Laminar Film Condensation on a Horizontal Disk with Suction at the Wall

J. S. CHIOU AND T. B. CHANG
Mechanical Engineering Department
National Cheng Kung University, Tainan, Taiwan, R.O.C.

(Received April 1993; revised and accepted December 1993)

Abstract—The problem of two-dimensional, steady-state film condensation on an isothermal horizontal disk with suction at the porous wall is studied for the case in which a cold plate faces upward. The dimensionless film thickness along the disk is found to be a function of parameter Ja/Pr (Jakob number/Prandtl number) and the suction parameter Sw. An essential part of the present analysis is the use of the condition that the boundary layer depth at the disk edge is equal to a critical(minimum) depth. The dimensionless heat transfer coefficients are also found to be functions of parameters Ja/Pr and Ra/Ja. Furthermore, the dimensionless heat transfer coefficient increases as suction parameter Sw increases.

Keywords—Horizontal disk condensation, Laminar condensation, Suction, Filmwise condensation.

NOMENCLATURE

\[ C_p \] specific heat at constant pressure
\[ g \] acceleration of gravity
\[ h \] heat transfer coefficient
\[ h_{fg} \] heat of vaporization
\[ Ja \] Jakob number defined in equation (7a)
\[ k \] liquid thermal conductivity
\[ L \] disk radius
\[ m \] condensate mass flux
\[ Nu \] Nusselt number defined in equation (16)
\[ P \] pressure
\[ P_{sat} \] saturation pressure
\[ Ra \] Rayleigh number
\[ R \] dimensionless coordinate defined by \( r/L \)
\[ Sw \] suction parameter defined in equation (7c)
\[ T \] temperature
\[ \Delta T \] saturation temperature minus wall temperature
\[ u, v \] radius and vertical velocity components
\[ r, y \] radius and vertical coordinate axes

\[ \delta \] condensate film thickness
\[ \delta_0 \] condensate film thickness at disk center
\[ \delta_{min} \] minimum film thickness at disk edge
\[ \eta \] dimensionless film thickness defined as \( \delta/\delta_0 \)
\[ \lambda \] defined by \( h_{fg} + \frac{3}{8} C_p \Delta T \)
\[ \mu \] liquid viscosity
\[ \rho \] liquid density

SUPERSCRIPTS
- indicates averaged quantity

SUBSCRIPTS
\[ 0 \] quantity at disk center
\[ c \] critical quantity
\[ \text{min} \] minimum quantity or quantity at the disk edge
\[ \text{sat} \] saturation property
\[ w \] quantity at wall
INTRODUCTION

The understanding of laminar film condensation phenomena is found to be important in the design of heat exchangers and other chemical process plants. It is also useful in modern manufacturing applications such as mass soldering process (also called condensation soldering) and chemical vapor deposition. The classical analysis of laminar film condensation on vertical or inclined surfaces was first performed by Nusselt [1]. In that analysis, the condensate film was assumed to be thin; convective and inertial effects were negligible. Within the condensate film, the temperature profile is linear.

Since the publication of Nusselt's paper in 1916, many other researchers had extended the analyses by either removing the overly restricted assumptions or performing the similar analyses on different geometries, a review for these analyses can be found in Merte's work [2].

For the cases with wall suction, Jain and Bankoff [3] and Yang [4] analyzed the laminar film condensation on a vertical plate with an uniform velocity at the porous wall. Lienhard and Dhir [5] tried to generalize the solution of film thickness for vertical film condensation with arbitrary specified suction velocity by using the Nusselt-Rohsenow approach. For the analyses with horizontal surfaces, the earliest study was due to Popov [6], who experimentally investigated the heat transfer rates of vapor condensation on a horizontal surface. Gerstmann and Griffith [7] then studied the condensation on the underside of a horizontal plate both theoretically and experimentally. The film condensation on the upper side of the horizontal plate was first investigated by Leppert and Nimmo [8,9]. However, they left out one point in their argument, that is, the condensate thickness at the plate edge is either assumed or specified by the particular overfall condition. Recently, Shigechi et al. [10] analyzed the same problem, their solution form of condensate thickness included the inclination angle of the vapor-liquid interface at the plate edge, and showed the film thickness decreased as the inclination angle increased.

In the present problem, saturated vapors are condensed on a horizontal disk, the condensates with the maximum thickness at the disk center are flowing radially outward. Under the influence of hydrostatic pressure gradient and suction velocity along the wall, parts of the condensates eventually fall off at the disk edge. In order to solve the film thickness and heat transfer coefficient on the disk, it is necessary to know the film thickness at the disk edge (the other boundary condition besides the one at the center). The thickness in this analysis is calculated by applying the minimum mechanical energy condition for the flowing condensates at the disk edge rather than arbitrary assumed. The concept of minimum mechanical energy was used in the hydraulic of open channel [11].

ANALYSIS

Consider a pure quiescent vapor at saturated state with temperature $T_{\text{sat}}$ and pressure $P_{\text{sat}}$ condensing on a horizontal isothermal porous disk, which has the temperature at $T_{w}$ and the diameter at $2L$ as shown in Figure 1. Under steady-state conditions, the laminar film condensation boundary layer, is established with the maximum depth at the center and the minimum (critical) depth at the disk edge. The flowrate of condensate within the boundary layer increases from zero at $r = 0$ to the maximum value at the edge of the disk. For this two-dimensional, steady-state, laminar film condensate flow on a horizontal disk, the general assumptions of Nusselt's classical analysis are made as follows:

1. The flow is steady and laminar.
2. The inertia within the film is negligible (for a creeping film flow).
3. The wall temperature and the vapor temperature are uniform and keep constant.
4. The kinetic energy within the film flow is negligible (for a creeping flow).
5. Physical properties in boundary layer are assumed constants.
The governing equations are:

continuity:
\[
\frac{1}{r} \frac{\partial}{\partial r}(ru) + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

r-momentum:
\[
0 = -\frac{\partial P}{\partial r} + \mu \left( \frac{\partial^2 u}{\partial y^2} \right), \tag{2}
\]

hydrostatic pressure:
\[
P = P_{\text{sat}} + \rho g (\delta - y). \tag{3}
\]

Substituting equation (3) into equation (2) yields
\[
\frac{\partial^2 u}{\partial y^2} = \frac{\rho g \delta}{\mu} \frac{d\delta}{dr}. \tag{4}
\]

By using the boundary conditions of \( u = 0 \) at \( y = 0 \) and \( \frac{\partial u}{\partial y} = 0 \) at \( y = \delta \) (no interfacial shear), we obtain
\[
u = \frac{\rho g \delta}{\mu} \frac{d\delta}{dr} \left( \frac{y^2}{2} - y\delta \right), \tag{4}
\]

assuming a linear temperature profiles through the condensate film yields
\[
k \frac{\partial T}{\partial y} = k \frac{T_{\text{sat}} - T_{w}}{\delta} = k \frac{\Delta T}{\delta}. \tag{5}
\]
The energy balance on a finite volume of liquid condensate at the radius of \( r \) can be written as

\[
2\pi r \, dr \left( k \frac{\partial T}{\partial y} \bigg|_{y=0} \right) - \frac{d}{dr} \left\{ \int_0^\delta \rho u [h_f g + C_p(T_{sat} - T)] r \, dy \right\} dr + 2\pi r dr \rho V_w (h_f g + C_p \Delta T) \tag{6}
\]

the left hand side is the conductive energy transfer from liquid film to the solid plate, and the first term on the right hand side is the net energy flux across the liquid films (from \( r \) to \( r + dr \)) while the second term is the energy flux flowing through the porous plate due to suction, equation (6) can be simplified as

\[
k \frac{\Delta T}{\delta} = \frac{1}{r} \frac{d}{dr} \left\{ \int_0^\delta \rho u [h_f g + C_p(T_{sat} - T)] r \, dy \right\} + \rho V_w (h_f g + C_p \Delta T), \tag{6a}
\]

substituting equation (4) into equation (6a) yields

\[
\frac{\delta}{r} \frac{d}{dr} \left( r^2 \frac{d\delta}{dr} \right) = - \frac{3k \Delta T \mu}{\rho^2 g \lambda} + 3d \frac{V_w \mu (\lambda + 0.625 C_p \Delta T)}{\rho g \lambda},
\]

where

\[
\lambda = h_f g + \frac{3}{8} C_p \Delta T.
\]

Defining the following parameters:

\[
Ja = \frac{C_p \Delta T}{\lambda}, \tag{7a}
Ra = \frac{\rho^2 g \mu I^3}{\mu^2}, \tag{7b}
S_w = \frac{Pr \rho V_w L}{Ja \mu} (1 + 0.625 Ja), \tag{7c}
\]

we may rewrite the above energy equation as

\[
\frac{\delta}{r} \frac{d}{dr} \left( r^2 \frac{d\delta}{dr} \right) = - \frac{3Ja}{Ra} L^3 + 3 \frac{Ja}{Ra} S_w L^2 \delta, \tag{8}
\]

and the boundary conditions of

\[
\frac{d\delta}{dr} = 0, \quad \text{at } r = 0, \tag{8a}
\delta = \delta_{\text{min}} (= \delta_c), \quad \text{at } r = L. \tag{8b}
\]

Since \( \delta_{\text{min}} \) is still unknown, equation (8) cannot be solved at this moment, however, we know the film thickness at the disk edge cannot be zero. In fact, it should be established according to the minimum mechanical energy principle [11], just like that used in open-channel hydraulics. The principle states that a fluid flowing across a horizontal plate under the influence of a hydrostatic pressure gradient and off the edge of the plate will adjust itself so that the rate of mechanical energy within the fluid will be minimum with respect to the boundary layer thickness at the plate. Hence, the minimum (critical) thickness can be calculated by setting the derivative of the mechanical energy with respect to \( \delta \) equal to zero for steady flow rate, which means

\[
\left[ \frac{\partial}{\partial \delta} \int_0^\delta 2\pi r \left( \frac{u^2}{2} + gy + \frac{P}{\rho} \right) \rho u \, dy \right]_{m_c} = 0, \tag{9}
\]

where \( m_c \) is the critical value of mass flow out of the disk edge.
The rate of condensed mass flux at any section \( r \) may then be found as follows:

\[
m = 2\pi r \int_0^6 \rho \mu \, dy = 2\pi r \frac{\rho^2 g}{\mu} \frac{d\delta}{dr} \left( -\frac{\delta^3}{3} \right),
\]

at the disk edge, it becomes

\[
m_c = 2\pi L \frac{\rho^2 g}{\mu} \frac{d\delta}{dr} \bigg|_{L} \left( -\frac{\delta_{\min}^3}{3} \right).
\]

(10)

By substituting equation (4) and equation (3) into equation (9), after integration, and with the aid of equation (10), we obtain the relation between the critical mass flow rate and the minimum film thickness at the disk edge as follows:

\[
\delta_{\min} = \left( \frac{54 m_c^2}{35 \rho^2 g} \right)^{1/3} \quad \text{or}
\]

\[
m_c = \left[ \frac{35}{54} \rho^2 g \delta_{\min}^3 \right]^{1/2}.
\]

(11a)

(11b)

Equating equation (10) and (11b) yields

\[
\frac{d\delta}{dr} \bigg|_{L} = - \left( \frac{35 \Pr L^3}{6 \Ra \delta_{\min}^3} \right)^{1/2}.
\]

(12)

Introducing the normalized variables \( R = r/L; \eta = \delta/\delta_0 \), equation (8) becomes

\[
\frac{\eta}{R} \frac{d}{dR} \left( R \eta^3 \frac{d\eta}{dR} \right) = -\frac{3J_0}{\Ra} \left( \frac{L}{\delta_0} \right)^5 + \frac{3}{\Ra} S_w \eta \left( \frac{L}{\delta_0} \right)^4,
\]

(13)

equation (12) becomes

\[
\frac{d\eta}{dR} \bigg|_{L} = - \left( \frac{35 \Pr}{6 \Ra \delta_{\min}^3} \right)^{1/2} \left( \frac{L}{\delta_0} \right)^{5/2},
\]

(14)
and the boundary condition of equation (8a) becomes

\[
\frac{d\eta}{dR} = 0, \quad \text{at} \quad R = 0.
\]

(15)

In the calculation, \( \eta \) is equal to one at \( R = 0 \).

From equation (13) and the corresponding boundary conditions, equation (14) and equation (15), we can obtain the dimensionless film thickness as a function of \( J_a, \Ra, \Pr, S_w \).

The numerical procedure is to guess a \( \delta_0 \) value and substitute \( \delta_0 \) into equation (13), with the aid of equation (15), \( \eta \) value along the radial direction can be obtained. After all of \( \eta \) values (at every grid points) have been calculated, we check equation (14) and obtain a new modified value for \( \delta_0 \). This process is repeated until

\[
\frac{d\eta}{dR} \bigg|_{L} + \left( \frac{35 \Pr}{6 \Ra \delta_{\min}^3} \right)^{1/2} \left( \frac{L}{\delta_0} \right)^{5/2} \leq \varepsilon, \quad \text{where} \quad \varepsilon = 10^{-4}.
\]

Since the slope of film thickness is steeper at the disk edge, the progressive finer grid size toward the edge (with 1200 total grid points) is used in the calculations.

The mean Nusselt number is then calculated as

\[
\overline{Nu} = \frac{L \overline{h}}{k},
\]

(16)

where

\[
\overline{h} = \frac{1}{\pi L^2} \int_0^L h(x) 2\pi r \, dr = \frac{1}{L^2} \int_0^L \delta(x) 2r \, dr - \frac{1}{L^2 \delta_0} \int_0^L 2r \, dr.
\]
RESULTS AND DISCUSSIONS

Figure 2 shows the distributions of dimensionless film thickness \( \eta \), \( \eta = (\delta)/(\delta_0) \) along the radial location with no suction at two different Ja/Pr values.

It is obvious that the minimum thickness at the disk edge increases as the parameter Ja/Pr increases. This is because more condensate will be formed as Ja/Pr becomes larger. If the surface tension energy is also considered in determining \( \delta_{\text{min}} \), the critical film thickness at the disk edge will become larger, as the surface tension force can hold up more condensate.

Figure 3a shows the distributions of dimensionless film thickness \( \eta \) near the edge of disk with constant Ja/Pr at three different suction parameters \( S_w \). The minimum thickness at the disk edge decreases as the suction parameter \( S_w \) increases. Figure 3b depicts the dimensional thickness along the radial direction. It is obvious that the film thickness is thinner whenever \( S_w \) is larger.

Since the thermal resistance increases as the liquid film becomes thicker, a thicker film will thus reduce the heat transfer coefficient. Figure 4 shows the Nusselt number increases as \( S_w \) increases, and linearly decreases as Ja/Pr increases. In fact, the relationship can be approximately correlated by

\[
\overline{N_u} \left( \frac{Ja}{Ra} \right)^{0.2} = 1.03 - 0.188 \left( \frac{Ja}{Pr} \right) + 0.0008S_w^{1.1}.
\]  

(17)

For the case of Ja/Pr < 0.1 and \( S_w < 200 \), the suction velocity is about 0.05 mm/sec when \( S_w = 100 \). For the case of no suction \( (S_w = 0) \), the water vapor condensing on a smooth plate at one atmosphere condition, the value of Ja/Pr is about 0.0025. The resulting Nusselt number is

\[
\overline{N_u} = 1.0295 \left( \frac{Ra}{Ja} \right)^{0.2}.
\]  

(18)

The experimental data on a finite-size horizontal plate by Nimmo and Leppert [5] is fitted into the following form:

\[
\overline{N_u} = 0.64 \left( \frac{Ra}{Ja} \right)^{0.2}.
\]  

(19)

Compare equation (18) and equation (19), the present value is about 60% higher than that of Nimmo's [5].
Figure 3a. Dimensionless film profile near the edge of the disk at different $S_w$.

Figure 3b. The distribution of film thickness.

Figure 4. Dimensionless heat transfer coefficient vs. $Ja/Pr$. 
Several reasons might contribute to this difference. For example, the solid plate analyzed here is a circular disk instead of flat plate, therefore, the condensed liquid will spread out wider and thinner. The Nusselt number is thus larger. Besides, the surface tension energy was neglected in calculating the critical film thickness, the obtained thickness should be smaller than the value if the surface tension effect had been included. Other factors, such as impurity, might deposit on the test plate after a prolong test will also reduce the heat transfer coefficient. Nevertheless, the present analysis and the test results all indicate that the mean Nusselt number increases with one fifth power of Ra/Ja instead of one fourth power, which was found by Nusselt [1] for vertical surfaces.

REFERENCES