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Neutrino masses and mixings in a predictive $SO(10)$ model with CKM CP violation

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Abstract

It has recently been shown that a minimal $SO(10)$ model with a single **10** and a single **126** Higgs field breaking B–L symmetry predicts large solar and atmospheric mixings in agreement with observations if it is assumed that the neutrino mass obeys the type II seesaw formula. No additional symmetries need to be assumed for this purpose. Understanding CP violation in the renormalizable version of the model, however, requires a significant non-CKM source. In this Letter we show that if we extend the model by the inclusion of a heavy **120**-dimensional Higgs field, then it can accommodate CKM CP violation while remaining predictive in the neutrino sector. Among the predictions are: (i) solar mixing angle in the observed range; (ii) θ_{13} in the range of 0.1 to 0.26; (iii) the Dirac phase close to maximal for the central value of the solar mixing angle.

1. Introduction

The simplest grand unified model for understanding small neutrino masses appears to be the $SO(10)$ model [1] for the following reasons: (i) it automatically brings in the right-handed neutrino, N_R, needed to implement the seesaw [2] mechanism since it fits in with other standard model fermions in the **16**-dimensional spinor representation (ii) it contains the $SU(4)_c$ symmetry [3] which relates the quark and lepton coupling parameters and in turn helps the predictivity of the model in the neutrino sector by reducing the number of parameters; (iii) it also contains the B–L symmetry [3,4] needed to keep the right-handed neutrino masses below the Planck scale and provides a group theoretic explanation of why neutrinos are necessarily Majorana particles.

While all these make the $SO(10)$ models appealing for neutrino mass studies, detailed quantitative predictions generally involve too many parameters limiting the predictive power unless extra symmetries (e.g., family symme-

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tries, etc.) are imposed on the theory. One exception to this is the class of models that uses only one **10** and one **126** Higgs multiplet to generate fermion masses [5]. The original set of papers on this model [5,6] used type I seesaw formula for neutrino mass is given by $\mathcal{M}_\nu = -M_\nu^D M_R^{-1} (M_\nu^D)^{\mathrm{T}}$, where M_ν^D is the Dirac mass of the neutrinos and M_R is the mass matrix of the right-handed neutrinos. These predictions are now in contradiction with experiments.

It was subsequently pointed out in Ref. [7] that if one uses type II seesaw formula for the neutrino masses [8] instead, the model automatically predicts large atmospheric mixing angle due to the fact that bottom quark and tau lepton masses converge towards each other when extrapolated to the GUT scale. The question remained whether this works for three generations and can lead to a realistic model for neutrinos. It was shown in Ref. [9] that the same b–τ mass convergence not only leads to a large solar mixing angle, but also to a small and detectable value for $U_{e3} \equiv \sin\theta_{13}$. A detailed numerical analysis was carried out that showed that the model is indeed in agreement with present neutrino data and in particular the prediction of a "large" value for θ_{13} which makes this model testable at the current as well as at the proposed long base line neutrino experiments.

In the three generation neutrino discussion in Ref. [9], the Yukawa couplings of fermions were assumed to be real and all CP-violating effects were assumed to originate from the supersymmetry breaking sector. It is, however, interesting to check if one can accommodate the CKM phase in the model by introducing phases in the couplings. A detailed investigation of the minimal model where CP violation is introduced through complex Yukawa couplings (as in the standard model) showed [10] that compatibility with neutrino data requires the CKM phase to be outside the first quadrant whereas the standard model CKM phase is in the first quadrant [11]. This would seem to imply that in order to understand observed CP violation in this model, one must invoke a significant non-CKM source for CP violation (as in the model with real Yukawa couplings), e.g., CP violation from the supersymmetry breaking sector. This could very well be true. However, since all observed CP-violating phenomena seem to be explainable by the CKM model, it is important to see whether one can explain both CKM CP violation and neutrino mixings by a minimal modification of this $SO(10)$ model. There are also other issues such as SUSY CP problem that one needs to address in the context of supersymmetry and it would be interesting to see how these can be addressed in this model.

In this Letter, we propose a very minimal way to incorporate CP violation into the model, which not only leads to a predictive model in the neutrino sector but also seems to have wider implication beyond just explaining CKM CP violation. For instance, the model presents a solution to the SUSY CP problem.

In order to attain our goal, we include a heavy **120** field with an extra Z_2 symmetry which we will call "parity" symmetry imposed on the model.[1] At energy scales below the mass of the **120** field, the effect of this field is to appear as a higher-dimensional contribution to the Yukawa couplings. This effective theory has the following properties. Despite the fact there are now three extra parameters in the model, the theory still remains predictive in the neutrino sector. Secondly, the mass matrices for quarks and leptons are hermitian, which therefore has the potential to solve other problems of supersymmetric models such as the SUSY CP problem. In this Letter we focus only on the neutrino sector.

The main results of this Letter are as follows: (i) using type II seesaw formula we are able to accommodate the CKM CP phase while keeping the model predictive in the neutrino sector; for example, we predict the solar mixing angle in the right range and $U_{e3} \geqslant 0.1$; (ii) the model has the potential to solve the SUSY CP problem and (iii) it predicts the Dirac phase of PMNS matrix to be near maximal for the central value of the solar mixing angle $\tan^2\theta_\odot \simeq 0.4$.

The Letter is organized as follows: in Section 2, we introduce the model with the inclusion of the **120** Higgs field and write down the fermion mass formulae in the general case; in Section 3, we impose parity symmetry on the model making it predictive in the neutrino sector; in Section 4, we discuss the predictions for neutrino mixings and Dirac CP phase for neutrinos; in Section 5, we present our conclusions and discuss the outlook for the model.

[1] Very different extensions of the model that use **120** but no symmetry have been discussed in Ref. [13].

2. $SO(10)$ model and CP violation

We start by writing down the Yukawa interactions of our model, which are responsible for the discussion of neutrino masses and mixings. The Yukawa superpotential involves the couplings of the **16**-dimensional matter spinor ψ_i with **10**- (H), $\overline{\mathbf{126}}$- $(\bar{\Delta})$, and **120**- (A) dimensional Higgs fields:

$$W_Y = \frac{1}{2}h_{ij}\psi_i\psi_j H + \frac{1}{2}f_{ij}\psi_i\psi_j\bar{\Delta} + \frac{1}{2}h'_{ij}\psi_i\psi_j A. \tag{1}$$

The Yukawa couplings, h and f, are symmetric matrices, whereas h' is an antisymmetric matrix due to $SO(10)$ symmetry. They are all complex matrices in general.

Once the $SO(10)$ symmetry breaks down to the standard model symmetry, we have four pairs of Higgs doublets arising from the H, $\bar{\Delta}$, and A Higgs fields. There may also be other Higgs doublets, e.g., in **210** multiplet. Under the $G_{422} = SU(4)_c \times SU(2)_L \times SU(2)_R$ decomposition we have the following representations that contain the Higgs doublets of up and down type: one pair arises from $H \supset (\mathbf{1}, \mathbf{2}, \mathbf{2})$, one pair comes from $\bar{\Delta} \supset (\mathbf{15}, \mathbf{2}, \mathbf{2})$, and two pairs come form $A \supset (\mathbf{1}, \mathbf{2}, \mathbf{2}) + (\mathbf{15}, \mathbf{2}, \mathbf{2})$. We assume that one pair of their linear combinations, H_u and H_d, remains massless (mass is $\sim O(v_{wk})$) and become the MSSM Higgs doublets. As for other pairs, they all have GUT scale masses. Using the light Higgs doublets, the MSSM Yukawa couplings below the GUT scale and the right-handed Majorana neutrino mass terms can be written as

$$W_Y \supset Y^u_{ij} Q_i U^c_j H_u + Y^d_{ij} Q_i D^c_j H_d + Y^e_{ij} L_i E^c_j H_d + Y^\nu_{ij} L_i N^c_j H_u + \frac{1}{2} f_{ij} L_i L_j \bar{\Delta}_L + \frac{1}{2} f_{ij} N^c_i N^c_j \bar{\Delta}^0_R, \tag{2}$$

where Q, U^c, D^c, L, E^c, N^c are the quark and lepton superfields which are all unified to the **16** spinor ψ field. $\bar{\Delta}_L$ is an $SU(2)_L$ triplet Higgs field and $\bar{\Delta}^0_R$ is a neutral component of $SU(2)_R$ triplet, both part of the $\overline{\mathbf{126}}$ field. Even though both of them have GUT scale masses we have included them with the MSSM superpotential because their VEVs lead to light neutrino masses via the seesaw mechanism.

The gauge coupling unification requires that the $\overline{\mathbf{126}}$ Higgs field acquires VEV at or close to the GUT scale. We also need to introduce a **126** Higgs field to satisfy the D-flat condition to maintain supersymmetry down to the weak scale. Though the **126** Higgs field does not couple to the fermions, the pair of Higgs doublets in the **126** mix with the doublets arising from the other Higgs multiplets, since **126** couples to the other Higgs multiplets with non-zero coupling. These five pairs of Higgs doublets, $H^{10}_{u,d}$, $\Delta_{u,d}$, $\bar{\Delta}_{u,d}$, $A^s_{u,d}$ and $A^{adj}_{u,d}$ are mixed and the light pair of Higgs doublet can be written as

$$(H_u, \ldots) = \left(H^{10}_u, \Delta_u, \bar{\Delta}_u, A^s_u, A^{adj}_u, \ldots\right) U_H, \tag{3}$$

$$(H_d, \ldots) = \left(H^{10}_d, \bar{\Delta}_d, \Delta_d, A^s_d, A^{adj}_d, \ldots\right) V_H, \tag{4}$$

where U_H and V_H are unitary matrices. The superscripts s and *adj* stand for $SU(4)_c$ singlet and adjoint pieces. We have temporarily ignored the doublets that may arise from other multiplets in the theory such as **210**. It is important to stress that in order to obtain one pair of MSSM Higgs doublets from five pairs at GUT scale, one needs to do a fine tuning of parameters. We have enough parameters in the Higgs superpotential that this is possible to achieve. We have also checked that we do not have any light color triplet fields.

The results given below remain unchanged in their presence. The Dirac mass matrices of quark and leptons are

$$M_u = M_{10} + M_{126} + M_{120}, \tag{5}$$

$$M_d = r_1 M_{10} + r_2 M_{126} + r_3 M_{120}, \tag{6}$$

$$M_e = r_1 M_{10} - 3r_2 M_{126} + A r_4 M_{120}, \tag{7}$$

$$M^D_\nu = M_{10} - 3M_{126} + A M_{120}, \tag{8}$$

where the three mass matrices in the expression are given by $M_{10} = h^* v_u (U_H)_{11}$, $M_{126} = c_1 f^* v_u (U_H)_{12}$, and $M_{120} = h'^* v_u ((U_H)_{14} + c_2 (U_H)_{15})$, where v_u is a vacuum expectation value of MSSM Higgs doublet H_u, and c_i

are Clebsch–Gordan (CG) coefficients. The coefficients r_i and A are written as

$$r_1 = \frac{(V_H)_{11}}{(U_H)_{11}} \cot\beta, \tag{9}$$

$$r_2 = \frac{(V_H)_{13}}{(U_H)_{12}} \cot\beta, \tag{10}$$

$$r_3 = \frac{(V_H)_{14} + c_2(V_H)_{15}}{(U_H)_{14} + c_2(U_H)_{15}} \cot\beta, \tag{11}$$

$$r_4 = \frac{(V_H)_{14} - 3c_2(V_H)_{15}}{(U_H)_{14} - 3c_2(U_H)_{15}} \cot\beta, \tag{12}$$

$$A = \frac{(U_H)_{14} - 3c_2(U_H)_{15}}{(U_H)_{14} + c_2(U_H)_{15}}, \tag{13}$$

where $\cot\beta$ is a ratio of vacuum expectation values of doublet Higgs fields, $\cot\beta = v_d/v_u$. The Majorana mass matrices of left- and right-handed neutrinos prior to seesaw diagonalization are given by

$$M_L = f^* v_L, \qquad M_R = f^* v_R, \tag{14}$$

where v_L and v_R are vacuum expectation values of $\bar{\Delta}_L$ and $\bar{\Delta}_R$, respectively. As already mentioned, since v_R is expected to be close to the GUT scale, this implies that v_L is $\sim v_{\text{weak}}^2/(\eta M_{\text{GUT}}) \ll v_{\text{weak}}$, where η is a coupling constant in the Higgs potential. The Majorana mass matrix of the heavy right-handed neutrino is proportional to M_{126}. The light neutrino mass matrix is given by the mixed type II seesaw formula,

$$\mathcal{M}_\nu^{\text{light}} = M_L - M_\nu^D M_R^{-1} \left(M_\nu^D\right)^{\mathrm{T}}. \tag{15}$$

As discussed in earlier papers [7,9], there are regions of the parameter space in the theory where the first term will dominate; we will call this the pure type II seesaw case. If on the other hand, we consider the parameter space where the second term is dominant we will call this type I seesaw. The bulk of our results will be for the pure type II case.

3. Parity invariance and a predictive model for neutrinos

In order to see if the model is predictive for neutrinos, let us count the number of parameters in the theory. In the basis, where M_{126} is real and diagonal, there are 3 real parameters in M_{126}, 6 complex parameters in M_{10} and 3 complex parameters in M_{120}. We also have 5 complex parameters in the Eqs. (9)–(13) as well as the VEVs of the Altogether, there are 31 real parameters in the fermion sector, and, therefore, we do not have any prediction for the neutrino mixings.

In order to be predictive in the leptonic sector of the model without imposing any flavor symmetry, we require the theory to be invariant under a parity symmetry. As we will see, it makes the Dirac mass matrices Eqs. (5)–(8) hermitian and leaves a total of 17 real parameters in the fermion sector making the model predictive. If we further require that the **120** Higgs field has a mass much higher than the GUT scale, its only manifestation is as an effective dimension four term in the superpotential. The reduces the number of parameters to 15 increasing the predictive power of the model. We explore both the cases with 17 and 15 parameters in a subsequent section.

We now define the parity transformation in the G_{422} basis. We write $SU(4)$ indices by μ, ν, $SU(2)_L$ indices by α, β and $SU(2)_R$ indices by $\dot{\alpha}, \dot{\beta}$. The $SO(10)$ spinor ψ and χ are decomposed as

$$\psi = \psi_{\mu\alpha} + \psi_{\dot{\alpha}}^{\mu}, \qquad \chi = \chi_{\mu\alpha} + \chi_{\dot{\alpha}}^{\mu}. \tag{16}$$

Bi-doublet Higgs fields in the **10**, $\overline{\mathbf{126}}$ and **120** are written as $H_{\alpha\dot{\alpha}}$, $\bar{\Delta}_\mu{}^\nu{}_{\alpha\dot{\alpha}}$, $A_{\alpha\dot{\alpha}}$, and $A_\mu{}^\nu{}_{\alpha\dot{\alpha}}$. Then the Yukawa interactions are written in the following (up to overall factors)

$$h\psi\chi H = h\left(\psi_{\mu\alpha}\chi^\mu_{\dot{\alpha}} + \psi^\mu_{\dot{\alpha}}\chi_{\mu\alpha}\right)H^{\alpha\dot{\alpha}} + \cdots, \tag{17}$$

$$f\psi\chi\bar{\Delta} = f\left(\psi_{\mu\alpha}\chi^\nu_{\dot{\alpha}} + \psi^\nu_{\dot{\alpha}}\chi_{\mu\alpha}\right)\bar{\Delta}_\nu{}^{\mu\alpha\dot{\alpha}} + \cdots, \tag{18}$$

$$h'\psi\chi A = h'\left(\psi_{\mu\alpha}\chi^\mu_{\dot{\alpha}} - \psi^\mu_{\dot{\alpha}}\chi_{\mu\alpha}\right)A^{\alpha\dot{\alpha}} + c_2 h'\left(\psi_{\mu\alpha}\chi^\nu_{\dot{\alpha}} - \psi^\nu_{\dot{\alpha}}\chi_{\mu\alpha}\right)A_\nu{}^{\mu\alpha\dot{\alpha}} + \cdots, \tag{19}$$

where c_2 is a CG coefficient. The Lagrangian is written as

$$\mathcal{L} = \int d^2\theta\, W + \int d^2\bar{\theta}\, \bar{W} \tag{20}$$

and

$$\mathcal{L} = \int d^2\theta\, h\left(\psi_{\mu\alpha}\chi^\mu_{\dot{\alpha}} + \psi^\mu_{\dot{\alpha}}\chi_{\mu\alpha}\right)H^{\alpha\dot{\alpha}} + \int d^2\bar{\theta}\, h^*\left((\psi_{\mu\alpha})^*\left(\chi^\mu_{\dot{\alpha}}\right)^* + \left(\psi^\mu_{\dot{\alpha}}\right)^*(\chi_{\mu\alpha})^*\right)\left(H^{\alpha\dot{\alpha}}\right)^* + \cdots. \tag{21}$$

We consider the symmetry under the following parity transformation,

$$\psi_{\mu\alpha} \leftrightarrow \left(\psi^{\mu\dot{\alpha}}\right)^*, \qquad d^2\theta \leftrightarrow d^2\bar{\theta}. \tag{22}$$

Of course, χ is also transformed in same manner.

In the Higgs sector, the transformations of the $(\mathbf{1}, \mathbf{2}, \mathbf{2})$ and $(\mathbf{15}, \mathbf{2}, \mathbf{2})$ sub-multiplets under G_{422} are:

$$H^{\alpha\dot{\alpha}} \leftrightarrow (H_{\alpha\dot{\alpha}})^*, \qquad \bar{\Delta}_\nu{}^{\mu\alpha\dot{\alpha}} \leftrightarrow \left(\bar{\Delta}_\mu{}^\nu{}_{\alpha\dot{\alpha}}\right)^*, \qquad A^{\alpha\dot{\alpha}} \leftrightarrow (A_{\alpha\dot{\alpha}})^*, \qquad A_\nu{}^{\mu\alpha\dot{\alpha}} \leftrightarrow \left(A_\mu{}^\nu{}_{\alpha\dot{\alpha}}\right)^*. \tag{23}$$

A consequence of the parity symmetry (23), is that the coupling matrices h and f real and symmetric and h' antisymmetric and imaginary; the parameters r_i $(i = 1, 2, 3, 4)$ and A in the Eqs. (9)–(13) are real. This considerably reduces the number of parameters in the theory and further makes the mass matrices for all charged fermions hermitian.

Let us clarify our motivation for introducing the **120** Higgs field. When the **120** Higgs field is absent, the fermion mass matrices are complex symmetric matrices in the absence of the parity symmetry and we have the following relation in the pure type II case

$$\mathcal{M}_\nu \propto M_d - r_1 M_u = U\left(V D_d V^{\mathrm{T}} - r_1 D_u\right)U^{\mathrm{T}}$$
$$\simeq U \begin{pmatrix} m_d e^{i\phi_d} + V_{us}^2 m_s e^{i\phi_s} & V_{us} m_s e^{i\phi_s} & V_{ub} m_b \\ V_{us} m_s e^{i\phi_s} & m_s e^{i\phi_s} & V_{cb} m_b \\ V_{ub} m_b & V_{cb} m_b & m_b - r_1 m_t \end{pmatrix} U^{\mathrm{T}}, \tag{24}$$

where m_c and m_u contributions in $(2, 2)$ and $(1, 1)$ elements are omitted, and ϕ_d and ϕ_s are complex phases in the diagonal matrix, D_d. If M_e is close to a diagonal matrix in the basis where M_u is diagonal, the maximal atmospheric mixing can be easily obtained when the $(3, 3)$ element is suppressed such that $|m_s e^{i\phi_s} - (m_b - r_1 m_t)| \ll 2V_{cb} m_b$. This suppression of $(3, 3)$ element is related with other observed facts such as bottom and tau mass convergence at GUT scale and $\Delta m^2_{\mathrm{sol}}/\Delta m^2_A \gg O(m_s^2/m_b^2)$. Assuming that the atmospheric mixing is maximal, we obtain the neutrino mass matrix, Eq. (24), as

$$\simeq U U_{23} \begin{pmatrix} m_d e^{i\phi_d} + V_{us}^2 m_s e^{i\phi_s} & -(V_{ub} - V_{us}V_{cb}) m_b/\sqrt{2} & (V_{ub} - V_{us}V_{cb}) m_b/\sqrt{2} \\ -(V_{ub} - V_{us}V_{cb}) m_b/\sqrt{2} & \epsilon m_0 & 0 \\ (V_{ub} - V_{us}V_{cb}) m_b/\sqrt{2} & 0 & m_0 \end{pmatrix} U_{23}^{\mathrm{T}} U^{\mathrm{T}}, \tag{25}$$

where m_0 and ϵm_0 are eigenvalues of (2-3) block, and $\epsilon^2 \sim \Delta m^2_{\mathrm{sol}}/\Delta m^2_A$ and $m_0 \sim 2m_s$. Thus, the solar mixing and 13 mixing are proportional to $|V_{ub}/V_{cb} - V_{us}|$ and $\tan 2\theta_{\mathrm{sol}} \sim \tan 2\theta_{13}/\epsilon$. Therefore, those mixing angles also depend on the KM phase, δ_{KM}. Since $V_{ub} = |V_{ub}|e^{-i\delta_{\mathrm{KM}}}$, the KM phase in the first quadrant gives a smaller value for solar mixing angle rather than in the second quadrant. In order to obtain the proper value of solar mixing angle, we have to choose a smaller value of ϵ. However, in the model without **120**, the ϵ parameter, which is a function of the strange quark mass, is constrained due to the fitting of three charged-lepton masses (especially electron mass), and we do not have proper fitting of the solar mixing angle data in the case where the KM phase is in the first quadrant. We can verify the situation in a precise analysis in the pure type II case [10]. In the type I case, things are more complicated, but it has been shown that it is not possible to fit the neutrino oscillation data in this model with the above minimal Higgs choice [12]. The mass squared ratio is constrained due to the charged-lepton mass fitting and it cannot be small enough when the KM phase is in the first quadrant. The mass squared ratio is a free parameter in the model with **120** since the additional parameter A in the sum rules Eqs. (5)–(8) can fit the electron mass, and therefore we can explain a smaller KM phase. Thus, we employ the **120** Higgs field to explain the large solar mixing angle along with the KM phase in the first quadrant. Interestingly, even though we have introduced a new Higgs field, the number of parameters is less than the minimal Higgs choice with most general CP phases due to the constraint of parity symmetry.

We further note that if the **120** field is heavier than the GUT scale, its effect on the physics at the GUT scale comes from a higher-dimensional operator of the form $\psi\Gamma\Gamma\Gamma\psi H\Phi/M$, where Φ is a **210** Higgs field, so that we get the relation $r_1 = r_3 = r_4$. In other words, the choice $r_1 = r_3 = r_4$ is not an ad hoc choice but can be guaranteed in a natural manner.

We now note that in the presence of the parity symmetry, since all the mass matrices are hermitian and also the μ-term and the gluino masses are real, the most dangerous graphs contributing to large electric dipole moment of the neutron are absent [14]. Therefore, this model has the potential to solve the so-called SUSY CP problem.

We also wish to recognize that the Z_2 CP symmetry we impose is broken at the GUT scale by the VEVs of **126**, **210** and **45** Higgs fields which break $SO(10)$ symmetry. The light MSSM doublet Higgs fields are no more CP eigenstates, and thus there is no cosmological domain wall problem at weak scale in the model.

4. Near bi-maximal solution for neutrino oscillation

As already noted in Section 3, under the parity symmetry, M_{10} and M_{126} are real symmetric matrices, M_{120} is a pure imaginary antisymmetric matrix, and the coefficients r_i and A are real parameters in the Eqs. (5)–(8). We can therefore rewrite the charged-lepton and Dirac neutrino mass matrices in terms of the other mass matrices as follows:

$$M_e = c_u \,\mathrm{Re}\, M_u + c_d \,\mathrm{Re}\, M_d + iAr_4 \,\mathrm{Im}\, M_u, \tag{26}$$

$$M_\nu^D = \mathrm{Re}\, M_u + \frac{3+c_d}{r_1} \mathrm{Re}(M_d - r_1 M_u) + iA \,\mathrm{Im}\, M_u, \tag{27}$$

where $c_u/(1-c_d) = r_1$, $-c_u/(3+c_d) = r_2$. The up- and down-type quark mass matrices are hermitian, and are written as

$$M_u = U D_u U^\dagger, \qquad M_d = UVD_dV^\dagger U^\dagger, \tag{28}$$

where $D_u = \mathrm{diag}(\pm m_u, \pm m_c, m_t)$, $D_d = \mathrm{diag}(\pm m_d, \pm m_s, m_b)$ and V is the CKM matrix, and U is a unitary matrix. We note that m_t and m_b component in the D_u and D_d can be made to be positive without loss of generality. Because of the parity symmetry, $M_d - r_3 M_u$ must be a real symmetric matrix. We fix the flavor basis as

$$M_d - r_3 M_u = U\big(VD_dV^\dagger - r_3 D_u\big)U^\dagger = \mathrm{diag}. \tag{29}$$

The unitary matrix U is determined by r_3, up to phase matrix P,

$$U = P\bar{U}, \quad P = \mathrm{diag}\big(e^{i\phi_1}, e^{i\phi_2}, 1\big). \tag{30}$$

The parameters are now 6 quark masses (with signatures), 3 mixing and 1 KM phase in the CKM matrix, and the coefficients c_d, c_u, ϕ_1, ϕ_2, r_3, r_4, A. There are 17 parameters in all. For example, the three charged-lepton masses can be used to determine c_d, c_u, Ar_4. The remaining 4 parameters give the neutrino oscillation parameters.

In the case where the **120** Higgs field is heavier than the GUT scale, it manifests itself as an effective dimension four operator of the form $\psi\psi H\Phi/M$. As a result, the Higgs doublets in **120** are decoupled. This leads to a reduction in the number of mixing parameters. This translates into the relation $r_1 = r_3 = r_4$ since the VEV ratio that defines $r_{1,3,4}$ is the same. In this case, there remain only two parameters describing the neutrino sector. They can be determined by two of the parameters from the neutrino oscillation data and the remaining neutrino parameters can then be predicted.

Interestingly, if we have the relation $r_1 = r_3$, the matrix $M_d - r_3 M_u$ is proportional to the light neutrino mass matrix M_L and the diagonalizing matrix U become close to MNSP matrix in the pure type II case. Therefore, if $r_1 = r_3 \sim m_b/m_t$ and the $(3,3)$ element of $M_d - r_1 M_u$ is suppressed, we have a large atmospheric mixing. The mass squared ratio is of the order of 10^{-2}, which is the right order seen in the experiment, only if $r_1 \sim m_b/m_t$ (otherwise, the mass squared ratio become $O(m_c^2/m_t^2)$ or $O(m_s^2/m_b^2)$). Furthermore, since the $(3,3)$ element of M_{126} is suppressed for that choice of r_1, the bottom-tau mass unification is satisfied and it is consistent with the renormalization group flow for the case of $\tan\beta \sim 50$.

Now let us study the prediction of the model in the case where $r_1 = r_3 = r_4$. In this case, we have 15 parameters in the model. After fixing the quark masses (with signatures) and the CKM parameters, we are left with 5 parameters, c_d, A, ϕ_1, ϕ_2, and r_1. Since mass squared ratio is a function of r_1, we can fix the parameter r_1 by the experimental value of $\Delta m^2_{\rm sol}/\Delta m^2_A$. The three charged-lepton masses can be used to fix c_d, A, and ϕ_2. As a result, the neutrino oscillation parameters, θ_A, $\theta_{\rm sol}$, $|U_{e3}|$, and one CP phase $\delta_{\rm MNSP}$ are predicted by only one phase parameter ϕ_1. Interestingly, the atmospheric mixing θ_A does not depend on the phase ϕ_1 very much, and the θ_A is really predicted when we fix the quark masses and mass squared ratio of light neutrino.

It should also be noted that the other arbitrariness in the model is due to the choice of the signs of different fermion masses, since the sign of a fermion mass is unobservable. We find that only for the two choices of the signs given below, we obtain acceptable solutions:

$$\text{(a)} \quad D_u = \mathrm{diag}(\pm,-,+), \qquad D_d = \mathrm{diag}(-,+,+), \qquad D_e = \mathrm{diag}(\pm,-,+), \tag{31}$$

$$\text{(b)} \quad D_u = \mathrm{diag}(\pm,-,+), \qquad D_d = \mathrm{diag}(+,+,+), \qquad D_e = \mathrm{diag}(\pm,+,+). \tag{32}$$

The solutions we present correspond only for these two choices of signatures.

In Fig. 1, we show the prediction of the atmospheric mixing $\sin^2 2\theta_A$ as a function of the mass squared ratio. The lines (a) and (b) in the figure are for the set of quark and lepton mass signatures given above which give acceptable solutions for neutrinos. We can obtain large atmospheric mixing angles with the proper choice of quark masses in their allowed range, mixings and KM phase [15] with the choice of the set of signatures, especially for case (a). The most important input parameter for obtaining a large atmospheric mixing is the strange quark mass. We show the strange mass dependence of θ_A in Fig. 2 in the case where the mass squared ratio $R \equiv \Delta m^2_\odot/\Delta m^2_A$ is 0.03. The strange mass in the figure is the running mass at 1 GeV. In order to obtain the experimental constraint $\sin^2 2\theta_A > 0.9$, we need the parameter region where the strange mass has a larger value. The bottom quark mass dependence is not negligible, and a larger value of bottom mass is preferred to obtain maximal atmospheric mixing.

After fixing all quark and lepton data and also the mass squared ratio, R, we can fit the solar mixing angle by choosing the free phase parameter ϕ_1. Then, $|U_{e3}|$ and $\delta_{\rm MNSP}$ are predicted. In Fig. 3, we show the correlation between solar mixing $\tan^2\theta_{\rm sol}$ and $|U_{e3}|$ by varying the phase parameter ϕ_1. We give two lines for the cases (a) and (b), and the two lines correspond to different mass squared ratios, $R = 0.02$ and $R = 0.07$. From the figure, we can see that $\Delta m^2_\odot/\Delta m^2_A = 0.07$ is not favored in the 3σ range of the experimental data of solar neutrino and U_{e3} for the case (a). In Fig. 4, we present the prediction of the $\sin\delta_{\rm MNSP}$ in the case where $R = \Delta m^2_\odot/\Delta m^2_A = 0.02$. The CP phase can be of any value in the range of the experimental data of solar neutrino. If we restrict the mass signatures to the (a) case, $\sin\delta_{\rm MNSP}$ could be predicted to be ± 0.9.

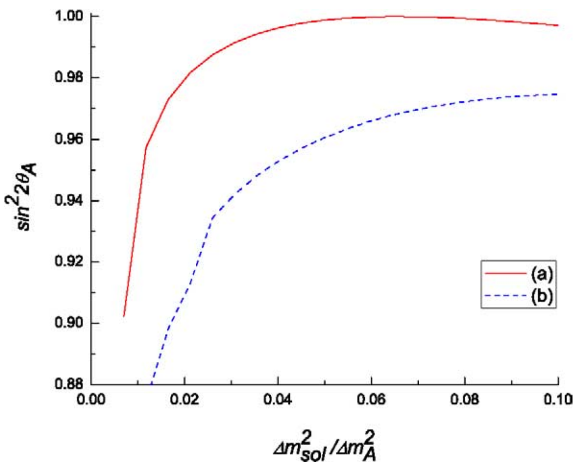

Fig. 1. The atmospheric mixing angle is plotted as a function of mass squared ratio. Predictions for different set of mass signatures (a) and (b) in Eqs. (31) and (32) are given.

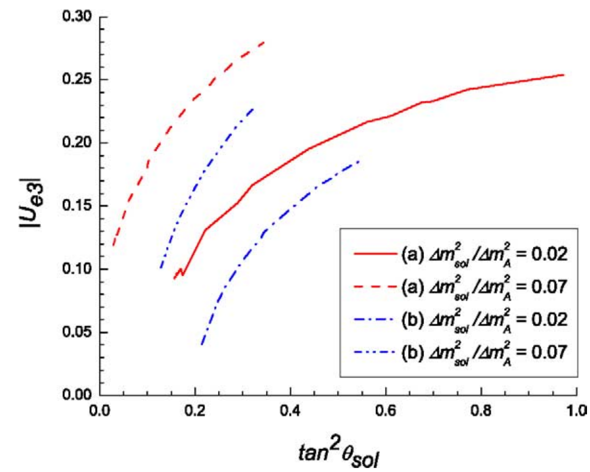

Fig. 2. The atmospheric mixing angle is plotted as a function of strange quark mass. The strange quark mass is given as a running mass at 1 GeV. The mass squared ratio is 0.03 in each set of mass signature, (a) and (b).

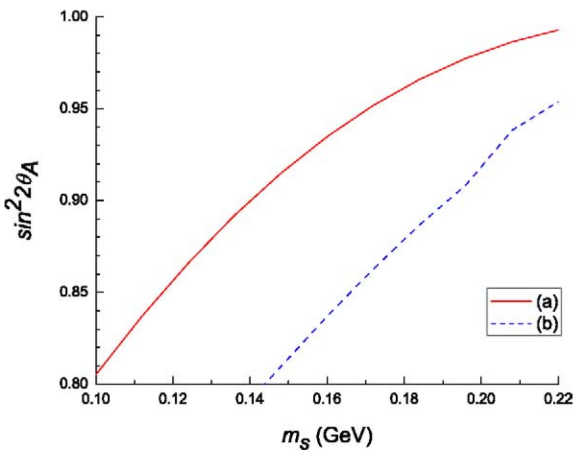

Fig. 3. The relation between solar mixing angle and $|U_{e3}|$ is plotted. Each line is plotted by varying free phase parameter ϕ_1. The experimentally allowed region in 3σ of recent data fitting is $0.3 < \tan^2\theta_{\rm sol} < 0.6$ and $|U_{e3}| < 0.26$ [16].

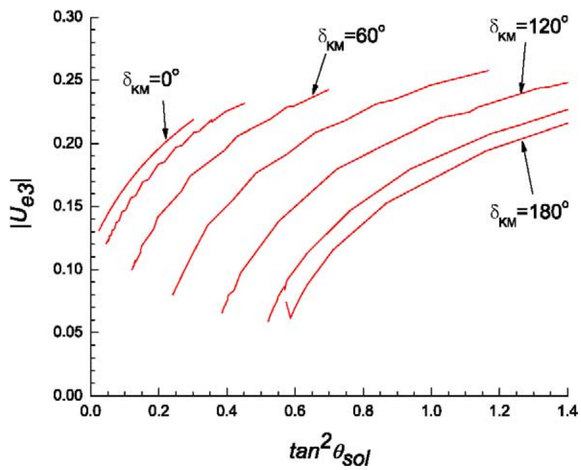

Fig. 4. The prediction of MNSP phase is plotted as a function of the solar mixing angle. These lines (a) and (b) are plotted in the case $\Delta m^2_{\rm sol}/\Delta m^2_A = 0.02$ for different mass signature.

The most interesting feature of this model is the prediction of $|U_{e3}|$. In Fig. 5, we show the bottom quark mass dependence of the $|U_{e3}|$ prediction. The bottom mass is defined as a running mass at m_b. Since a large SUSY correction to the bottom quark mass can be induced in the large $\tan\beta$ case, the running bottom mass can be large, and the larger bottom mass gives smaller value of $|U_{e3}|$. In a similar way, larger $\tan\beta$ predicts smaller $|U_{e3}|$ since the larger $\tan\beta$ gives larger bottom quark mass at GUT scale. In Fig. 6, we show the plot of $|U_{e3}|$. Each point is dotted for different quark mass and mixings which are randomly generated in the experimentally allowed region. We can see that the model predicts a lower limit for $|U_{e3}|$ of about 0.1.

In Fig. 7, we can see the KM phase dependence of the prediction of the model. The lines in the figure are drawn by changing ϕ_1 for different values of KM phases. The mass squared ration is set to be 0.02. For a smaller KM

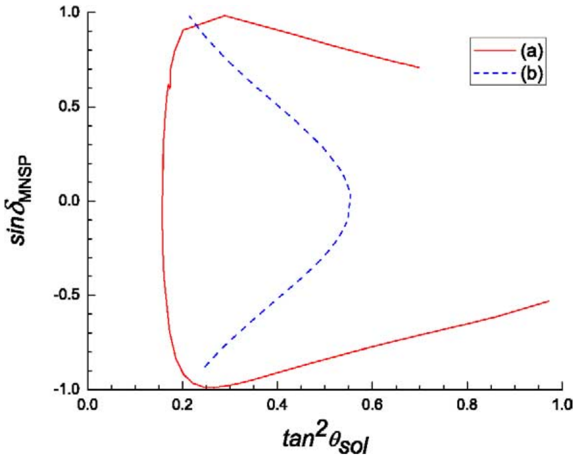

Fig. 5. The relation between solar mixing angle and $|U_{e3}|$ is plotted as dots for different bottom quark masses. The bottom quark mass is given as a running mass at m_b. This plot is given in the case of mass signature (a) and mass squared ratio is 0.02.

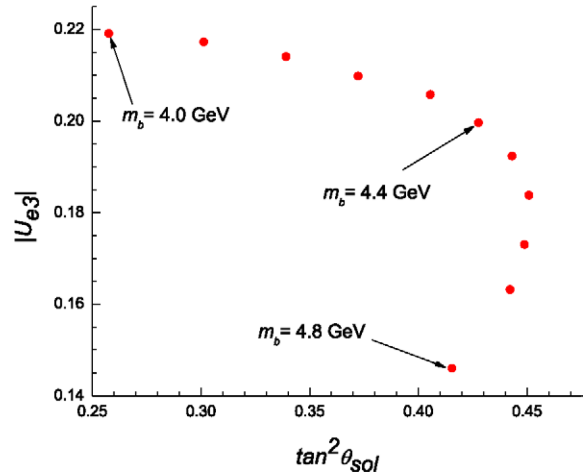

Fig. 6. The prediction of $|U_{e3}|$ and solar mixing angle is plotted as dots for randomly generated quark masses (with signature) and mixings in the experimentally allowed region. The lower bound of $|U_{e3}|$ exists in this model.

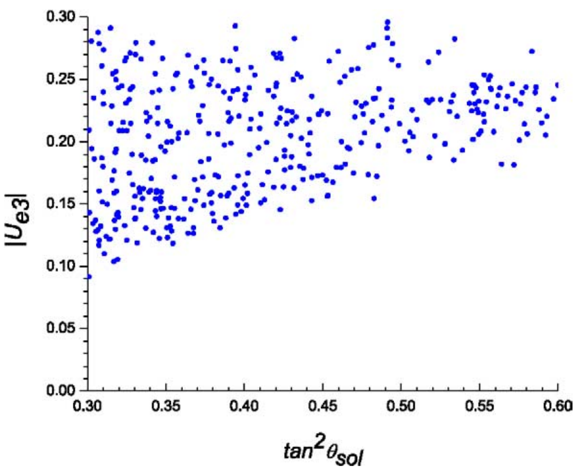

Fig. 7. The relation between solar mixing angle and $|U_{e3}|$ is plotted for various KM phases. Each plot is given in the case of mass signature (a) and mass squared ratio is 0.02.

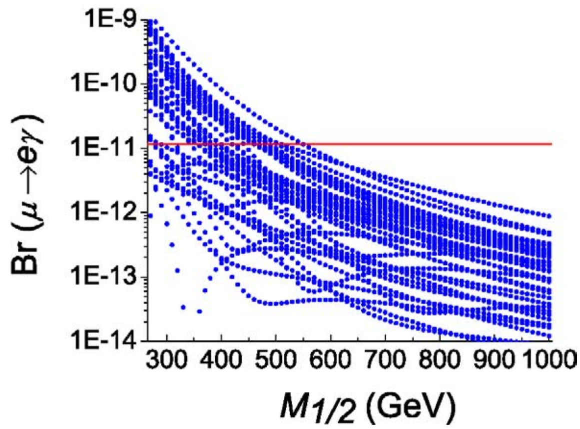

Fig. 8. The BR$[\mu \to e\gamma]$ is plotted as a function of $m_{1/2}$ for $\tan\beta = 10$ in type II.

phase, the lines shift to the smaller solar mixing. In this model, the experimentally allowed solar mixing can be obtained in the first quadrant KM phase, contrary to the minimal model without the **120** Higgs field. We can see that a smaller value of $|U_{e3}|$ can be allowed for the larger values of KM phase.

Since all the parameters of the model are now determined, it can be used to make other predictions. As an example, we have calculated the magnitude of lepton flavor-violating process $\mu \to e + \gamma$ in the model (see Fig. 8). Note that the predictions are in the range currently being probed by experiments [17]. We use the mSUGRA universal boundary conditions at the GUT scale, i.e., m_0 (universal scalar mass), $m_{1/2}$ (universal gaugino mass), A_0 (universal trilinear mass). The other two parameters are sign of μ and $\tan\beta$. The dots in the plots are produced for various model points generated by fitting the fermion masses and mixing angles. We can see that the BR of

$\mu \to e\gamma$ can be large for smaller values of $m_{1/2}$. The lightest neutralino is the dark matter candidate in this model and we satisfy the 2σ range of the recent relic density constraint $\Omega_{\rm CDM} = 0.1126^{+0.008}_{-0.009}$ [18] in the parameter space. When we satisfy the relic density constraint, the m_0 gets determined. We choose $A_0 = 0$ and $\mu > 0$. The right-handed masses have hierarchies and therefore get decoupled at different scales. The flavor-violating pieces present in Y_ν and f induces flavor violations into the charged lepton couplings and into the soft SUSY breaking masses, e.g., m^2 terms, etc. Also, an additional symmetry between the GUT and the v_R scale (type I) helps to induce flavor violation [12]. The electric dipole moment of electron is smaller than the experimental reach in the range of parameter space showed in the figure.

We comment that in the case of type I seesaw, the large mixing solution is a sharp resonance solution and the solution is not stable to predict the mixings contrary to the type II seesaw.

5. Discussion

In this Letter we consider the prediction of neutrino masses and mixings for an $SO(10)$ model where fermion masses receive contribution from the presence of three Higgs multiplets **10**, **120** and $\overline{\mathbf{126}}$. We impose a parity symmetry on the model, so that it has very few parameters which enables prediction of two mixing angles and all the CP phases in the neutrino mixing matrix. The advantage of this model over the most minimal $SO(10)$ model is that now the CKM phase is in the first quadrant as required by the standard model analysis of all observed hadronic CP violation. We also wish to emphasize that this one of the few models in the literature that can predict leptonic CP phases.

As far as experimental tests of this model are concerned, the parameter $U_{e3} \equiv \sin\theta_{13}$ is predicted to be large like the most minimal $SO(10)$ model [7,9] but is somewhat smaller, i.e., $U_{e3} \geqslant 0.1$. This can be tested in the next round of planned long baseline experiments. We also predict that the Dirac phase for neutrinos can be maximal. Furthermore, the model has the potential to solve SUSY CP problem due to the fact that all fermion masses are hermitian. The model also predicts observable amount of LFV in muon decay.

Note added

After this work was completed and was being prepared for publication, two papers appeared [19,20] which also include the effect of **120** Higgs field on fermion masses in the $SO(10)$ model.

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References

[1] H. Georgi, in: C.E. Carlson (Ed.), Particles and Fields, AIP, New York, 1975;
H. Fritzsch, P. Minkowski, Ann. Phys. 93 (1975) 193.

[2] T. Yanagida, in: O. Sawada, A. Sugamoto (Eds.), Proceedings of the Workshop on the Unified Theory and Baryon Number in the Universe, KEK, Tsukuba, 1979;
S.L. Glashow, Cargese Lectures, 1979;
M. Gell-Mann, P. Ramond, R. Slansky, in: D. Freedman, P. van Nieuwenhuizen (Eds.), Supergravity, North-Holland, Amsterdam, 1979;
R.N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.

[3] J.C. Pati, A. Salam, Phys. Rev. D 10 (1974) 275.
[4] R.N. Mohapatra, R.E. Marshak, Phys. Rev. Lett. 44 (1980) 1316.
[5] K.S. Babu, R.N. Mohapatra, Phys. Rev. Lett. 70 (1993) 2845, hep-ph/9209215.
[6] L. Lavoura, Phys. Rev. D 48 (1993) 5440, hep-ph/9306297;
D.G. Lee, R.N. Mohapatra, Phys. Lett. B 324 (1994) 376, hep-ph/9310371;
B. Brahmachari, R.N. Mohapatra, Phys. Rev. D 58 (1998) 015001, hep-ph/9710371;
K. Matsuda, Y. Koide, T. Fukuyama, Phys. Rev. D 64 (2001) 053015, hep-ph/0010026;
K. Matsuda, Phys. Rev. D 69 (2004) 113006, hep-ph/0401154;
K. Matsuda, Y. Koide, T. Fukuyama, H. Nishiura, Phys. Rev. D 65 (2002) 033008, hep-ph/0108202;
K. Matsuda, Y. Koide, T. Fukuyama, H. Nishiura, Phys. Rev. D 65 (2002) 079904;
T. Fukuyama, N. Okada, JHEP 0211 (2002) 011, hep-ph/0205066;
T. Fukuyama, T. Kikuchi, N. Okada, Phys. Rev. D 68 (2003) 033012, hep-ph/0304190;
T. Fukuyama, T. Kikuchi, N. Okada, hep-ph/0306025.
[7] B. Bajc, G. Senjanovic, F. Vissani, Phys. Rev. Lett. 90 (2003) 051802, hep-ph/0210207.
[8] G. Lazarides, Q. Shafi, C. Wetterich, Nucl. Phys. B 181 (1981) 287;
R.N. Mohapatra, G. Senjanovic, Phys. Rev. D 23 (1981) 165.
[9] H.S. Goh, R.N. Mohapatra, S.P. Ng, Phys. Lett. B 570 (2003) 215, hep-ph/0303055.
[10] H.S. Goh, R.N. Mohapatra, S.P. Ng, Phys. Rev. D 68 (2003) 115008, hep-ph/0308197.
[11] See, for example, A.J. Buras, Lectures given at 41st Schladming School in Theoretical Physics, Schladming, Austria, 2003, hep-ph/0307203.
[12] B. Dutta, Y. Mimura, R.N. Mohapatra, Phys. Rev. D 69 (2004) 115014, hep-ph/0402113.
[13] N. Oshimo, Phys. Rev. D 66 (2002) 095010, hep-ph/0206239.
[14] R.N. Mohapatra, A. Rasin, Phys. Rev. D 54 (1996) 5835, hep-ph/9604445;
K.S. Babu, B. Dutta, R.N. Mohapatra, Phys. Rev. D 61 (2000) 091701, hep-ph/9905464;
S. Abel, S. Khalil, O. Lebedev, Nucl. Phys. B 606 (2001) 151, hep-ph/0103320.
[15] K. Hagiwara, et al., Particle Data Group Collaboration, Phys. Rev. D 66 (2002) 010001.
[16] G.L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, A.M. Rotunno, Phys. Rev. D 69 (2004) 017301, hep-ph/0308055.
[17] M. Aoki, Review talk at the Neutrino2004 Conference, Paris.
[18] C.L. Bennett, et al., Astrophys. J. Suppl. 148 (2003) 1, astro-ph/0302207.
[19] S. Bertolini, M. Frigerio, M. Malinsky, hep-ph/0406117.
[20] W.-M. Yang, Z.-G. Wang, hep-ph/0406221.