Neutralino relic density in a supersymmetric $U(1)'$ model

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Abstract

We study properties of the lightest neutralino (χ^0) and calculate its cosmological relic density in a supersymmetric $U(1)'$ model with a secluded $U(1)'$ breaking sector (the S-model). The lightest neutralino mass is smaller than in the minimal supersymmetric standard model; for instance, $m_{\chi^0} \lesssim 100$ GeV in the limit that the $U(1)'$ gaugino mass is large compared to the electroweak scale. We find that the $Z\chi^0\chi^0$ coupling can be enhanced due to the singlino components in the extended neutralino sector. Neutralino annihilation through the Z-resonance then reproduces the measured cold dark matter density over broad regions of the model parameter space.

1. Introduction

Recent mapping of the Cosmic Microwave Background (CMB) anisotropy [1] has provided precision information on the densities of matter and dark energy in the Universe. The major part of the matter is non-relativistic and non-baryonic (cold and dark). When the Sloan Digital Sky Survey (SDSS) data on large scale structure are analyzed in combination with the Wilkinson Microwave Anisotropy Probe (WMAP) data, a cold dark matter (CDM) relic density

$$\Omega_{\text{CDM}}h^2 = 0.12 \pm 0.01 (\text{SDSS} + \text{WMAP}) \tag{1}$$

is found [2]. This very restrictive range for $\Omega_{\text{CDM}}h^2$ has significant impact on the allowed masses and couplings of cold dark matter particles.

A neutral, stable, massive particle that interacts weakly is a natural candidate for CDM. The Standard Model (SM) does not have a particle with these properties but a supersymmetric model with R-parity conservation, one of the best motivated new physics possibilities, does. In the Minimal Supersymmetric Standard Model (MSSM) the lightest neutralino is the favored lightest supersymmetric particle (LSP).

The lightest neutralinos existed abundantly in the early Universe in thermal equilibrium with other particles, where their pair annihilations were balanced by pair creation. As the Universe cooled, the neutralino density became Boltzmann-suppressed. Deviation from thermal equilibrium began when the temperature reached the freeze-out temperature $T_f \simeq m_{\chi^0}/20$. After the temperature dropped to $\sim \frac{1}{5}T_f$, the annihilation rate became equal to the expansion rate, and the neutralino relic density was $n_{\chi^0} = H/\langle \sigma v \rangle$, where H is the Hubble expansion rate at that temperature. Here $\langle \sigma v \rangle$ is the thermally averaged cross section times neutralino velocity [3]. The remaining CDM density relative to the critical density is

$$\Omega_{\chi^0} = n_{\chi^0} m_{\chi^0}/\rho_c = H m_{\chi^0}/(\langle \sigma v \rangle \rho_c). \tag{2}$$

A viable CDM model must reproduce the recent precise measurement of Ω_{CDM}, preferably without fine-tuning of the model parameters. The full test of neutralino dark matter can be accomplished with the direct detection of the LSP in collider experiments and/or in elastic scattering experiments at underground detectors. The parameters of the model must quantitatively explain $\Omega_{\chi^0} h^2 \simeq 0.12$ and the direct detection rates.

There have been extensive theoretical studies of the relic density [4–6]. In the minimal supergravity model (mSUGRA) annihilations of stable supersymmetric particles can reproduce the right order of magnitude. However, the allowed regions of mSUGRA parameter space are becoming increasingly constrained [7] by the combination of the recent relic density and the LEP data and now allow only a few regions of parameter space including (i) pair annihilation of neutralinos with dominant bino composition through A^0-, H^0-resonances at high $\tan \beta$ (the so-called A-funnel) [8], (ii) neutralino–stau coannihilation [9], and (iii) annihilation of neutralinos with mixed gaugino–higgsino components in the focus point region [10]. A nearly pure bino LSP state can give the right size of the relic density [11] and also satisfy the required radiative electroweak symmetry breaking (EWSB) in unified models [12]. In $SO(10)$ grand unified supergravity models with Yukawa unification, it is more difficult to obtain the required relic density, but it is still possible with specific scalar mass patterns [13].

The MSSM suffers from the μ-problem [14] and the lack of a sufficiently strong first order phase transition for electroweak baryogenesis (EWBG) over most of the parameter space.[1] The missing ingredients of possible TeV-scale new physics that would cure these problems may modify the properties of the CDM candidate. The Next-to-Minimal Supersymmetric extension of the Standard Model (NMSSM) [16] avoids the problems by introducing an additional SM singlet Higgs and a discrete symmetry $\mathbf{Z_3}$, but it then suffers from the cosmological domain wall problem [17].

A natural extension of the MSSM that avoids the above difficulties is a supersymmetric $U(1)'$ model, with Higgs singlets to break the additional Abelian symmetry spontaneously at the TeV scale.[2] Additional $U(1)$ gauge symmetries are predicted in many types of new theories, including superstrings [20], grand unification, extra dimensions [21], dynamical symmetry breaking [22], and the little Higgs model [23]. In this Letter we investigate the properties of the lightest neutralino and evaluate its relic density[3] in an extended model of the MSSM with an extra $U(1)$ gauge symmetry and four extra Higgs singlets ($S, S_1, S_2,$ and S_3) [25]. The superpotential is[4]

$$W = h_s S H_1 H_2 + \lambda_s S_1 S_2 S_3, \tag{3}$$

where h_s and λ_s are dimensionless parameters. We call this the S-model.

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[1] To have a strong first-order phase transition in the MSSM, the light Higgs mass should be only slightly above the LEP experimental bound and the light stop should be lighter than the top [15].

[2] For another extension of the MSSM using a discrete symmetry but free of domain wall problem, see the nMSSM model [18,19].

[3] For an earlier study of the relic density in a $U(1)'$ model with one singlet, in a different approach and framework, see Ref. [24].

[4] For a $U(1)'$ model with only one singlet, see Ref. [26]. It is more difficult to obtain $M_{Z'} \gg M_Z$ in such models.

The S-model solves the μ-problem by generating an effective μ parameter

$$\mu_{\text{eff}} = h_s \langle S \rangle. \tag{4}$$

The model is free from the domain wall problem since there is no discrete symmetry [25–27]. The S-model can also provide a sufficiently strong first-order phase transition and new sources of CP violation for EWBG [28]. The Z' has a large mass[5] generated by singlet Higgs fields, $S_{1,2,3}$, which acquire large (TeV scale) VEVs for small λ_s because of an almost F and D flat direction. These multiple singlets allow μ_{eff} to be of the electroweak scale while keeping the Z' heavier than the experimental limit.

Electroweak symmetry breaking is driven by electroweak scale trilinear terms. This leads to $\tan\beta \equiv v_2/v_1 \simeq 1$, while solutions without unwanted global minima at $\langle H_i^0 \rangle = 0$ typically have $\langle S \rangle \lesssim 1.5 \langle H_i^0 \rangle$ [25]. However, both of these conditions can be relaxed somewhat [31]. All dimensionful supersymmetry-breaking parameters are close to the electroweak scale. The squark and slepton masses are similar to those of the MSSM, and the soft supersymmetry breaking Higgs and singlet masses are smaller than the Higgs doublet mass in the MSSM.

In our analysis, we consider the limit with $M_1' \gg M_{1,2}$, where M_1', M_1, and M_2 are the gaugino masses for $U(1)'$, $U(1)_Y$, and $SU(2)_L$, respectively [25,31]. This limit suppresses the effects of the $U(1)'$ Higgs charges and greatly reduces the number of free parameters in the neutralino mass matrix. The allowed mass range of the lightest neutralino then is limited ($m_{\chi^0} \lesssim 100$ GeV) compared to that of the MSSM, due to the large component of the singlino in the neutralino, which we assume is the LSP. While the small mass of the LSP makes the heavy fermion and gauge boson channels of the MSSM irrelevant, it opens up new relevant channels, particularly the Z-resonance and the light Higgs resonance channels. The lighter Higgs scalar (h^0) in this model is a mixture of the Higgs doublets and singlets, and it can be much lighter than the LEP bound of $m_{h^0} > 115$ GeV that applies to the SM-like Higgs bosons [25,31].

In the MSSM, Z-resonance annihilation is not likely to be a relevant channel because the $Z\chi^0\chi^0$ coupling is small when $\tan\beta \sim 1$. However, when the singlino component is introduced into the neutralino sector, the $Z\chi^0\chi^0$ coupling can be enhanced to give sufficiently large $\chi^0\chi^0$ annihilation. This Z-resonance annihilation alone, with suitable parameter values, can reproduce the acceptable cold dark matter relic density in most of the allowed χ^0 mass range in the S-model.

2. Mass and coupling of the lightest neutralino

We consider a scenario with a massive Z' and VEVs of S_i ($i = 1, 2, 3$) that are large compared to other electroweak scale parameters. There is an approximate decoupling of the neutralinos associated with the Z' and the S_i, and the effective neutralino mass matrix for the remaining neutralinos in the basis of $\{\tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0, \tilde{S}\}$ is[6]

$$\mathcal{M}_{\chi^0} = \begin{pmatrix} M_1 & 0 & -g_1 v_1/2 & g_1 v_2/2 & 0 \\ 0 & M_2 & g_2 v_1/2 & -g_2 v_2/2 & 0 \\ -g_1 v_1/2 & g_2 v_1/2 & 0 & -h_s s/\sqrt{2} & -h_s v_2/\sqrt{2} \\ g_1 v_2/2 & -g_2 v_2/2 & -h_s s/\sqrt{2} & 0 & -h_s v_1/\sqrt{2} \\ 0 & 0 & -h_s v_2/\sqrt{2} & -h_s v_1/\sqrt{2} & 0 \end{pmatrix}, \tag{5}$$

where $e = g_1 \cos\theta_W = g_2 \sin\theta_W$. The VEVs of the Higgs doublets are $\langle H_i^0 \rangle \equiv \frac{v_i}{\sqrt{2}}$ with $\sqrt{v_1^2 + v_2^2} \simeq 246$ GeV and the VEV of the Higgs singlet is $\langle S^0 \rangle \equiv \frac{s}{\sqrt{2}}$. The mass eigenstates are ordered as $m_{\chi^0} = m_{\chi_1^0} < m_{\chi_2^0} < \cdots < m_{\chi_5^0}$. This mass matrix leads to a kind of see-saw mechanism [19,25,33] that makes the lightest neutralino mass very

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[5] The CDF limit is $M_{Z'} \gtrsim 500$–800 GeV, depending on the model [29,30].

[6] In this limit, the neutralino matrix is basically the same as that of the one-singlet models and shares many of the same properties [19,32].

small. In our analysis we scanned over the parameters

$\tan\beta = 1.03, 1.5, 2.0, 2.5,$

$h_s = 0.1$ to 0.75 (in steps of 0.01),

$M_2 = -500$ to $500\,\text{GeV}$ (in steps of 1 GeV),

$s = 100$ to $500\,\text{GeV}$ (in steps of 1 GeV).

We use the gaugino mass unification relations ($M_1 = \frac{5}{3}\frac{g_1^2}{g_2^2}M_2 \simeq 0.5M_2$), but take $M_1' \gg M_2$. The $U(1)'$ charge dependence vanishes from the effective mass matrix for large M_1'.

The MSSM mass matrix[7] corresponds to dropping the last row and column in the matrix of Eq. (5) and taking $\mu = h_s s/\sqrt{2}$. The MSSM upper bound on m_{χ^0} is very sensitive to the value of M_2, while that of the S-model depends sensitively on the value of h_s. In both models, the upper bound on the lightest neutralino mass has its largest value when $\tan\beta$ is 1.03 (or 1).[8] When $\tan\beta \sim 1$, the m_{χ^0} bound in the S-model increases almost linearly with h_s. We choose our upper bound of h_s to be 0.75 as advocated in Ref. [25] to keep the running of h_s finite up to the Planck scale [19,25].

We find that

$$m_{\chi^0} \lesssim 99\,\text{GeV}. \tag{6}$$

The maximum value of m_{χ^0} occurs with $\tan\beta = 1.03$, $h_s = 0.75$, $M_2 = -183\,\text{GeV}$ and $s = 130\,\text{GeV}$ for the S-model[9] while for the MSSM the maximum $m_{\chi^0} \lesssim 254\,\text{GeV}$ occurs for $\tan\beta = 1.03$, $M_2 = -500\,\text{GeV}$ and $\mu \simeq 261\,\text{GeV}$.

The small LSP mass makes the Z-resonance contribution to the relic density relevant in the S-model. For $Z\chi_i^0\chi_j^0$ and $Z'\chi_i^0\chi_j^0$ interactions, the general Lagrangian is given by [33]

$$\mathcal{L} = \frac{1}{4}g_Z Z_\mu\big(\bar{\tilde{H}}_1^0\gamma^\mu\gamma^5\tilde{H}_1^0 - \bar{\tilde{H}}_2^0\gamma^\mu\gamma^5\tilde{H}_2^0\big)$$
$$+ \frac{1}{4}g_{Z'}Z_\mu'\big(2Q_1\bar{\tilde{H}}_1^0\gamma^\mu\gamma^5\tilde{H}_1^0 + 2Q_2\bar{\tilde{H}}_2^0\gamma^\mu\gamma^5\tilde{H}_2^0 + 2Q_S\bar{\tilde{S}}\gamma^\mu\gamma^5\tilde{S}\big) \tag{7}$$

which, in terms of neutralino mass eigenstates, can be written as

$$\mathcal{L}_{Z(Z')\chi^0\chi^0} = \frac{1}{2}g_Z Z_\mu\bar{\chi}_i^0\gamma^\mu\big(O_{ij}''^L P_L + O_{ij}''^R P_R\big)\chi^0 + \frac{1}{2}g_{Z'}Z_\mu'\bar{\chi}_i^0\gamma^\mu\big(I_{ij}''^L P_L + I_{ij}''^R P_R\big)\chi^0. \tag{8}$$

Here the couplings are defined as

$$O_{ij}''^L \equiv -\frac{1}{2}N_{i3}N_{j3}^* + \frac{1}{2}N_{i4}N_{j4}^*, \qquad O_{ij}''^R \equiv \frac{1}{2}N_{i3}^*N_{j3} - \frac{1}{2}N_{i4}^*N_{j4}, \tag{9}$$

$$I_{ij}''^L \equiv -Q_1 N_{i3}N_{j3}^* - Q_2 N_{i4}N_{j4}^* - Q_S N_{i5}N_{j5}^*, \tag{10}$$

$$I_{ij}''^R \equiv Q_1 N_{i3}^*N_{j3} + Q_2 N_{i4}^*N_{j4} + Q_S N_{i5}^*N_{j5}, \tag{11}$$

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[7] The MSSM parameter range considered here (especially for $\tan\beta$) is not experimentally allowed (e.g., by the lightest Higgs mass), but is chosen to demonstrate the effects of adding the $U(1)'$.

[8] Since the exact value $\tan\beta = 1$ would make the coupling of $Z\chi^0\chi^0$ vanish in both the MSSM and the S-model, we choose $\tan\beta = 1.03$ instead of 1.

[9] The upper bound on m_{χ^0} in the different limits are: (i) for $M_1' \gg M_2$, $s_i \sim \mathcal{O}(\text{EW})$, $m_{\chi^0} \lesssim 100\,\text{GeV}$; (ii) for $M_1' = M_1$, $s_i \gg \mathcal{O}(\text{EW})$, $m_{\chi^0} \lesssim 280\,\text{GeV}$; (iii) for $M_1' = M_1$, $s_i \sim \mathcal{O}(\text{EW})$, $m_{\chi^0} \lesssim 100\,\text{GeV}$. We scanned 100 to 1000 GeV for M_2 and s_i and assumed $h_s \lesssim 0.75$ with $\lambda_s \lesssim 0.2$. For the $U(1)'$ couplings, various E_6 model charge assignments (χ, ψ, η) and the GUT motivated coupling constant $g_{Z'} = \sqrt{\frac{5}{3}}g_1$ were assumed. For a full 9×9 neutralino mass matrix, see Refs. [25,31].
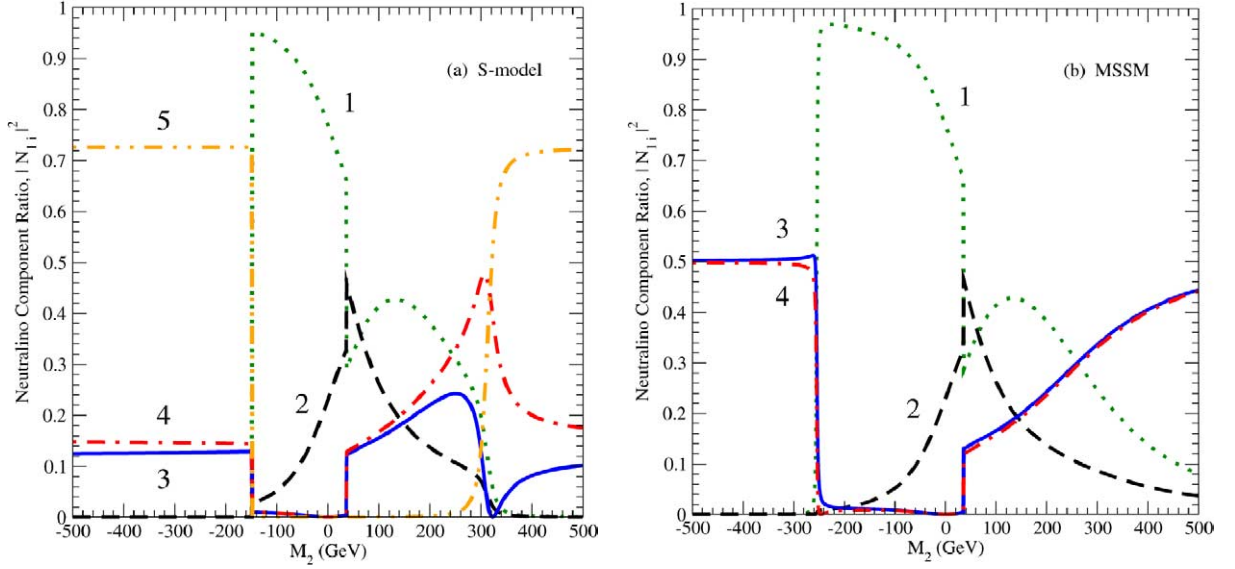
Fig. 1. The fraction of the lightest neutralino ($|N_{1i}|^2$) versus M_2 for (a) the S-model and (b) the MSSM. All components are presented: $|N_{11}|^2$ ($\tilde{B}$, dot), $|N_{12}|^2$ ($\tilde{W}_3$, dash), $|N_{13}|^2$ ($\tilde{H}_1^0$, solid), $|N_{14}|^2$ ($\tilde{H}_2^0$, dash-dot), and $|N_{15}|^2$ ($\tilde{S}$, dash-dot-dot). The curve for $|N_{1i}|^2$ is labeled by i. Fixed values of $h_s = 0.75$, $\tan\beta = 1.03$ and $s = 250$ GeV are illustrated. The difference of $|N_{13}|^2$ and $|N_{14}|^2$ is large when the singlino component is present even though $\tan\beta \sim 1$.

where the matrix N relates the weak and mass bases. Q_1, Q_2 and Q_S are the $U(1)'$ charges of the H_1^0, H_2^0 and S, respectively. The $Z\chi^0\chi^0$ coupling $O_{11}''^L = -O_{11}''^R = -\frac{1}{2}(|N_{13}|^2 - |N_{14}|^2)$ is composed of the two doublet higgsino components coupling to the Z boson. A massive (TeV-scale) Z' would not provide sufficiently large annihilation for our light neutralinos, and thus in our analysis we use only the Z-resonance annihilation.

Fig. 1 presents the $|N_{1i}|^2$ with $i = 1$ ($\tilde{B}$, dot), 2 ($\tilde{W}_3$, dash), 3 ($\tilde{H}_1^0$, solid), 4 ($\tilde{H}_2^0$, dash-dot), 5 ($\tilde{S}$, dash-dot-dot), for $M_2 = -500$ to 500 GeV and a fixed set of values $h_s = 0.75$, $\tan\beta = 1.03$ and $s = 250$ GeV. The value of each $|N_{1i}|^2$ is shown versus M_2 in (a) the S-model and (b) the MSSM. We note that in the region around $M_2 \sim 300$ GeV the singlino component $|N_{15}|^2$ increases, and $|N_{13}|^2$ deviates substantially from $|N_{14}|^2$ making the $Z\chi^0\chi^0$ coupling large. This does not happen in the MSSM for the same parameters. In addition, for $M_2 \lesssim -150$ GeV, the singlino dominates and the difference of $|N_{13}|^2$ and $|N_{14}|^2$ in the S-model is larger than that in the MSSM. As an example, the components of the LSP for $M_2 = 330$ GeV are

$$\chi^0 = 0.18\tilde{B} - 0.11\tilde{W}_3 - 0.09\tilde{H}_1^0 - 0.58\tilde{H}_2^0 + 0.78\tilde{S} \quad (S\text{-model}), \tag{12}$$

$$\chi^0 = 0.46\tilde{B} - 0.27\tilde{W}_3 + 0.60\tilde{H}_2^0 - 0.60\tilde{H}_3^0 \qquad (\text{MSSM}). \tag{13}$$

The large difference in $|N_{13}|^2$ and $|N_{14}|^2$ for the S-model is remarkable in view of the fact that $\tan\beta = 1$ makes the $\tilde{H}_1^0$ and $\tilde{H}_2^0$ parts in the mass matrix of Eq. (5) the same up to the sign, which leads one to expect $|N_{13}|^2 - |N_{14}|^2 \sim 0$, as is the case in the MSSM (Fig. 1(b)). Thus, the addition of the singlino component plays a critical role in enhancing the difference of $|N_{13}|^2$ and $|N_{14}|^2$, which is particularly important for the χ^0 in the S-model to generate sufficient annihilation mediated by the Z boson. The suppressed coupling (by $\tan\beta \sim 1$) can still be large enough to give an acceptable relic density. Fig. 1 also shows that the gaugino components $|N_{11}|^2$ (bino) and $|N_{12}|^2$ (wino), especially the bino, are dominant for relatively small values of $|M_2|$ in both models.

We present $|O_{11}''| \equiv |O_{11}''^L| = |O_{11}''^R|$ for the S-model and the MSSM in Fig. 2. For the same values of $h_s = 0.75$ and $s = 250$ GeV, we select 3 different $\tan\beta$ values of (a) $\tan\beta = 1.03$, (b) $\tan\beta = 2.0$, and (c) $\tan\beta = 2.5$. For each $\tan\beta$, the $Z\chi^0\chi^0$ coupling is much larger in the S-model than in the MSSM in most of the parameter space.
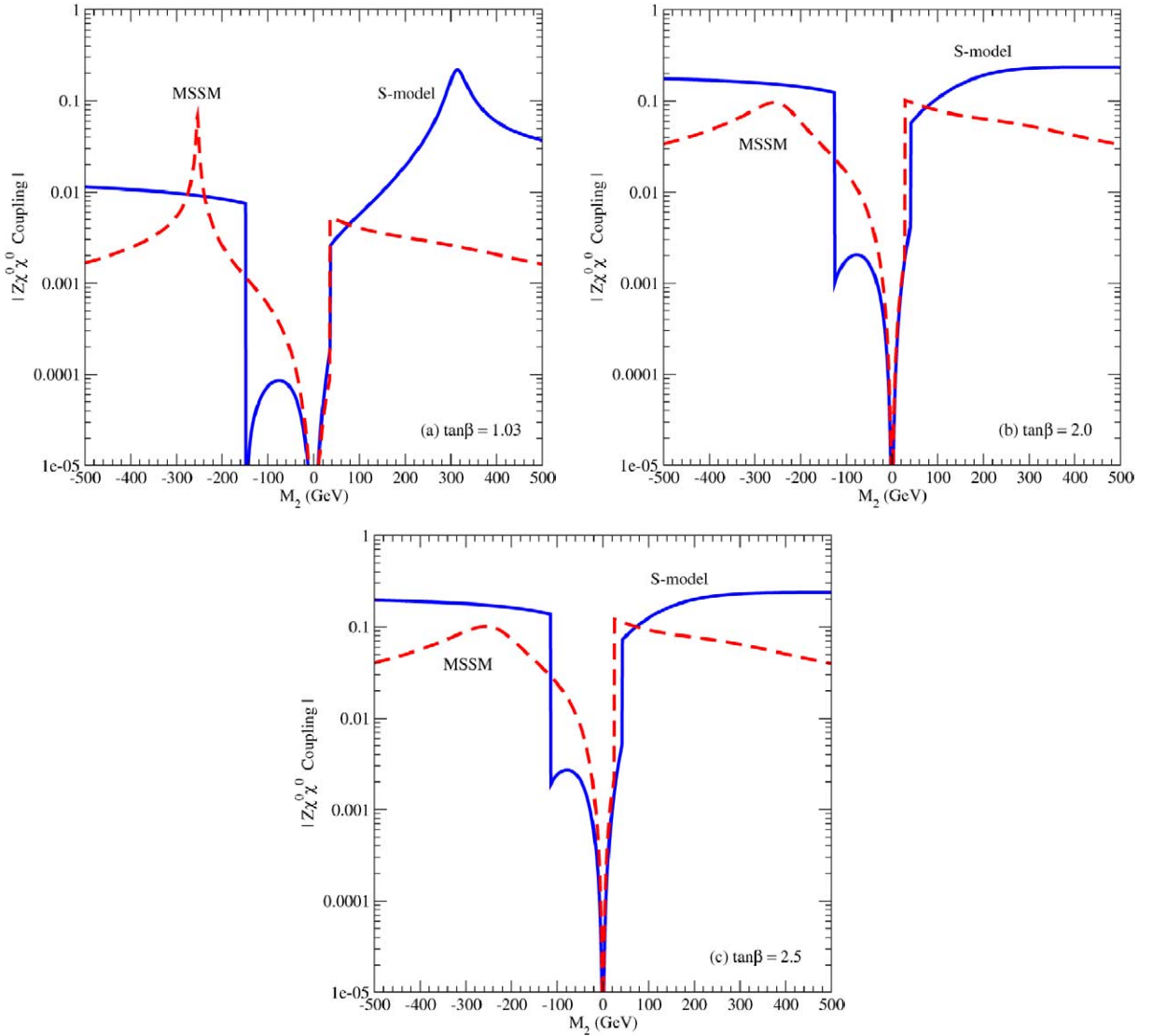
Fig. 2. The $Z\chi^0\chi^0$ coupling $|O''_{11}|$ versus M_2 for (a) $\tan\beta = 1.03$, (b) $\tan\beta = 2.0$, and (c) $\tan\beta = 2.5$, in the S-model (solid) and in the MSSM (dash). Fixed values of $h_s = 0.75$ and $s = 250$ GeV are used. The coupling is much larger in the S-model than in the MSSM in most of the parameter space.

We numerically checked that this feature holds for other choices of the parameter values ($0.4 \lesssim h_s \lesssim 1.0$, 100 GeV $\lesssim s \lesssim 1000$ GeV, $0.5 \lesssim \tan\beta \lesssim 2.5$).

For $\tan\beta \sim 1$ (Fig. 2(a)), the $Z\chi^0\chi^0$ coupling in the S-model is at its maximum for $M_2 \simeq 300$ GeV and there is a relatively small but still noticeable peak in the MSSM at $M_2 \simeq -250$ GeV. The χ^0 relic density depends on not only the coupling but also whether m_{χ^0} is near $M_Z/2$.

Fig. 3(a) shows the χ^0 mass versus M_2 and (b) $|O''_{11}|$ versus the LSP mass. The same parameter values as Figs. 1 and 2(a) are used. The LSP mass in the S-model is almost constant for $M_2 \lesssim -200$ GeV and $M_2 \gtrsim 300$ GeV, and it is smaller than in the MSSM. The lightest neutralino masses in both models have practically identical dependence
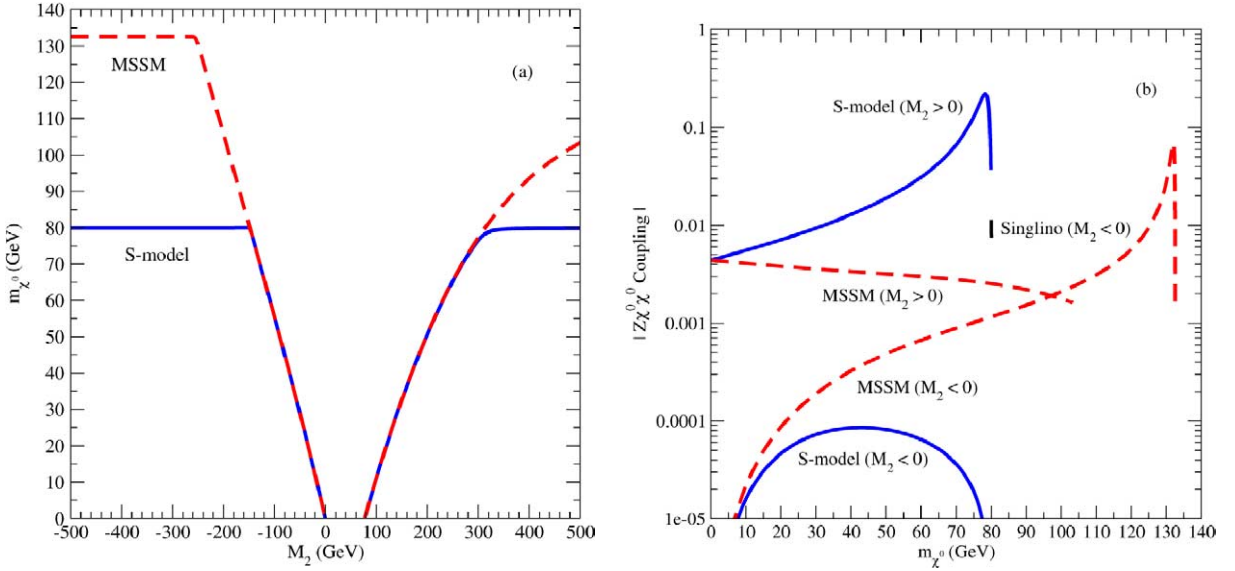
Fig. 3. (a) The lightest neutralino mass as a function of M_2 and (b) the $Z\chi^0\chi^0$ coupling ($|O_{11}''|$) versus the lightest neutralino mass in the S-model (solid) and the MSSM (dash). Fixed values of $h_s = 0.75$, $\tan\beta = 1.03$ and $s = 250$ GeV are used. The S-model has a smaller m_{χ^0} bound, and, for $M_2 > 0$, larger $Z\chi^0\chi^0$ coupling than the MSSM.

on M_2 before they reach the flat curves in our parameter choice ($\tan\beta \simeq 1$). Enhancements of couplings in the S-model and the MSSM are found around $m_{\chi^0} \sim 80$ GeV and 130 GeV, respectively, that is, for the flat parts of the mass curves in Fig. 3(a).

3. Neutralino annihilation mediated by the Z boson

The relic density of the lightest neutralino is inversely proportional to the annihilation cross section of χ^0 pairs. When kinematically allowed, the neutralino pairs annihilate into pairs of fermions, gauge bosons, and Higgs bosons through s, t, and u channel diagrams. In general, the annihilation cross section is greatly enhanced by the Z boson or the Higgs boson (ϕ^0) poles, and the relic density is correspondingly suppressed when $2m_{\chi^0} \sim M_Z$ or $2m_{\chi^0} \sim M_\phi$.

The Higgs masses and couplings in the S-model depend on several free parameters. Moreover, the light Higgs width is very narrow and a fine-tuning of parameters would be required for neutralino annihilation through ϕ^0 to generate the appropriate relic density, and the other Higgs bosons would often be too heavy to have resonance effects because the LSP is light [31]. We are therefore interested in the case that the $Z\chi^0\chi^0$ coupling is sizable and Z mediated annihilation alone can lead to an acceptable relic density ($\Omega_{CDM}h^2$). A more complete investigation is in progress to include all Higgs resonance effects. We require a lower bound on m_{χ^0} of $M_Z/2$ so that $Z \to \chi^0\chi^0$ does not affect the Z width significantly.[10] Then the lightest neutralino mass in the S-model is constrained to the range

$$46 \lesssim m_{\chi^0} \lesssim 100 \text{ GeV} \tag{14}$$

with the upper bound from Eq. (6).

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[10] Since $\Delta\Gamma_{inv} < 2.0$ MeV (95% C.L.) [34], smaller m_{χ^0} is possible [19]. For reasonably smaller masses, our results are unchanged.

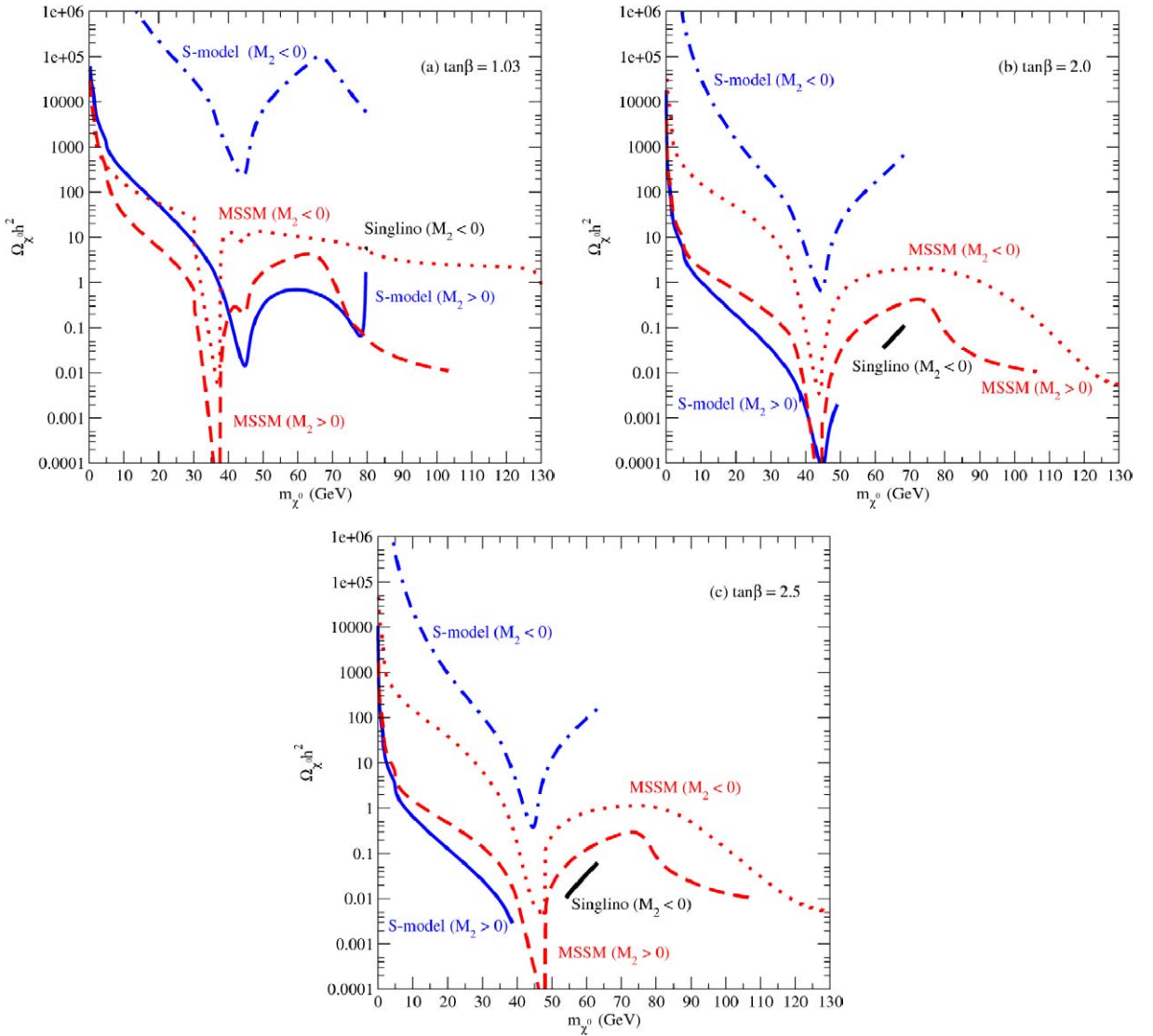Fig. 4. The neutralino relic density ($\Omega_{\chi^0} h^2$) versus m_{χ^0} in the S-model [$M_2 > 0$ (solid) and $M_2 < 0$ (dash-dot)] and the MSSM [$M_2 > 0$ (dash) and $M_2 < 0$ (dot)] for (a) $\tan\beta = 1.03$, (b) $\tan\beta = 2.0$, and (c) $\tan\beta = 2.5$. The isolated black curve corresponds to the singlino-dominated flat part of the S-model in Fig. 3 with $M_2 < 0$. Fixed values of $h_s = 0.75$ and $s = 250$ GeV are used. The acceptable relic density is $\Omega_{\chi^0} h^2 \sim 0.1$.

We evaluate the annihilation cross section including interferences among all diagrams by calculating the amplitude of each diagram with the helicity amplitude formalism, then numerically evaluate the full matrix element squared. The neutralino relic density is calculated with relativistic Boltzmann averaging, neutralino annihilation threshold effects and Breit–Wigner poles [5,6,8].

We show the neutralino relic density ($\Omega_{\chi^0} h^2$) versus m_{χ^0} in Fig. 4 for both the S-model[11] and the MSSM for the $\tan\beta$ choices (a) $\tan\beta = 1.03$, (b) $\tan\beta = 2.0$, and (c) $\tan\beta = 2.5$. We included all possible channels for

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[11] For similar relic density results in the nMSSM, see Ref. [19]. For the NMSSM relic density calculations, see Ref. [35].

the MSSM calculation; for the S-model we included only the kinematically relevant s-channel processes with the $Z\chi^0\chi^0$ coupling, $\chi^0\chi^0 \to Z \to f_i\bar{f}_i$, W^+W^- (f_i is a SM fermion). We scanned over $-500 < M_2 < 500$ GeV for $h_s = 0.75$, and $s = 250$ GeV to calculate m_{χ^0} and evaluate the $Z\chi^0\chi^0$ coupling. There are several interesting aspects to note.

- When $m_{\chi^0} \simeq M_Z/2$, there is a dip in the curve of relic density versus m_{χ^0} since there is a peak in the annihilation cross section enhanced by the Z-pole.
- In the S-model, a positive value of M_2 gives two acceptable CDM density regions (around the Z pole and the enhanced coupling region such as $m_{\chi^0} \sim 80$ GeV in the $\tan\beta = 1.03$ case) while a negative M_2 leads to $\Omega_{\chi^0}h^2 \gtrsim 1$ except for the singlino-dominated region.
- The MSSM can also give an acceptable CDM density, but the model is excluded for $\tan\beta \sim 1$ by the LEP Higgs bound, owing to its small Higgs mass. The S-model Higgs contains a singlet component and the light Higgs can be compatible with the LEP constraint [31].
- The small isolated region for relatively large m_{χ^0} corresponds to $M_2 < 0$ with the lightest neutralino being singlino-dominated in the S-model.[12] As observed in Fig. 1, a sudden sizable deviation of $|N_{13}|^2$ from $|N_{14}|^2$ in a singlino dominated region ($M_2 \lesssim -150$ GeV for $\tan\beta = 1.03$) provides a sudden drop of $\Omega_{\chi^0}h^2$ from the rest of the $M_2 < 0$ curve.

Fig. 5 presents ranges of neutralino relic density in regions of the M_2-s plane in the S-model. We choose $h_s = 0.75$ with (a) $\tan\beta = 1.03$, (b) $\tan\beta = 2.0$, and (c) $\tan\beta = 2.5$. Also shown is the region excluded by the LEP 2 chargino mass bound with $m_{\chi_1^+} < 104$ GeV [36]. The parameter points at which the light chargino is the LSP were omitted. The tree level mass for the lighter chargino ($m_{\chi_1^+} < m_{\chi_2^+}$) is evaluated with the chargino mass matrix

$$m_{\chi^\pm} = \begin{pmatrix} M_2 & \sqrt{2}M_W\sin\beta \\ \sqrt{2}M_W\cos\beta & \mu_{\text{eff}} \end{pmatrix}.$$

We present the S-model neutralino relic density in 3 regions: $0.09 < \Omega_{\chi^0}h^2 < 0.15$ (filled square), $\Omega_{\chi^0}h^2 < 0.09$ (open circle), and $0.15 < \Omega_{\chi^0}h^2 < 1.0$ (cross). The 3σ range[13] of the CDM relic density of Eq. (1) is $0.09 < \Omega_{\chi^0}h^2 < 0.15$; however if there are other sources of dark matter in addition to the lightest neutralino, the range $\Omega_{\chi^0}h^2 < 0.09$ would be relevant. Due to the finite grid, the filled square should be understood to be on the boundary of the open circle and the cross.

There are three separate regions that have an acceptable CDM density (filled square): (P1) Near the Z pole, (P2) enhanced coupling region ($\tan\beta \approx 1$ case), and (P3) isolated singlino region. In general, the $Z\chi^0\chi^0$ coupling is enhanced by the singlino component. There appears a sudden peak of the enhancement (P2) when $\tan\beta \approx 1$ as we can see from Fig. 2(a). In the enhanced coupling region (P2), even when the χ^0 mass is significantly distant from the Z pole, an acceptable relic density can be obtained. The isolated singlino region (P3) is singlino-dominated and happens for $M_2 < 0$.

For $\tan\beta = 1.03$, there is a small region in the M_2-s plane with $M_2 > 0$ that satisfies the relic density and LEP chargino mass constraints. The solution in this region is due to the enhanced $Z\chi^0\chi^0$ coupling. For $\tan\beta = 2.0$, there is a large acceptable region with $M_2 < 0$. With $M_2 > 0$ most of the parameter regions that give an acceptable relic density are excluded by the LEP 2 chargino search. For $\tan\beta = 2.5$, there is a large region with $M_2 > 0$ that reproduces the observable relic density and is consistent with the chargino mass limits.

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[12] The case in which the lightest neutralino is mostly singlino-like (e.g., $\tan\beta = 1.03$, $m_{\chi^0} \sim 80$ GeV for both $M_2 < 0$ and $M_2 > 0$), is qualitatively similar to those studied in Ref. [24].

[13] Since we are using tree level masses and couplings, we allow rather conservative 3σ range for the allowed CDM relic density.
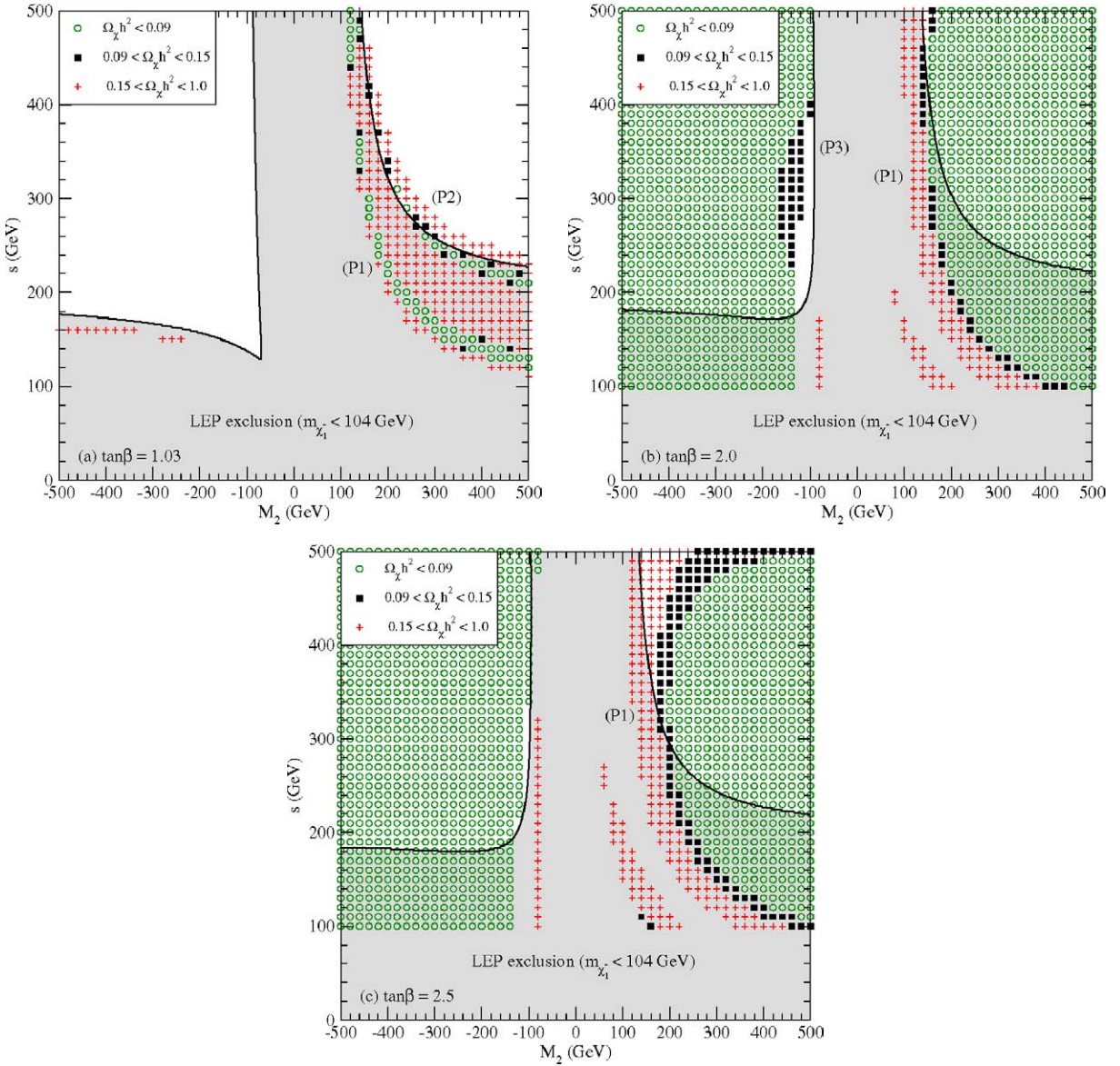
Fig. 5. Regions of the S-model neutralino relic density in the M_2-s plane (with s scanned only above 100 GeV) for $0.09 < \Omega_{\chi^0}h^2 < 0.15$ (filled square; 3σ allowed range), $\Omega_{\chi^0}h^2 < 0.09$ (open circle), and $0.15 < \Omega_{\chi^0}h^2 < 1.0$ (cross). Three representative values of $\tan\beta$ are chosen: (a) $\tan\beta = 1.03$, (b) $\tan\beta = 2.0$, and (c) $\tan\beta = 2.5$, and a fixed value of $h_s = 0.75$ is used. The shaded region of the parameter space (bounded by solid curves) is excluded by the LEP 2 chargino mass limit ($m_{\chi_1^+} \lesssim 104$ GeV). There exist sizable regions (filled square) in the parameter space consistent with the relic density constraint outside of the chargino mass exclusion boundary.

We numerically checked, with suitable parameter values including different h_s, that the S-model can reproduce the observed relic density for most of the theoretically allowed m_{χ^0} range without violating the LEP constraints. However, for a relatively large neutralino mass, i.e., $m_{\chi^0} \approx 80$ to 100 GeV, it becomes hard to satisfy both the

relic density and the LEP constraints since those m_{χ^0} values occur only with $\tan\beta$ close to 1 and h_s close to its maximum (0.75), but then the chargino constraint becomes severe, as can be seen in Fig. 5(a).

4. Conclusions

We have studied the properties of the lightest neutralino in a supersymmetric $U(1)'$ model with a secluded $U(1)'$ breaking sector [25]. In this model, $\tan\beta \sim 1$ is required for natural electroweak symmetry breaking. In general, the model allows a lightest neutralino mass up to about 300 GeV (depending on the charge assignments and input mass limits) with the gaugino unification assumption $M_{1'} = M_1 = \frac{5}{3}\frac{g_1^2}{g_2^2}M_2$, but only up to $m_{\chi^0} \lesssim 100$ GeV in the limit with $M_{1'} \gg M_1 = \frac{5}{3}\frac{g_1^2}{g_2^2}M_2$. We quantitatively studied the S-model in the limit where M_1', s_i are much larger than the electroweak scale. In this limit the $Z\chi^0\chi^0$ coupling is enhanced compared to the MSSM due to the singlino component. This allows the χ^0 pair annihilation rate via the Z resonance channel alone to reproduce the observed relic density of the cold dark matter. In addition, the doublet-singlet mixed nature of the Higgs bosons allows the lightest Higgs bosons to have a small mass without violating the LEP constraint on SM-like Higgs mass. The S-model explains the relic density over a considerable fraction of the parameter space.

Acknowledgements

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