 Dortmund, Germany
\textsuperscript{b}Institute of Machining Technology, Technische Universität Dortmund, Baroper Straße 301, 44227 Dortmund, Germany
\* Corresponding author. Tel.: +49-231-755-7204; fax: +49-231-755-4387. E-mail address: wonggasem@statistik.uni-dortmund.de.

Abstract

The turning process, one of the most popular material removal processes in industry, has several performance measures which are usually found to be correlated, such as tool wear, cutting force and surface finish. In order to apply optimization methods, such as the desirability index, the conditional independence assumption is usually made. However, this assumption rarely holds true in real world applications and the optimal solution obtained might be biased towards the performance measures which have strong positive correlations with the others. Despite the fact that the desirability index has been developed and frequently applied in industry for a long time, only a few studies have been carried out to solve optimization problems with correlated objectives. The modified desirability index which provides a solution for integrating the expert’s preferences and the correlation information of the performance measures into the overall performance index, the principal component analysis (PCA) based desirability index (DI), has been only recently developed.

In this paper, an optimization using the PCA-based DI is demonstrated based on empirical models of hard turning of AISI 6150 steel in which uncertainties are propagated by model errors. The results show that the degree of importance of each performance measure has been adjusted by the integration of the covariance information into the overall performance index.

1. Introduction

Turning is one of the most popular material removal processes which is commonly performed on CNC machines. The common performance measures of turning processes are surface roughness, tool wear, processing time and passive force which are generally found to be correlated. In the optimization of turning operations, the desirability index, an optimization method, is frequently applied. The concept of desirability transforms the process performance measures from their units of measurement into the desirability scores on the dimensionless [0,1] scale and combines them into the desirability index (DI). However, in the formulation of DI, desirability scores are assumed to be independent, regardless of the existence of their correlations. Consequently, the independence assumption is generally violated in the optimization and biased optimization results can be obtained.

Principal component analysis (PCA) is a useful mathematical technique applied frequently in the Taguchi method to handle correlations between performance measures as well as to allow these performance measures to be transformed into a single performance index. Despite the fact that PCA has been popularly applied in Taguchi method as in [1,2], the implementation of PCA in the desirability index has been only recently developed.

In this paper, the optimization procedure based on PCA and desirability concepts, the PCA-based desirability index [3] is applied to optimize the correlated measures of a turning operation. The overall performance index that is defined as the weighted composite of the principal component scores is monotonically increasing in the desirability scores.
According to the relationship between Pareto optimality and monotonicty of the index optimization which has been provided by Arnaud and Corinne [4], the optimality of the solutions obtained can be guaranteed by the PCA-based desirability index. In addition, uncertainties have been also taken into account in the optimization model to ensure the optimal performance under the dynamic operating conditions.

2. Principal component analysis-based desirability index

The desirability index approach originated by Harrington [5], is an optimization technique widely used for solving multi-objective optimization problems (MOPs). It allows the process performance measures in different scales to be transformed into the dimensionless desirability scores through desirability functions (DFs). The overall performance index called desirability index (DI) can be obtained by combining all desirability scores; thus, the MOP is converted to a single-objective optimization problem (SOP) which is less complex.

2.1. Desirability functions and desirability index

Desirability functions (DFs) are functions that utilize expert knowledge or preference to convert the performance measures to the desirability scores which are on the dimensionless [0,1] scale. The closer the value of the desirability score to 1, the better performance and the closer to 0, the more unfavorable the performance. For any performance measure which has only a lower or an upper specification limit, the following Harrington DF [5] can be used for the transformation:

$$d_j(Y_j) = e^{-a d_j(Y_j)}$$

$$Y_j = -\ln(\ln(d_j(Y_j))) = b_{0j} + b_{1j} Y_j.$$  \hspace{1cm} (2)

The constants $b_{0j}$ and $b_{1j}$ in equation 2 can be determined by the solution of a system of two linear equations with two values of $Y_j$ and their corresponding values of $d_j(Y_j)$.

The desirability index (DI) is an index that is used to indicate the overall performance of the process. It can be obtained by using either the arithmetic (DIa) or geometric mean (DIG) of the desirability scores.

$$DI_a = \frac{\sum_{j=1}^{m} w_j d_j}{\sum_{j=1}^{m} w_j}$$ \hspace{1cm} (3)

$$DI_g = \left( \prod_{j=1}^{m} d_j^{w_j} \right)^{1/\sum w_j}$$ \hspace{1cm} (4)

In equation 3 and 4, $d_j$ denotes the desirability score of the $j^{th}$ measure, $w_j$ denotes the weight of $d_j$ which is assigned according to the degree of importance of the $j^{th}$ measure, and $m$ is the total number of performance measures or objectives in the optimization.

2.2. Principal component analysis (PCA)

Principal component analysis (PCA) is an approach used to handle correlated data. The PCA transformation projects the correlated variables onto the uncorrelated principal components (PCs) through the linear combinations of the original component variables. It has been frequently applied in optimizations using the Taguchi method.

2.3. PCA-based desirability index

The PCA-based desirability index (DI PCA) has been developed based on the idea of implementing PCA in the desirability index. The correlations between the desirability scores are decorrelated, and transformed into the uncorrelated principal components (PCs) by the PCA transformation. Then, PCs which have different scales are converted into PC scores on the [0, 1] scale. Based on the principle of weighted-PCA [1], PC scores are combined into DI PCA which represents the overall performance of the process. The primary improvement over the existing PCA-based indices in Taguchi method is that, DI PCA is strictly monotonically increasing in the value of desirability scores; thus, dominance relations of the solutions are not changed by the transformations and the non-dominated solutions are obtained from index optimization.

Suppose that there are $m$ measures to be optimized and $n$ experimental runs in the experimental design. The procedure of PCA-based DI can be described as follows:

Step 1 : Assign the desirability score to each measure. Let $Y_{kj}$ denote the value of the $j^{th}$ measure in the $k^{th}$ experiment, the appropriate desirability function (DF) according to the variable type and specifications of $Y_{kj}$ is to be applied to compute the desirability score $d_j(Y_{kj})$.

Step 2 : Perform the PCA transformation. Since all $d_j(Y_{kj})$ share the same scale, the PCA transformation can be applied to transform $d_j(Y_{kj})$ into the uncorrelated principal components $Z_{kj}$. For every $k^{th}$ experiment, $d_j(Y_{kj})$ are transformed into $Z_{kj}$ using:

$$Z_{kj} = \sum_{j=1}^{m} a_{kj} d_j(Y_{kj})$$ \hspace{1cm} (5)
where \(a_{ij}\) denotes the \(j^{th}\) element of the \(i^{th}\) eigenvector which is derived from the \(m \times m\) covariance matrix of \(d_j(Y_{kj})\).

Step 3: Calculate the principal component score \(N_{ki}\) for each \(Z_{ki}\), using the following equations:

\[
N_{ki} = \frac{Z_{ki}}{Z_{\text{ideal}(i)}}
\]

for every \(i^{th}\) eigenvector that has either all positive or all negative elements, and

\[
N_{ki} = \frac{1}{2} \left( \frac{\Psi_{ki} - \Psi_{\text{ideal}(i)}}{Z_{\text{ideal}(i)} - \Psi_{\text{ideal}(i)}} \right)
\]

for every \(i^{th}\) eigenvector that has both positive and negative elements, with:

\[
Z_{\text{ideal}(i)} = \sum_{j=1}^{m} a_{ij}
\]

\[
\Psi_{ki} = \sum_{j=1}^{m} a_{ij} d_j(Y_{kj}) I_{(0,1)}(a_{ij})
\]

\[
\Psi_{\text{ideal}(i)} = \sum_{j=1}^{m} a_{ij} I_{(0,1)}(a_{ij})
\]

\[
I_{(0,1)}(a_{ij}) = \begin{cases} 1 & \text{if } a_{ij} \in (0,1] \\ 0 & \text{if } a_{ij} \notin (0,1] \end{cases}
\]

where \(\Psi_{ki}\) is the product of the linear combination of the \(i^{th}\) eigenvector’s positive elements and the corresponding \(d_j(Y_{kj})\). The ideal value of \(Z_i\) and \(\Psi_i\) can be obtained when all \(d_j(Y_{kj})=1\), in other words, when all desirability scores are absolutely desirable. The value of \(N_{ki}\) is in the interval \([0,1]\) and is monotonically increasing in the value of \(d_j(Y_{ki})\) which means automatically that the higher \(N_{ki}\) reflects the higher performance.

Step 4: Combine all PC scores into PCA-based desirability index \(D_{\text{PCA}}\), using the following equation:

\[
D_{\text{PCA}} = \sum_{i=1}^{m} \lambda_i \sum_{i=1}^{n} N_{ki}
\]

where \(\lambda_i\) is the corresponding eigenvalue of the \(i^{th}\) eigenvector. Since \(\lambda_i\) and \(N_{ki}\) are non-negative, and \(N_{ki}\) increases monotonically when the values of \(d_j(Y_{ki})\) increase, \(D_{\text{PCA}}\) is a monotonic transformation of \(d_j(Y_{ki})\).

Step 5: Search for the operating parameters and conditions that maximize the value of \(D_{\text{PCA}}\) by using an optimization algorithm.

3. Case study and result analysis

The case study of this paper is an optimization of the hard turning operation of AISI 6150 steel. The empirical models of [6] which are formulated from a total 157 experiments with 15 different parameter-value sets, are used in the optimization. The prediction errors are estimated from the experiment data of [6], and are assumed to be distributed normally with mean zero, being independent and identically distributed (i.i.d.).

3.1. Optimization model

The controllable parameters of this case study are feed \(f\), depth of cut \(a_p\) and cutting speed \(v_c\), with the same parameter ranges as performed by [6]: \(f = 0.05, 0.06, \ldots, 0.16\) mm, \(a_p = 0.05, 0.06, \ldots, 0.16\) mm and \(v_c = 100, 101, \ldots, 200\) m/min.

The performance measures of interest are the passive force \(F_p\) [N], the width of flank wear land on the minor cutting edge \(V_Bm\) [\(\mu m\)] and the cutting time \(t\) [s]. The quality of surface finish is controlled through the constraint that the predicted 95th percentile of surface roughness depth \(R_{z95}\) \(\leq 3\) \(\mu m\) must be satisfied. Due to the fact that in the real turning applications, a certain volume of material removal would be required in order to produce a finished product, a constant volume of material removal of \(20,000\) \(\mu m^{3}\) is designated. On the other hand, if this volume is not fixed as a constant, a biased optimization result could be obtained, for example, a small tool wear might be generated from a small volume of material removal by decreasing depth of cut when the cutting path length or cutting time is constant. The optimization of this turning operation is to find the best combination of \(f\), \(a_p\) and \(v_c\) which minimize \(F_p\), \(V_Bm\) and \(t\) and satisfy the constraint \(R_{z95}\) \(\leq 3\) \(\mu m\).

3.2. Optimization procedure

As \(F_p\), \(V_Bm\) and \(t\) are measures which have a one-sided specification, the one-sided Harrington’s desirability function will be applied to transform these performance measures into the desirability scores \(d_1(F_p)\), \(d_2(V_Bm)\) and \(d_3(t)\). The configurations for Harrington’s functions is defined as following:

\[
(F_p^{(1)}, d_1^{(1)}) = (30, 0.99), (F_p^{(2)}, d_1^{(2)}) = (100, 0.5), (V_Bm^{(1)}, d_2^{(1)}) = (0, 0.99), (V_Bm^{(2)}, d_2^{(2)}) = (100, 0.01) \quad \text{and} \quad (t^{(1)}, d_3^{(1)}) = (0, 0.99), (t^{(2)}, d_3^{(2)}) = (600, 0.01)
\]

where \(d_1^{(1)}\) denotes the desirability score of the first performance measure for the first linear equation and \(d_1^{(2)}\) for the second linear equation as defined in equation 2. By minimizing \(F_p\), the dynamic stability of the turning operation as well as the surface finish of the product can be enhanced. However, \(F_p\) will not cause process instability before a specific threshold is exceeded. Thus,
Fp will not be strictly minimized (d1(2)=-0.5) and an Fp value which is lower than 30 N is assumed to provide no performance improvement (d1(1)=0.99). While the values VBm of 100 μm or above are considered as tool failure [6], and the value t of 600 s or above is selected as totally undesirable processing time which will be strictly minimized (d2(2), d2(2)=0.01). After the constants b0j and b1j in equation 2 have been solved from the two linear equations, d1(Fp), d2(VBm) and d3(t) can be obtained by using equation 1.

Next, the principal component analysis (PCA) transformation is to be performed on d1(Fp), d2(VBm) and d3(t) which are correlated variables, dimensionless and share the same scale. For the transformation, the covariance information of d1(Fp), d2(VBm) and d3(t) is required. In order to obtain a non-biased covariance information, a 5th factorial experimental design matrix is selected for generating the data of performance measures in order to ensure uniform coverage of the parameter space. According to the constraint Rz95 ≤ 3 μm, the feasible feed range is limited to f ≤ 0.1 mm. Since it is known that parameters with f > 0.1 violate the constraint, the range of f used to generate the covariance information will be restricted from 0.05 mm to 0.1 mm. The uncertainty analysis is performed using the Monte-Carlo method with 1 million iterations in which the values of Fp, VBm and t are generated according to the Monte-Carlo method with 1 million iterations in which the values of Fp, VBm and t are generated according to the experimental design in each iteration. The expected value of correlation coefficients of Fp, VBm and t estimated from the Monte-Carlo method are shown in table 1.

### Table 1. The expected value of correlation coefficients of performance measures

<table>
<thead>
<tr>
<th>Variable</th>
<th>Fp</th>
<th>VBm</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fp</td>
<td>1</td>
<td>-0.2086</td>
<td>-0.1828</td>
</tr>
<tr>
<td>VBm</td>
<td>-0.2086</td>
<td>1</td>
<td>0.8975</td>
</tr>
<tr>
<td>t</td>
<td>-0.1828</td>
<td>0.8975</td>
<td>1</td>
</tr>
</tbody>
</table>

Unexpectedly, the correlation between Fp and VBm is found to be negative which contradicts the positive correlation found in [6-8]. Possible causes are the following: First most of the relationship between specific performance measures are analyzed in experiments in which operating parameters are constants or vary by one parameter at a time, and the correlations shown in table 1 are determined from the factorial experimental design in which various combinations of operating parameters are involved, so that the effects of the operating parameters may dominate the mechanical dependency between Fp and VBm. Second, the performance measures in this study are interpolated for the operation with a fixed volume of material removal of which differs from the experiment performed in [6-8].

Third, in this case study, VBm is defined as the width of flank wear land on the minor cutting edge which differs from the flank wear found on the major cutting edge VBc; thus, the correlation between Fp and VBm cannot be compared to Fp and VBc which found in [7, 8]. A good explanation for the negative correlation between Fp and t is that, increase in f or ap would increase Fp due to the larger tool-chip contact surface whereas t becomes shorter due to the higher rate of material removal. For this case, the effects of Vc on Fp is generally known to be so small and is dominated by the effects from f and ap; thus, can be neglected. The strong positive correlation between VBm and t can be explained in that by increasing either f or ap, the cutting path length and t become shorter, and due to the shorter period of workload less VBm is generated.

### Table 2. The expected value of covariances of desirability scores

<table>
<thead>
<tr>
<th>Variable</th>
<th>d1(Fp)</th>
<th>d2(VBm)</th>
<th>d3(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>d1(Fp)</td>
<td>0.0711</td>
<td>-0.0293</td>
<td>-0.0468</td>
</tr>
<tr>
<td>d2(VBm)</td>
<td>-0.0293</td>
<td>0.0470</td>
<td>0.0599</td>
</tr>
<tr>
<td>d3(t)</td>
<td>-0.0468</td>
<td>0.0599</td>
<td>0.1244</td>
</tr>
</tbody>
</table>

### Table 3. The eigenvalues and eigenvectors for PCA transformation

<table>
<thead>
<tr>
<th>Principal component</th>
<th>Eigenvalue</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>0.1839</td>
<td>-0.4392</td>
</tr>
<tr>
<td>Second</td>
<td>0.0444</td>
<td>0.8935</td>
</tr>
<tr>
<td>Third</td>
<td>0.0141</td>
<td>-0.0940</td>
</tr>
</tbody>
</table>

Table 2 shows the expected value of covariances of desirability scores which will be used in the PCA transformation and the diagonal elements represent the variances of the desirability scores. The eigenvalues and the eigenvectors which are derived from the covariance matrix shown in table 2, are listed in tables 3. Using the formula in equation 5, d1(Fp), d2(VBm) and d3(t) are transformed into the principal components (PCs) Z1, Z2 and Z3. In order to integrate these PCs into the PCA-based desirability index (DIPCA), a transformation to the PC scores must be performed using equation 6 and 7. Finally, the overall performance index, DIPCA can be obtained using equation 8.

When there is no uncertainty in the optimization problem, only the operating parameter set that maximize the value of DIPCA is to be searched. However, in real turning operation, there are uncertainties on the performance measures and the obtained DIPCA is no longer a static variable. Therefore, not only the expected value of DIPCA, but also the worst case representative, the 5th percentile which is estimated by Monte-Carlo method, are selected as the second objective in the optimization. As a consequence, the optimization...
problem is no longer a single objective optimization problem. The optimization is performed by using an algorithm for finding the Pareto solutions from all possible combinations of parameters. Assuming equal importance for all desirability scores (\(w_1 = w_2 = w_3 = 1\)), the arithmetic (\(DI_a\)) and geometric mean (\(DI_g\)) of the desirability scores, are used to compare the optimization results with \(DI_{PCA}\).

3.3. Results and analysis

The optimal solutions obtained from each index are listed in table 4 in which \(E(DI)\) denotes the expected value of the indices \(DI_{PCA}\), \(DI_g\) and \(DI_a\), and \(DI_{05}\) denotes the 5th percentile of each index. It can be clearly observed that the highest cutting speed \(v_c = 200\) m/min is preferred by all indices, which is supported by the results in [6] which the effects of \(v_c\) on \(VB_m\) are found to be small; hence, increase in \(v_c\) would generate only slightly more \(VB_m\), but prominently decrease \(t\) while improving simultaneously \(F_p\) and \(Rz\). The optimal results from \(DI_g\) and \(DI_a\) share similar combinations such that \(f = 0.09\) mm, \(a_p = 0.23\)–0.24 mm and \(v_c = 200\) m/min. While the smaller feed \((f = 0.06\)–0.07 mm) is preferred by \(DI_{PCA}\) which results in a better \(F_p\) but a longer \(t\) with an approximately 4 percent larger \(VB_m\) as shown in table 4. In the worst case, the result \(F_{p95} = 110\) N and \(VB_{m95} \approx 86\) \(\mu\)m can be expected from the optimal solution from \(DI_g\) and \(DI_a\), and the result \(F_{p95} \approx 102\) N and \(VB_{m95} \approx 89\) \(\mu\)m from \(DI_{PCA}\). Since the upper specification limit of all performance measures are not exceeded, all optimal solutions shown in table 4 satisfy the preference described in section 3.2.

The surface plots of \(DI_{PCA}\), \(DI_g\) and \(DI_a\) at the optimal cutting speed \(v_c = 200\) m/min are illustrated in figure 1a, 1b and 1c respectively in which the upper surface represents the expected value of the index, the lower surface represents the 5th percentile of each index, and the points on the upper surface are the locations of the optimal solutions that are listed in table 4. By comparing the surface plot in figure 1a and 1c to figure 1b, it can be seen that the indices \(DI_{PCA}\) and \(DI_a\) lack the ability to detect when one or more values of desirability scores approach zero, as it can be observed that the value of \(E(DI_{PCA})\) and \(E(DI_a)\) does not drop to zero when \(a_p < 0.1\) mm in which the value of \(d_2(VB_m)\) and \(d_3(t)\) are approaching zero. In the case that the optimal solutions obtained from the optimization may contain a zero desirability score, the minimal acceptable value for desirability scores should be taken as criterion.

<table>
<thead>
<tr>
<th>(f)</th>
<th>(a_p)</th>
<th>(v_c)</th>
<th>(E(F_p))</th>
<th>(E(VB_m))</th>
<th>(E(t))</th>
<th>(E(DI))</th>
<th>(DI_{05})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.07</td>
<td>0.25</td>
<td>200</td>
<td>89.3247</td>
<td>82.3173</td>
<td>337.0605</td>
<td>0.6139</td>
<td>0.5029</td>
</tr>
<tr>
<td>0.06</td>
<td>0.26</td>
<td>200</td>
<td>85.9098</td>
<td>83.0076</td>
<td>371.2128</td>
<td>0.6065</td>
<td>0.5069</td>
</tr>
<tr>
<td>0.09</td>
<td>0.24</td>
<td>200</td>
<td>97.7759</td>
<td>79.147</td>
<td>275.1475</td>
<td>0.4921</td>
<td>0.3687</td>
</tr>
<tr>
<td>0.09</td>
<td>0.23</td>
<td>200</td>
<td>96.4884</td>
<td>79.5637</td>
<td>286.6209</td>
<td>0.4911</td>
<td>0.3701</td>
</tr>
<tr>
<td>0.09</td>
<td>0.24</td>
<td>200</td>
<td>97.7759</td>
<td>79.147</td>
<td>275.1475</td>
<td>0.5533</td>
<td>0.4571</td>
</tr>
<tr>
<td>0.09</td>
<td>0.23</td>
<td>200</td>
<td>96.4884</td>
<td>79.5637</td>
<td>286.6209</td>
<td>0.5527</td>
<td>0.4576</td>
</tr>
<tr>
<td>0.08</td>
<td>0.25</td>
<td>200</td>
<td>94.4203</td>
<td>80.6873</td>
<td>297.3696</td>
<td>0.5515</td>
<td>0.4595</td>
</tr>
<tr>
<td>0.07</td>
<td>0.26</td>
<td>200</td>
<td>90.9508</td>
<td>81.7507</td>
<td>324.7461</td>
<td>0.5462</td>
<td>0.4601</td>
</tr>
</tbody>
</table>

Fig. 1. (a) the surface plot of \(DI_{PCA}\) at \(v_c = 200\) m/min; (b) the surface plot of \(DI_g\) at \(v_c = 200\) m/min; (c) the surface plot of \(DI_a\) at \(v_c = 200\) m/min
The different contributions of \( d_1(F_p) \), \( d_2(VB_m) \) and \( d_3(t) \) on \( DI_{PCA} \) and \( DI_a \) can be found when comparing figure 1a with 1c. From the parameter range \( a_p > 0.3 \) mm, the relatively low value of \( d_1(F_p) \) can be expected due to the excessive \( F_p \) generated from large \( a_p \) and the value of \( E(DI_{PCA}) \) drops remarkably deeper than \( E(DI_a) \) which can be interpreted as that the value of \( d_1(F_p) \) has more contribution in \( DI_{PCA} \) that in \( DI_a \). In contrast, the degree of importance of \( d_2(VB_m) \) and \( d_3(t) \) is dwindled in \( DI_{PCA} \) as the value of \( E(DI_{PCA}) \) is slightly higher than \( E(DI_a) \) for the parameters with \( a_p < 0.1 \) mm where the values of \( d_2(VB_m) \) and \( d_3(t) \) drop significantly.

In overview, the results show that the optimal operation parameters obtained from the indices share similar ranges, \( f = 0.06–0.09 \) mm, \( a_p = 0.23–0.26 \) mm and \( v_c = 200 \) m/min in which the best balance for \( F_p \), \( VB_m \), and \( t \) as well as the \( R_{z95} \leq 3 \) \( \mu \)m constraint satisfaction can be found. Due to the integration of the covariance information, the optimal solutions obtained from the \( DI_{PCA} \) show a slightly different trend from \( DI_g \) and \( DI_a \).

4. Conclusion and discussion

The optimization of a hard turning operation of AISI 6150 steel using the developed desirability index, principal component analysis based desirability index (PCA-based DI), has been demonstrated in this paper. The results show that the optimal solutions obtained from PCA-based DI slightly differ from the solutions obtained from the existing desirability indices, the arithmetic mean of desirability scores (\( DI_a \)) and the geometric means of desirability index (\( DI_g \)), due to the integration of covariance information in PCA-based DI. The passive force \( F_p \) which has no positive correlations with the other measures, has the highest contribution in the overall performance when the covariance information is utilized. Hence, the result obtained from PCA-based DI agrees with the expectation that the performance measures which have no positive correlations with the others should have more contribution and vice versa in the desirability index. The low-sensitivity to low values (non-zero) of desirability scores of PCA-based DI as well as \( DI_g \) also allows the measures with low desirability scores to be compensated by the other high desirability scores which is very difficult for \( DI_a \).

The concept of the PCA-based DI can be applied not only in the optimization of the turning process, but also in any optimization problems which have correlated objectives. In addition, the benefit of using strictly monotone overall performance indices such as PCA-based DI over the PCA-based indices in the Taguchi method is that the optimality can be assured through the monotonicity of the performance index.

References