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Latitude Based Model for Tilt Angle Optimization for Solar Collectors in the Mediterranean Region

Hassane Darhmaoui, Driss Lahjouji*

Al Akhawayn University in Ifrane, Hassan II Avenue, Ifrane 53000, Morocco

Abstract

This paper inspects the different parameters that intervene in the determination of the optimal tilt angle for maximum solar energy collection. It proposes a method for calculating the optimal tilt angle based upon the values of the daily global solar radiation on a horizontal surface. A computer program using the mathematical model to calculate the solar radiation incident on an inclined surface as a function of the tilt angle is implemented. Four years data of daily global solar radiation on a horizontal surface in 35 sites in different countries of the Mediterranean region is used. The program assumes a due south orientation of the collectors and it determines the optimal tilt angle for maximum solar radiation collection for sites in the Mediterranean region. A regression analysis using the results of the computer simulation is conducted to develop a latitude based tilt angle optimization mathematical model for maximum solar radiation collection for the sites. We tested both a linear and a quadratic model (of the form ax^2+bx) for representing the relationship between the annual optimal tilt angle and the site's latitude. The quadratic model is better; it provides very high prediction accuracy. 99.87% of the variation in the annual optimal tilt angle is explained by the variability in site's latitude with an average residual angle of only 0.96° for all 35 sites studied. It also gives an average percentage decrease in the annual solar radiation of only 0.016% when compared with actual optimal tilt angles.

Keywords: Tilt angle optimization; solar radiation; regression analysis; latitude based model; mediteranian countries

* Corresponding author. Tel.: +212-668-865-286.
E-mail address: d.lahjouji@aui.ma.

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1. Introduction

The orientation, also known as the azimuth angle and the tilt angle of the collector's surface with respect to the horizontal, strongly influence the performance of solar based conversion system because they determine the amount of solar radiation reaching the inclined surface of the collector [1]. In fact, several factors like the global radiation on a horizontal surface, the ground reflectance and the day of the year constitute the parameters of a complex function that determine the amount of solar radiation incident on an inclined surface at any time.

Solar radiation data is usually measured in the form of global and diffuse radiation on a horizontal surface at the latitude of interest. A sine qua non condition to the calculation of solar radiation incident on a tilted surface is the determination of the relative amount of the beam and the diffuse radiation contained in the measured horizontal global radiation. Solar collectors are tilted to maximize solar energy collected; accordingly, it is important to determine the optimal tilt angle at which maximum solar energy is collected. The tracking systems, that follow the direction of the sun on its daily sweep across the sky, allow the maximization of solar radiation incident on the collector's surface. A gain of 40% in solar radiation incident on the collector is achieved if a two axis tracking system is adopted instead of a fixed collector [2]. However, tracking systems are expensive, need energy for their operation and are not always applicable especially for small scale systems.

As a result, optimally orienting the collector maximizes the solar energy collected. In the northern hemisphere, it is generally known that the optimal tilt angle depends on the day of the year, the latitude and the optimal collector orientation is due south. Tilt angle optimization has been extensively addressed in many articles quantitatively and qualitatively [3-5]. Other articles approach the issue analytically [2] [6-8]. For heating seasons, Chiou JP et al. [6] developed a method for calculating the optimal tilt angle for due south collectors. The effects of latitude, solar reflectivity, and clearness index were considered by Elsayed MM [7] in determining the optimal tilt angle analytically. Tang R and Wu T [9] used the monthly horizontal radiation to develop a simple mathematical procedure allowing the determination of the optimal tilt angle. In the literature, there are many papers that, using only the latitude, give different recommendations for the optimal tilt angle [2-10]. Some studies [11-12] suggest that the optimal tilt angle is the latitude ±15°. (Latitude ±15°) ±15° is the optimal tilt angle suggested by Masters GM [13]. In summer months, the optimal tilt angle is usually 15° less than the latitude, whilst in winter months; it is 15° more than the latitude. From the literature, it can be noticed that the optimal tilt angle is location specific [14-19].

This paper examines the theoretical aspects that determine the optimal tilt angle for maximum solar energy collection and presents a method for calculating it. This method takes into account the anisotropy of the diffuse radiation in the circumsolar region plus an isotropically distributed component from the rest of the sky dome. A computer program using the mathematical equation of the presented model is implemented. The computer program is used in determining the optimal tilt angle of 35 sites in the Mediterranean region. A regression analysis using site's latitude and its corresponding optimal tilt angle is conducted to develop a mathematical model that allows the determination of the optimal tilt angle at which maximum solar radiation is collected using only site's latitude.

The remainder of this paper is organized as follows. In the second section, the different solar radiation prediction models are discussed. The third section describes the procedure followed to calculate solar radiation incident on an inclined surface as a function of the tilt angle. The fourth section presents the methodology adopted, followed by the fifth section that presents and discusses the results of the regression analysis, and finally the last section concludes the paper.

2. Solar Radiation Prediction Models

In the last few years, many models that allow calculating global radiation on a tilted surface from the available data on a horizontal surface have been presented [20-21]. Some of these models require special measurements [22]; and some apply to only specific cases. These models calculate the beam and the ground reflected radiation incident on an inclined surface using the same method. The only difference among these models is the treatment of the sky diffuse radiation. The estimation models are either isotropic [20] or anisotropic [21] depending on the assumption of the sky diffuse component.

Many papers address the issue of converting sky diffuse radiation from a horizontal surface into an inclined surface [23-24]. The assumption commonly used in calculating diffuse radiation incident on inclined surface from the value of the diffuse component on a horizontal surface is that sky radiation is always isotropically distributed [21, 25-26]. However, theoretical and experimental results show that the isotropic distribution assumption is inaccurate [27]. As a result, and because of the strong forward scattering effect of aerosols, the sky diffuse radiation should be considered anisotropic [28-29]. In fact, G. A. Kamali et al [23] compared all the isotropic and anisotropic models using Root Mean Square Errors (RMSE) and Mean Bias Errors (MBE), and found that Reindl's model (anisotropic model) provides the most accurate prediction for south facing surfaces. As a result, the Reindl's model is adopted in this paper.

3. Incident Solar Radiation on an Inclined Surface

Correlation procedures are required to obtain insolation values on a tilted surface from horizontal radiation because most published meteorological data give the global radiation on a horizontal surfaces. The monthly average daily total radiation on titled surface is dependent on the direct beam, diffuse, and ground reflected components. Thus, the incident total radiation on tilted surface at a slope angle from the horizontal is given by:

$$H_T = H_B + H_D + H_R \tag{1}$$

where H_T, H_B, H_D, and H_R are respectively the monthly average daily total, beam, diffuse, and reflected radiation on a tilted surface in (MJ/m²/day).

3.1. Beam Radiation Incident on a Tilted Surface

The monthly average daily beam radiation received on a tilted surface can be expressed as:

$$H_B = \left(H_g - H_d\right)R_b \tag{2}$$

where H_g and H_d are respectively the monthly average daily global and diffuse radiation on a horizontal surface in (MJ/m²/day), and R_b is the ratio of the average daily beam radiation on a tilted surface to that on a horizontal surface.

A simple physical based method using the monthly average clearness index (K_T), developed by A. de Miguel et al [30], is used to for estimating the monthly average daily diffuse radiation on a horizontal surface (H_d) from the monthly average daily global radiation on a horizontal surface (H_g). The monthly average clearness index (K_T) is given by [25]:

$$K_T = \frac{H_g}{H_0} \tag{3}$$

whereH_0 is the monthly average daily extraterrestrial radiation on a horizontal surface in (MJ/m²/day). It is expressed as:

$$H_{0}=\left(\frac{24*3600}{\pi*10^{6}}\right)G_{on}\left(\cos\emptyset\cos\delta\sin\omega_{S}+\frac{\pi}{180}\omega_{S}\sin\emptyset\sin\delta\right) \tag{4}$$

and

$$G_{on}=G_{sc}\left(1+0.033\cos\left(\frac{360*n}{365}\right)\right) \tag{5}$$

where G_{sc} is the solar constant (1367 W/m²), $\emptyset$ is the latitude of the site in (degree), δ is the solar declination angle in (degree), ω_S is the sunshine hour angle in (degree), n is the day of the year starting from the first of January ($n = 1$).

The solar declination angle (δ) is given by [25]:

$$\delta=23.45\sin\left[360*\frac{(284+n)}{365}\right] \tag{6}$$

The sunshine hour angle (ω_S) is given by:

$$\omega_{S}=\cos^{-1}(-\tan(\emptyset)*\tan(\delta)) \tag{7}$$

The value of the monthly average daily diffuse radiation on a horizontal surface depends on three different ranges of clearness index. The resulting correlations are given in Table 1.

Table 1. Clearness Index Ranges

Ranges	Expression
$K_T \leq 0.13$	$\frac{H_d}{H_g}=0.952$
$0.13 < K_T \leq 0.8$	$\frac{H_d}{H_g}=0.868+1.335K_T-5.782K_T^2+3.721K_T^3$
$K_T > 0.8$	$\frac{H_d}{H_g}=0.141$

It is assumed that there is no atmosphere in estimating the value of the ratio of the average daily beam radiation on a tilted surface to that on a horizontal surface. For due south surfaces in the northern hemisphere, sloped towards the equator, the equation for R_b is expressed as:

$$R_{b}=\frac{\cos(\emptyset-\beta)\cos\delta\sin\omega'_{S}+\left(\frac{\pi}{180}\right)\omega'_{S}\sin(\emptyset-\beta)\sin\delta}{\cos\emptyset\cos\delta\sin\omega_{S}+\left(\frac{\pi}{180}\right)\omega_{S}\sin\emptyset\sin\delta} \tag{8}$$

where β is the tilt angle of the surface of the collector with respect to the horizontal in (degree), and ω'_S is the sunset hour angle for the tilted surface in (degree). It is given by:

$$\omega'_{S}=\min\begin{bmatrix}\cos^{-1}(-\tan(\emptyset)*\tan(\delta))\\\cos^{-1}(-\tan(\emptyset-\beta)*\tan(\delta))\end{bmatrix} \tag{9}$$

where “min” means the smaller of the two values in the bracket.

3.2. Ground Reflected Radiation Incident on a Tilted Surface

The monthly average daily ground reflected radiation incident on an inclined surface in (MJ/m²/day) is given by:

$$H_R = H_g \rho \frac{(1 - \cos\beta)}{2} \tag{10}$$

where ρ is the solar reflectivity.

3.3. Diffuse Radiation Incident on a Tilted Surface

Isotropic and anisotropic models are the models used in estimating the ratio of the diffuse solar radiation on an inclined surface to that on a horizontal surface. The intensity of diffuse sky radiation in the isotropic model is assumed to be uniform over the sky dome. As a result, the diffuse radiation incident on a tilted surface depends on the fraction of the sky dome seen by it [31]. The anisotropic models assume the anisotropy of the diffuse radiation in the circumsolar region (sky near the solar disk), an isotropically distributed component from the rest of the sky dome and diffuse radiation from the horizon [32].

The sky diffuse radiation incident on an inclined surface in (MJ/m²/day) is given by:

$$H_D = R_d H_d \tag{11}$$

where R_d is the ratio of the average daily diffuse radiation on a tilted surface, to that on a horizontal surface expressed by Reindl as [33]:

$$R_d = \frac{H_b}{H_0} R_b + \left(1 - \frac{H_b}{H_0}\right)\left(\frac{1+\cos\beta}{2}\right)\left[1 + \sqrt{\frac{H_b}{H_g}}\sin\left(\frac{\beta}{2}\right)^3\right] \tag{12}$$

where H_b is the monthly average daily beam radiation on a horizontal surface in (MJ/m²/day).

3.4. Total Radiation Incident on a Tilted Surface

The total radiation incident on an inclined surface in (MJ/m²/day) is given by:

$$H_T = (H_g - H_d)R_b + H_g \rho \frac{1 - \cos\beta}{2} + H_d R_d \tag{13}$$

The direct and diffuse components of global radiation on a horizontal surface are needed for estimating global radiation on tilted surfaces.

4. Methodology

The NASA Climatology Resource for Agro-climatology website is used for obtaining the daily global solar radiation on a horizontal surface for all the selected sites for a period of four years (starting January 1st, 2008 and ending December 31st, 2011). The daily global radiation on a horizontal surface for February 29th, 2008 is removed. The remaining 1460 values of the daily global radiation on a horizontal surface are daily averaged to obtain the daily average global radiation on a horizontal surface between 2008 and 2011. In case the value of a daily global radiation is missing, the average of the daily global radiation on a horizontal surface for the day before and the day after the missing value is used instead. The overall error is small and doesn't affect the findings because the daily global radiation on a horizontal surface is averaged over four years. Mathematica 8.0 computer program using the abovementioned model is implemented to determine the annual optimal tilt angle for maximum solar radiation collection for each one of the 35 sites. A multiple regression analysis using the results of the computer simulation is conducted to develop a latitude based tilt angle optimization mathematical model for maximum solar energy collection in the Mediterranean region.

5. Regression Analysis

Table 2 summarizes the optimal tilt angle (degree), the annual solar radiation (MJ/m²), the predicted optimal tilt (degree) as well as the predicted annual solar radiation (MJ/m²) for each one of the 35 sites in the Mediterranean region selected for the study. In general, the annual optimal tilt angle increases with increasing latitude. This remark reinforces the claim that the annual optimal tilt angle depends on site's latitude. The general trend shows that the annual solar radiation decreases with increasing tilt angle. However, no conclusion can draw about it because the annual solar radiation varies considerably from site to site and even for sites at the same latitude. Although Monaco and Sarajevo have almost the same latitude (43.73° N and 43.51° N respectively), their annual solar radiation is 7177.77 MJ/m² and 5710.92 MJ/m² respectively. It represents an important difference of 1466.85 MJ/m².

Table 2. Recapitulative Table

Country	City	Latitude (° N)	Optimal Tilt Angle (°)	Annual Solar Radiation (MJ/m²)	Predicted Optimal Tilt Angle (°)	Predicted Annual Solar Radiation (MJ/m²)
Egypt	Luxor	25.69	28.1	9262.77	27.4	9262.19
Morocco	Smara	26.73	28.3	8725.67	28.0	8725.59
Egypt	Sharm Sheikkh	27.86	30.1	9291.24	29.0	9289.83
Morocco	Agadir	30.41	32.2	8391.50	31.3	8390.78
Libya	Syrte	31.20	31.3	8459.52	31.9	8459.17
Egypt	Alexandria	31.20	31.3	8686.46	31.9	8686.19
Palestine	Gaza	31.41	32.2	8623.50	32.1	8623.48
Libya	Tripoli	32.90	34.1	8353.38	33.1	8352.35
Syria	Damascus	33.51	33.1	8212.87	33.7	82.12.52
Algeria	Mecheria	33.55	35.3	8341.85	33.7	8339.40
Lebanon	Beirut	33.88	31.9	7767.32	33.8	7764.47
Cyprus	Nicosia	35.16	34.1	7873.21	35.0	7872.59
Morocco	Larache	35.18	34.9	7965.97	35.0	7965.95
Greece	Heraklion	35.32	32.5	7681.01	35.1	7675.73
Syria	Latakia	35.52	35.1	8076.53	35.2	8076.52
Tunisia	Tunis	35.41	37.2	8043.73	35.2	8040.29
Malta	Valetta	35.89	35.6	8143.29	35.3	8143.19
Tunisia	Bizerte	37.27	37.0	7746.87	36.5	7746.63
Spain	Seville	37.38	36.5	7304.04	36.6	7304.03
Turkey	Isparta	37.76	36.9	7263.43	36.7	7263.38
Italy	Marsala	37.80	37.4	7769.41	36.7	7769.04

Greece	Athens	37.97	35.7	7478.28	36.8	7477.32
Turkey	Bursa	40.18	36.0	6319.33	38.6	6315.18
Spain	Madrid	40.40	39.2	7095.51	38.6	7095.28
Albania	Vlorë	40.46	38.1	6989.54	38.7	6989.24
Greece	Thessaloniki	40.63	37.7	6554.41	38.7	6553.75
Italy	Naples	40.83	38.5	7081.13	38.8	7081.04
Montenegro	Podgorica	42.47	39.5	5806.41	40.0	5806.28
Bosnia & Herzegovina	Sarajevo	43.51	41.8	5710.92	40.7	5710.24
Monaco	Monaco	43.73	43.9	7177.77	40.7	7169.68
Italy	Florence	43.77	40.5	6134.86	40.7	6134.83
Italy	Milan	45.46	43.0	6045.56	41.8	6044.77
France	Lyon	45.76	43.7	5783.48	41.9	5781.53
Croatia	Zagreb	45.81	39.1	5365.38	41.9	5361.90
Slovenia	Ljubljana	46.05	42.1	5325.49	42.3	5325.46

Figure 1 shows a scatterplot of the annual optimal tilt angle for maximum solar energy collection with respect to site's latitude as well as the plot of the regression model. The data points are not exactly clustered about a straight line but instead follow a line with a very slight curvature. Because the data points have a curvilinear pattern that appears to be a simple curve, there is a quadratic relationship between the dependent variable (i.e. annual optimal tilt angle for maximum solar radiation collection) and the independent variable (i.e. site's latitude). As a result, the quadratic model is used to fit the data points. A majority of data points are clustered around the plot of the quadratic regression model with the exception of two points with latitudes around 44 °N and 45° N that appear to be potential outliers.

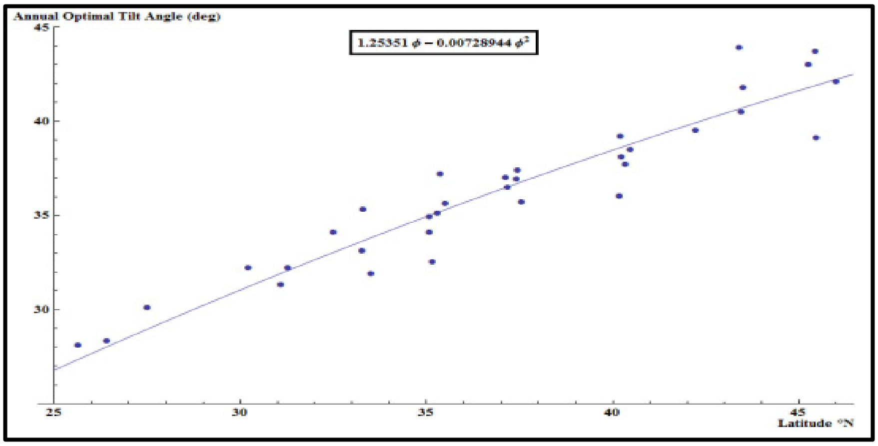

Fig. 1. Scatterplot of the annual optimal tilt angle with respect to site's latitude and plot of the quadratic regression model

Table 3. Parametric table

	Estimate	Standard Error	t-Statistic	P-Value
Latitude²	-0.00728944	0.00114203	-6.3829	$3.13988*10^{-7}$
Latitude	1.25351	0.0443511	28.2633	$1.05372*10^{-24}$

From table 3, the quadratic regression model that allows the determination of the annual optimal tilt angle for maximum solar energy based on site's latitude in the Mediterranean region is given by the following mathematical model:

$$\hat{Y}_i = 1.25351\emptyset_i - 0.00728944\emptyset_i^2 \quad (14)$$

where $\hat{Y}_i$ is the predicted annual optimal tilt for maximum solar radiation collection in (degree) for site "i", $\emptyset_i$ is the latitude in (degree N) for site "i" where $\emptyset_i$ ranges between 25°N and 46°N, b_2 = -0.00728944 represents the coefficient of the quadratic effect, and b_1 = 1.25351 represent the coefficient of the linear effect. The coefficient of the quadratic effect provides information about both the direction and the steepness of the curvature. A small negative coefficient of determination indicates a slight downward curvature.

Table 4. ANOVA table

	DF	SS	MS	F-Stat
Model	2	46173.1	23086.6	13220.21
Error	33	57.6282	1.74631	
Corrected Total	34	589.495		

From the value of F-Stat (table 4), it can that at least one of the terms of the quadratic model is useful for predicting the annual optimal tilt angle for maximum solar radiation collection; and hence, the quadratic model is useful. From the values of t-Statistic and P-value of "Latitude" (table 3), it can be concluded that the model is significantly improved by including the coefficient of the linear effect. Based on P-value and t-Statistic value of "Latitude²" (table 3), it can be concluded that the quadratic model is significantly better than the linear model for representing the relationship between the annual optimal tilt angle and site's latitude. The coefficient of multiple determination r^2 = 0.998753indicates that 99.87% of the variation in the annual optimal tilt angle is explained by the quadratic relationship with site's latitude. Only 0.13% of the sample variability in the annual optimal tilt angle is due to factors other than what is accounted for by the quadratic regression model.

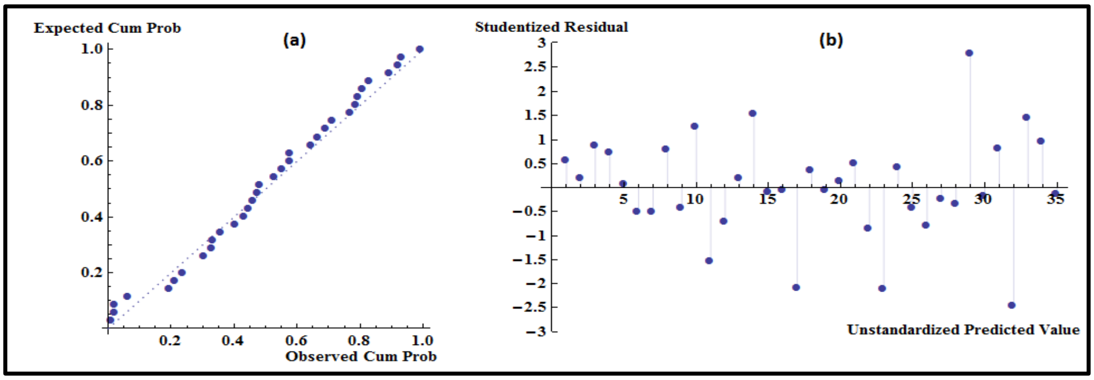

Fig. 2. (a) Normal Probability Plot; (b) Studentized Residual Plot

From figure 2.a, the normal plot of the residuals shows the data points close to a diagonal line; thus, the normality distribution assumption is satisfied. Each of the studentized residual plots in figure 2.b shows a random scatter of points with constant variability within three standard deviations from the mean residual of zero with a large majority of points varying within one standard deviation. As a result the equal variance assumption is satisfied.

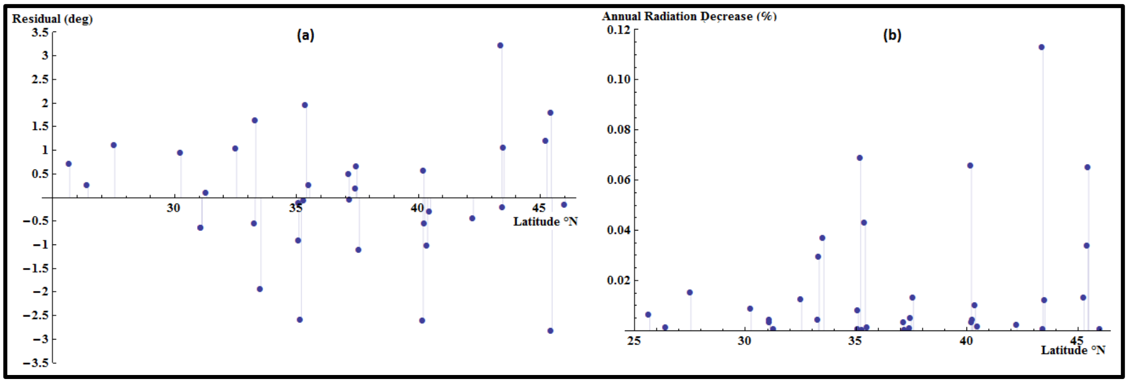

Fig. 2. (a) Optimal Tilt Angle Residual Plot; (b) Annual Solar Radiation Percentage Decrease Plot

Figure 3.a is a plot of the residual of the predicted optimal tilt angle from the actual optimal tilt angle with respect to site's latitude for the 35 sites in the Mediterranean region. The residuals vary modestly over sites' latitudes. The residuals vary in terms of absolute value between a minimum of 0.1° and a maximum of 3.2° with an average residual of 0.96°. It means that the quadratic model provides an accurate prediction of the annual optimal tilt for maximum solar radiation in the Mediterranean region. Figure 3.b is a plot of the percentage decrease in the incident annual solar radiation on a tilted surface when the predicted optimal tilt angles are used instead of the actual ones. For most cases (i.e. 27 out of 35) have an annual solar radiation percentage decrease less than 0.02%. The annual solar radiation percentage decrease varies between a minimum of $1.24 * 10^{-4}$% and a maximum of 0.11% with an average percentage decrease of 0.016%. This very small percentage decrease reinforces the accuracy of the quadratic model in the Mediterranean region.

6. Conclusion

In this paper, we developed a mathematical model for determining the optimal tilt angle for maximizing the total solar radiation incident on an inclined surface, using the global solar radiation incident on a horizontal surface. The paper also presents a quadratic regression model that allows the prediction of the annual optimal tilt angle for maximum solar radiation collection for sites in the Mediterranean region. The obtained quadratic regression model satisfies all the statistical tests and assumptions and provides an accurate approximation of the annual optimal tilt angle for maximum solar radiation collection. In fact, 99.87% of the variation in the annual optimal tilt angle is explained by the variability in site's latitude. For the 35 sites in the Mediterranean region selected for the study, using the quadratic model gives an optimal tilt angle average residual of only 0.96°. It also gives an average percentage decrease in the annual solar radiation of only 0.016% compared with actual optimal tilt angles.

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