CP violation in the charged pion energy spectra of the decays $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$

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Abstract

CP violation leads to a difference between the parameters $g^+$ and $g^-$ that characterise the energy distributions of the “odd” pion in the decays $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$ and $K^\pm \rightarrow \pi^\pm\pi^\pm\pi^\mp$. We argue that for the first decay, the asymmetry $\Delta g = (g^+ - g^-)/(g^+ + g^-)$ is fixed at a value around $\Delta g = 2 \times 10^{-6}$, whereas for the second decay, the asymmetry $\Delta g$ may be one order of magnitude larger.

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It is well known that the strength of direct CP violation in the $K_L \rightarrow 2\pi$ decays, as determined by the parameter $\epsilon'$, is crucially depending on the fact that the QCD penguin (QCDP) and the electroweak penguin (EWP) contributions partially cancel one another [1]. Thus, it is not difficult to understand that before the experimental value $\epsilon'/\epsilon = (1.67 \pm 0.26) \times 10^{-3}$ [2] was available the theoretical predictions for $\epsilon'/\epsilon$ were one order of magnitude smaller than this value [3], or very uncertain, leading to values of this ratio varying all over the range $10^{-4} \leq \epsilon'/\epsilon \leq 10^{-3}$ [4,5]. In the present note we discuss some consequences for the $K^\pm \rightarrow 3\pi$ decays.

In [6–8], it was found that contrary to the case of $\epsilon'$, in $K^\pm \rightarrow \pi^\pm\pi^\pm\pi^\mp$ decay, the EWP contribution enhances the QCDP contribution. But in order to estimate the magnitude of the CP-violating effect, it was necessary to resort to unreliable theoretical estimates of the QCDP and the EWP contributions (see Ref. [8]).

For the $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$ decay, the situation is cleaner, because as explained in the present note, the CP-odd asymmetry $\Delta g$ in this case turns out to be proportional to practically the same combination of QCDP and EWP contributions as in $\epsilon'$. Consequently, $\Delta g$ can be estimated reliably using the known value of $\epsilon'$. For the $K^\pm \rightarrow \pi^\pm\pi^\pm\pi^\mp$ decay, on the other hand, we argue that $\Delta g$ may be one order of magnitude larger than in the $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$ decay. This conclusion differs from those proposed in Refs. [9,10].

Our investigation is based on the effective $\Delta S = 1$ nonleptonic Lagrangian proposed in Ref. [11],

$$\mathcal{L}(\Delta S = 1) = \sqrt{2} G_F \sin \theta_C \cos \theta_C \sum_i c_i O_i,$$

(1)

where the $O_i$ are four-quark operators, defined as

$$O_1 = \bar{s}_L Y_{\mu}d_L \cdot \bar{u}_L Y_{\mu}u_L - \bar{s}_L Y_{\mu}u_L \cdot \bar{u}_L Y_{\mu}d_L,$$

$$O_2 = \bar{s}_L Y_{\mu}d_L \cdot \bar{u}_L Y_{\mu}u_L + \bar{s}_L Y_{\mu}u_L \cdot \bar{u}_L Y_{\mu}d_L + 2\bar{s}_L Y_{\mu}d_L \cdot \bar{d}_L Y_{\mu}d_L + 2\bar{s}_L Y_{\mu}d_L \cdot \bar{s}_L Y_{\mu}s_L,$$

$$O_3 = \bar{s}_L Y_{\mu}d_L \cdot \bar{u}_L Y_{\mu}u_L + \bar{s}_L Y_{\mu}u_L \cdot \bar{u}_L Y_{\mu}d_L + 2\bar{s}_L Y_{\mu}d_L \cdot \bar{d}_L Y_{\mu}d_L - 3\bar{s}_L Y_{\mu}d_L \cdot \bar{s}_L Y_{\mu}s_L,$$

$$O_4 = \bar{s}_L Y_{\mu}d_L \cdot \bar{u}_L Y_{\mu}u_L + \bar{s}_L Y_{\mu}u_L \cdot \bar{u}_L Y_{\mu}d_L - \bar{s}_L Y_{\mu}d_L \cdot \bar{d}_L Y_{\mu}d_L.$$
\[ O_5 = \bar{s}_L \gamma \mu \lambda^a d_L \left( \sum_{q=u,d,s} \bar{q}_R \gamma \mu \lambda^a q_R \right), \]
\[ O_6 = \bar{s}_L \gamma \mu \gamma \nu q_R \left( \sum_{q=u,d,s} \bar{q}_R q_R \right). \]  

(2)

For our study of CP violation, we must add two more four-quark operators,
\[ O_7 = \frac{3}{2} \bar{s}_L \gamma \mu (1 + \gamma_5) d_L \left( \sum_{q=u,d,s} e_q \bar{q}_R \gamma \mu (1 - \gamma_5) q_R \right), \]
\[ O_8 = -12 \sum_{q=u,d,s} e_q (\bar{s}_L q_R)(\bar{q}_R d_L). \]  

(3)

where \( e_q \) is the quark-charge matrix.

The operators \( O_{5-8} \) arise from the QCD penguin diagram and the operators \( O_{7,8} \) arise, analogously, from the electroweak penguin diagram. The Wilson coefficients \( c_{5-8} \) contain the imaginary parts necessary for CP violation. The bosonization of the operators \( O_{1-8} \) can be achieved by exploiting the relations between di-quark field operators and pseudoscalar fields as represented in [12], and the reordering relations in colour and spinor spaces as from [13].

Representing the \( K \rightarrow 2\pi \) amplitudes in the form
\[ M(K^0 \rightarrow \pi^+\pi^-) = A_0 e^{i\delta_0} - A_2 e^{i\delta_2}, \]
\[ M(K^0 \rightarrow \pi^0\pi^0) = A_0 e^{i\delta_0} + 2A_2 e^{i\delta_2}, \]  

(4)

this approach yields
\[ A_0 = \kappa \left[ c_1 - c_2 - c_3 + \frac{32}{9} \beta (\Re \tilde{c}_5 + i \Im \tilde{c}_5) \right], \]
\[ A_2 = \kappa \left[ c_4 + i \frac{2}{3} \beta A^2 \Im \tilde{c}_7 (m_K^2 - m_\pi^2)^{-1} \right]. \]  

(5)

(6)

Here, \( \delta_0 \) and \( \delta_2 \) are the pion-pion scattering phase shifts in the isospin \( T = 0 \) and \( T = 2 \) channels, and the remaining parameters are
\[ \kappa = G_F F_\pi \sin \theta_C \cos \theta_C \frac{m_K^2 - m_\pi^2}{\sqrt{2}}, \]
\[ \beta = \frac{2m_\pi^4}{A^2(m_u + m_d)^2}, \]
\[ \tilde{c}_5 = c_5 + \frac{3}{16} \tilde{c}_6, \quad \tilde{c}_7 = c_7 + 3c_8, \quad \Lambda \approx 1 \text{ GeV}. \]

Since \( \tilde{c}_7/\tilde{c}_5 \sim \alpha_{\text{em}} \) and small, we have neglected the EWP contribution to \( A_0 \).

From data on \( K \rightarrow 2\pi \) rates one can deduce the values of the real parts of the amplitudes \( A_0 \) and \( A_2 \) [14], i.e.,
\[ c_4 = 0.328, \]
\[ c_1 - c_2 - c_3 + \frac{32}{9} \beta \Re \tilde{c}_5 = -10.13. \]  

(7)

(8)

Furthermore, if as suggested by Refs. [11,13], we assume \( c_1 - c_2 - c_3 = -2.89 \), then we have in addition \( -\frac{32}{9} \beta \Re \tilde{c}_5 = -7.24. \)

Using the definition of the parameter \( \epsilon' \),
\[ \epsilon' = \epsilon e^{i(\delta_2 - \delta_0)} \left[ -\frac{\Re A_0}{\Re A_2} + \frac{\Re A_2}{\Re A_0} \right] A_2 / A_0, \]  

(9)

and its experimental value, we deduce
\[ -\frac{\Re \tilde{c}_5}{\Re \tilde{c}_7} \left( 1 - \Omega + 24.4 \frac{\Re \tilde{c}_7}{\Im \tilde{c}_5} \right) = (1.63 \pm 0.16) \times 10^{-4}. \]  

(10)

The new parameter \( \Omega \) takes into account effects of isospin violation, coming from the quark mass difference \( m_d \neq m_u \) and the electromagnetic interaction. As a result hereof, the physical state vector of the isovector \( I = 1 \) neutral pi-meson acquires an admixture of states with isospin \( I = 0 \),
\[ |\pi^0_{\text{phys}}\rangle = |\pi^0\rangle + \lambda |\eta\rangle + \lambda' |\eta'\rangle. \]  

(11)

For a recent review see Ref. [15].

As a consequence of mixing there are alternative contributions to the \( K \rightarrow \pi^0\pi^0 \) decays. The weak-interaction Lagrangian of Eq. (1) can in a first step induce the transitions \( K^0 \rightarrow \pi^0 \eta(\eta') \) which, in a second step, are followed by the transitions \( \eta(\eta') \rightarrow \pi^0 \) induced by the isospin mixing of Eq. (11). Thus, in the tree approximation, the isospin decompositions of the \( K^0 \rightarrow \pi^+\pi^- \) and \( K^0 \rightarrow \pi^0\pi^0 \) amplitudes change into
\[ \langle \pi^+\pi^- | H_w | K^0 \rangle = (A_0 - \gamma)(I=0) - (A_2 - \gamma)(I=2) \]
\[ = A_0 - A_2, \]
\[ \langle \pi^0\pi^0 | H_w | K^0 \rangle = (A_0 - \gamma)(I=0) + 2(A_2 - \gamma)(I=2) \]
\[ = A_0 + 2A_2 - 3\gamma. \]

Here, \(-\gamma\) is the matrix element coming from the mixing. It is the same for both isospin channel amplitudes and \(1/3\) of the amplitude for the \( K^0 \rightarrow \eta(\eta') \rightarrow \pi^0\pi^0 \) transition.

Now, in the absence of EWP contributions, the combination
\[ -\frac{\Re A_0}{\Re A_2} \frac{\Im \gamma}{\Im \tilde{c}_5} \left[ 1 - \frac{\Re A_0}{\Re A_2} \frac{\Im \gamma}{\Im \tilde{c}_5} \right] \]
\[ \equiv -\frac{\Re A_0}{\Re A_0}[1 - \Omega], \]  

(12)

(13)

of Eq. (9) transforms into
\[ \beta \Re \tilde{c}_5 \left[ 1 + (261.1 \pm 2.1) \frac{\Re \tilde{c}_7}{\Im \tilde{c}_5} \right] = (3.56 \pm 0.61) \times 10^{-4}. \]  

(14)

Below, we shall see that the numerical result of Eq. (14) leads to a reliable estimate of the asymmetry parameter \( \Delta g \) in the \( K^\pm \rightarrow \pi^0\pi^0 \) decay.

Let us now turn to the \( K^\pm \rightarrow 3\pi \) decays. Applying the same techniques as above and taking into account the appearance of
CP-even imaginary parts due to strong $\pi\pi$ final-state rescattering, we get in leading $p^2$ approximation for the $\tau$ and $\tau'$ decay amplitudes:

$$M(K^\pm(k) \to \pi^\pm(p_1)\pi^\pm(p_2)\pi^\mp(p_3)) = \tilde{k}\left[1 + ia + \frac{1}{2}g_eY(1 + ib^r \pm id_{\text{KM}}^r) + \cdots\right]. \quad (15)$$

$$M(K^\pm(k) \to \pi^0(p_1)\pi^0(p_2)\pi^\mp(p_3)) = \frac{k}{2}\left[1 + ia + \frac{1}{2}g_eY(1 + ib^r \pm id_{\text{KM}}^r) + \cdots\right]. \quad (16)$$

The indices $\tau$ and $\tau'$ refer to the decay modes of the kaon. The parameters $a$, $b^r$ and $b'^r$ arise from the strong pion–pion rescattering and are consequently CP-even. The $d_{\text{KM}}^r$ are CP-odd imaginary terms produced by the Kobayashi–Maskawa phase. Furthermore, $Y$ is a kinematic factor, $Y = (s_3 - s_0)/m_\pi^2$, with $s_3 = (k - p_3)^2$ and $s_0 = 4m_K^2 + m_\pi^2$.

In $K^\pm \to \pi^\pm\pi^\mp\pi^\mp$ decay, the parameter values are

$$a = 0.12, \quad b^r = 0.71, \quad g_\tau = -\frac{3m_\pi^2}{m_K^2}(1 + 9c_4/c_0),$$

$$c_0 = c_1 - c_2 - c_3 - c_4 + \frac{32}{9}\beta\text{Re}\tilde{c}_5 = -10.46,$$

$$\tilde{k} = G_Fm_K^2 \sin\theta_C \cos\theta_C c_0/3\sqrt{2},$$

and for the CP-odd contribution we get

$$d_{\text{KM}}^r = -\frac{32}{9}\beta\text{Im}\tilde{c}_5\frac{9c_4}{c_0(c_0 + 9c_4)} \times \left[1 + \frac{3A^2(c_0 + 9c_4)}{16m_K^2c_4}(1 + \frac{12c_4m_K^2}{A^2(c_0 + 9c_4)})\text{Im}\tilde{c}_7\right] = -2\frac{16c_4}{c_0(c_0 + 9c_4)}\beta\text{Im}\tilde{c}_5(1 - 14.36\frac{\text{Im}\tilde{c}_7}{\text{Im}\tilde{c}_5}). \quad (17)$$

In $K^\pm \to \pi^0\pi^0\pi^\pm$ decay, two parameters are different, i.e.,

$$b'^r = 0.49, \quad g_\tau' = \frac{6m_\pi^2}{m_K^2}(1 - 9c_4/2c_0),$$

as is the CP-odd contribution

$$d_{\text{KM}}^r = \frac{32}{9}\beta\text{Im}\tilde{c}_5\frac{9c_4/2}{c_0(9c_4 - 9c_4/2)} \times \left[1 - \frac{3A^2(c_0 - 9c_4/2)}{8m_K^2c_4}\right] \times \left[1 - \frac{c_0m_K^2}{2(c_0 - 9c_4/2)(m_K^2 - \frac{m_\pi^2}{2})}\text{Im}\tilde{c}_7\right] = \frac{16c_4}{c_0(c_0 - 9c_4/2)}\beta\text{Im}\tilde{c}_5\left(1 + 27.8\frac{\text{Im}\tilde{c}_7}{\text{Im}\tilde{c}_5}\right). \quad (18)$$

The slope parameters $g_\tau^\pm$ in $\tau$ decay are defined by the equation

$$|M(K^\pm(k) \to \pi^\pm(p_1)\pi^\pm(p_2)\pi^\mp(p_3))|^2 \sim [1 + g_\tau^\pm Y + \cdots] \quad (19)$$

with a similar definition for $g_\tau'^\pm$ in $\tau'$ decay. The CP-asymmetry parameters $\Delta g_{\tau,\tau'}$ in the two decays are defined as

$$\Delta g_\tau = \frac{g_\tau^+ - g_\tau^-}{g_\tau^+ + g_\tau^-} = \frac{a\tilde{d}_{\text{KM}}^r}{1 + ab^r}.$$  

$$\Delta g_\tau' = \frac{g_\tau'^+ - g_\tau'^-}{g_\tau'^+ + g_\tau'^-} = \frac{a\tilde{d}_{\text{KM}}^r}{1 + ab'^r}. \quad (20)$$

Discussing first $\tau'$ decay, we realise when comparing Eqs. (18) and (14) that the linear combinations of QCDP and EWP contributions appearing in these expressions are very similar. In fact, at $\Omega = 0.124$ the two combinations are identical. Thus, exploiting our knowledge of the experimental value of $\epsilon'$ we predict for the asymmetry parameter of Eq. (20)

$$\Delta g_\tau' = (1.8 \pm 0.24) \times 10^{-6}. \quad (21)$$

At $\Omega_{\text{eff}} = 0.060 \pm 0.077$ from Ref. [19] $\Delta g_\tau' = (1.71 \pm 0.29) \times 10^{-6}. \quad (22)$$

We conclude that due to the close resemblance of the expressions for $\epsilon'$ and $\Delta g_\tau'$ decay our prediction for $\Delta g_\tau$ should be quite robust.

The CP-asymmetry parameter in $\tau$ decay is most easily discussed via the ratio

$$\frac{-\Delta g_\tau}{\Delta g_\tau'} = 2\left(\frac{c_0 - 9c_4/2}{c_0 + 9c_4}\right)\left(\frac{1 + ab^r}{1 + ab'^r}\right) \times \frac{1 - 14.36\text{Im}\tilde{c}_7/\text{Im}\tilde{c}_5}{1 + 27.8\text{Im}\tilde{c}_7/\text{Im}\tilde{c}_5}. \quad (23)$$

which is obtained by combining Eqs. (17)–(19). Therefore,

(a) if the EWP contributions do not play any significant role in direct CP violation, i.e., when $\text{Im}\tilde{c}_7/\text{Im}\tilde{c}_5$ is negligibly small, then

$$-\Delta g_\tau/\Delta g_\tau' = 3.1 \quad \text{or} \quad -\Delta g_\tau \geq 0.56 \times 10^{-5}; \quad (24)$$

(b) if the EWP contribution cancels half of the QCD contribution in $\epsilon'$ (see Refs. [20,21]), then

$$-\Delta g_\tau = 7.8\Delta g_\tau' \geq 1.3 \times 10^{-5}. \quad (25)$$

The above results are obtained in leading $p^2$ approximation. The role of $p^4$ corrections for $\Delta g_\tau$ were studied in Refs. [6, 8], and they were found to increase the value of $\Delta g_\tau$ by 23%. For $\Delta g_\tau'$ the corresponding investigation has not yet been performed. But one effect can be seen at once. According to Refs. [6,8], the corrections of order $p^4$ increase the rescattering parameter $a$ in Eq. (20) by 30%. Thus, we expect the corrected value of $\Delta g_\tau'$ to lie in the range $(1.8-2.5) \times 10^{-6}$.

Finally, we remark once more that our numerical results not only differ from those reported in [9], which are $-\Delta g_\tau = (2.3 \pm 0.6) \times 10^{-6}$ and $\Delta g_\tau' = (1.3 \pm 0.4) \times 10^{-6}$, but also from the more recent ones reported in [10], which are $-\Delta g_\tau = (2.4 \pm 1.2) \times 10^{-5}$ and $\Delta g_\tau' = (1.1 \pm 0.7) \times 10^{-5}$. Both investigations were performed within the framework of chiral
perturbation theory. In Ref. [10] attempts were made to estimate contributions of order $p^4$, but the predicted value for $\Delta g_\tau'$ has large uncertainties.

Our results strongly suggest, that accurate measurements of $\Delta g_\tau$ and $\Delta g_\tau'$ should clarify the relative importance of QCDP and EWP mechanisms in direct CP violation.

References