Non-perturbative Kähler potential, dilaton stabilization and Fayet–Iliopoulos term

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Abstract

We study the dilaton stabilization in models with anomalous $U(1)$ symmetry by adding specific string-motivated, non-perturbative corrections to the tree-level dilaton Kähler potential. We find that the non-perturbative effects can stabilize the dilaton at a desirably large value. We also observe that the size of Fayet–Iliopoulos term is reduced at the stabilized point.
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Dilaton and moduli fields play an important role in superstring theory as well as extra-dimensional models. Within the framework of 4D string models, couplings like gauge and Yukawa couplings are determined by vacuum expectation values (VEVs) of these fields. In heterotic models, for example, the gauge coupling g is determined as $1/g^2 = \langle \mathrm{Re}(S) \rangle$ by the VEV of the dilaton field S. However, in 4D models with $N = 1$ supersymmetry (SUSY) these fields have perturbatively flat potential, and their VEVs are undetermined. Thus, how to stabilize their VEVs is an important problem. The non-perturbative superpotential due to gaugino condensations is a plausible origin for sta-

bilizing their VEVs. However, in the case with a single gaugino condensation and the tree-level Kähler potential,

$$K_0(S + \bar{S}) = -\ln(S + \bar{S}), \tag{1}$$

the dilaton VEV cannot be stabilized at a finite value, but runs away to infinity.

Several models have been proposed to stabilize the dilaton VEV. The models with double or more gaugino condensations, i.e., the so-called racetrack models, can stabilize the dilaton VEV [1]. The problem of the racetrack type models is that the stabilized value of the dilaton tends to be too small compared with the value $\mathrm{Re}(S) = 1/g^2 \approx 2$, which is suggested by the unified gauge coupling in the minimal supersymmetric Standard Model. A certain degree of fine-tuning is necessary to realize the dilaton stabilization at weak coupling region.

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Another possibility for the dilaton stabilization is to assume non-perturbative Kähler potential of the dilaton field [2,3], as was studied in Refs. [3–7]. With a certain form of non-perturbative Kähler potential, a single gaugino condensation can stabilize the dilaton at a finite value. Moreover, the dilaton VEV of $O(1)$ can be realized for a reasonable choice of parameters, although one has still to fine-tune parameters so that the tree-level vacuum energy vanishes.

On the other hand, it is usually true that D-terms in the scalar potential do not play any essential role on dilaton stabilization, because the dilaton field appears as an overall factor in D-terms. There can happen, however, an exception, that is, the case with D-term for an anomalous $U(1)$ symmetry. Most of 4D string models have anomalous $U(1)$ symmetries [8–10], whose anomalies can be canceled by the Green–Schwarz (GS) mechanism. In heterotic models, the dilaton field transforms non-linearly like $S \to S + 2i\delta_{GS}\Lambda_A$ under anomalous $U(1)$ transformation $V_A \to V_A + i\Lambda_A - i\bar{\Lambda}_A$, where δ_{GS} is a GS coefficient and V_A is the anomalous $U(1)$ vector multiplet. It follows that the dilaton Kähler potential is a function $K(s)$ of gauge-invariant combination $s \equiv S + \bar{S} - 2\delta_{GS}V_A$. Accordingly, the anomalous $U(1)$ D-term contains the Fayet–Iliopoulos (FI) term

$$\xi = \delta_{GS}\langle K_S \rangle M^2, \tag{2}$$

where M is the reduced Planck scale and K_S is the first derivative of the dilaton Kähler potential. If we take the tree-level Kähler potential $K_0(s)$ and assume that $\mathrm{Re}(S) = O(1)$, we have $\xi^{1/2}/M = 10^{-1}$–10^{-2}. (Hereafter we take the $M = 1$ unit.) In general, the magnitude of the FI term depends on the dilaton VEV as well as the form of dilaton Kähler potential. Therefore, the anomalous $U(1)$ D-term can play a nontrivial role in dilaton stabilization, as was suggested before in Refs. [11,12].

The dilaton-dependent FI term has also several phenomenologically interesting aspects. For example, the ratio of the FI term to the Planck mass squared can be an origin of coupling hierarchies [13,14]. The FI term can also be used to break SUSY [15–18] as well as to mediate SUSY-breaking effects to scalar mass terms [19–23]. Furthermore, in the D-term inflation scenario, the FI term is a dominant term

in the vacuum energy driving the inflation [24].[1] In these applications, the size of the FI term, which is determined as Eq. (2) in the heterotic case, is quite important.

In this Letter, we study the dilaton stabilization mechanism in which a dominant role is played by the dilaton-dependent FI term (2) due to non-perturbative Kähler potential. In this scenario, the dilaton VEV can easily be stabilized at weak coupling, $\mathrm{Re}(S) = O(1)$, as we will see below. Similar studies have been done in Refs. [16,17], where the superpotential due to gaugino condensation is also added to stabilize the dilaton VEV. In our case, however, we do not assume such dilaton-dependent superpotential. This means that the dominant part of scalar potential V is given by $V \sim (\delta_{GS}K_S)^2$. As a result, the dilaton VEV is stabilized around the point satisfying $K_S = 0$. This minimum corresponds to the point discussed before from the viewpoint of maximally enhanced symmetry [12]. Moreover, we will present an example of dilaton-dependent superpotential that does not spoil the dilaton stabilization through the anomalous $U(1)$ D-term so that the resulting FI term has a suppressed value compared with the value expected from the tree-level Kähler potential.

Basically it is difficult to stabilize the dilaton only through the D-term scalar potential if the Kähler potential takes the tree-level form (1). To realize it, we assume that non-perturbative effects generate another term in the dilaton Kähler potential. Of course, it is not clear, at present, which type of terms would be generated by non-perturbative physics. Therefore, for illustrating purpose, we use the following ansatz for non-perturbative potential [5],

$$K_{np}(S + \bar{S}) = d(S + \bar{S})^{p/2} e^{-b(S+\bar{S})^{1/2}}, \tag{3}$$

where d, p and b are real constants. It is required that $b > 0$, for the non-perturbative term must vanish in the weak coupling limit, $\mathrm{Re}(S) = 1/g^2 \to \infty$. Then, in models with an anomalous $U(1)_A$, we consider the total Kähler potential of dilaton,

$$K^{(I)}(s) = K_0(s) + K_{np}(s). \tag{4}$$

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[1] See, Refs. [25,26] for D-term inflation scenarios in type I models.

Alternatively, the total dilaton Kähler potential of the form

$$K^{(\mathbb{I})}(s) = \ln\left(e^{K_0(s)} + e^{K_{\rm np}(s)}\right) \tag{5}$$

has also been discussed in the literature. We also give comments on the case with $K^{(\mathbb{I})}(s)$.

Now let us explain our setting. The total Kähler potential takes the form

$$K = K(S + \bar{S} - 2\delta_{\rm GS} V_A) + K\left(\Phi^i, \bar{\Phi}^{\bar{i}}\right) + \sum_\kappa K_{\kappa\bar{\kappa}}\left(\Phi^i, \bar{\Phi}^{\bar{i}}\right) \bar{\phi}^{\bar{\kappa}} e^{2q_\kappa^A V_A} \phi^\kappa + \cdots, \tag{6}$$

where the first term is the dilaton Kähler potential $K^{(\rm I)}$ or $K^{(\mathbb{I})}$. In the second and third terms, Φ^i are gauge singlet moduli fields other than the dilaton field, and ϕ^κ stand for matter fields with $U(1)_A$ charge q_κ^A. The ellipsis denotes terms including gauge multiplets other than $U(1)_A$ and higher order terms of ϕ^κ. For superpotential W, we first consider the model in which W does not include the dilaton field,

$$W = W\left(\Phi^i, \phi^\kappa\right), \tag{7}$$

unlike the non-perturbative term generated by gaugino condensation. This is an important assumption and we will come back to this point later.

Under the above setting, the scalar potential is given by

$$V = e^K \left[\frac{1}{K_{S\bar{S}}} |K_S W|^2 \right.$$
$$+ \left(K^{-1}\right)^{I\bar{J}}$$
$$\times (K_I W + W_I)(K_{\bar{J}} \bar{W} + \bar{W}_{\bar{J}}) - 3|W|^2 \right]$$
$$+ \frac{1}{2\,{\rm Re}(S)} \left(\delta_{\rm GS} K_S - \sum_\kappa q_\kappa^A K_{\kappa\bar{\kappa}} |\phi^\kappa|^2\right)^2 + \cdots, \tag{8}$$

where $K_{S\bar{S}}$ is the Kähler metric of the dilaton field, and subindices I, J represent derivatives with respect to the Φ^i or ϕ^κ. Here the ellipsis denotes D-terms other than the $U(1)_A$ D-term. A solution of the stationary condition $\partial V/\partial S = 0$ is given by

$$K_S = 0, \qquad \Delta \equiv \sum_\kappa q_\kappa^A K_{\kappa\bar{\kappa}} |\phi^\kappa|^2 = 0. \tag{9}$$

The first equation is the condition of vanishing FI term, from which the dilaton is stabilized as we shall

see shortly. We have assumed that the second condition in Eq. (9) also satisfies F-flatness conditions. Actually, this solution corresponds to vanishing F-term of S and vanishing $U(1)_A$ D-term, so that SUSY is unbroken in the dilaton sector. At this point (9), the second derivative of V is written as

$$\left.\frac{\partial^2 V}{\partial S \partial \bar{S}}\right|_{K_S = \Delta = 0}$$
$$= \left\langle K_{S\bar{S}} V + 2K_{S\bar{S}} e^K |W|^2 + \frac{\delta_{\rm GS}^2 K_{S\bar{S}}^2}{{\rm Re}(S)} \right\rangle. \tag{10}$$

On the right-hand side of this equation, the first term can be neglected when the (tree-level) vacuum energy is taken to be approximately zero. (Note that the vacuum energy contribution from the dilaton sector vanishes at $K_S = \Delta = 0$.) Moreover, the second derivative $K_{S\bar{S}}$ must be positive because it determines a normalization of kinetic term of the dilaton. We find that the right-hand side of Eq. (10) are positive at $K_S = \Delta = 0$, and thus Eq. (9) corresponds to a local minimum of the scalar potential V.

Let us discuss a concrete example. We consider the Kähler potential $K^{(\rm I)}$. Its first derivative with respect to the dilaton is obtained as

$$K_S^{(\rm I)}(s) = -\frac{1}{s} + \frac{d}{2} s^{p/2 - 1} e^{-bs^{1/2}} \left[p - bs^{1/2}\right]. \tag{11}$$

The solutions to the equation $K_S^{(\rm I)} = 0$ behave differently for $d < 0$ case and $d > 0$ case. When p and b are positive and fixed, the $d < 0$ case can lead to larger value of ${\rm Re}(S)$ than the $d > 0$ case. For example, in the case with $p = b = 1$ and $d = -e^2$, the dilaton VEV is stabilized as ${\rm Re}(S) = 2$, while we obtain ${\rm Re}(S) = 0.125$ in the case with $p = b = 1$ and $d = 8e^{1/2}$. Since we are interested in the solution ${\rm Re}(S) = O(1)$, we will mainly consider the case with $d < 0$ and give a brief comment for $d > 0$ later.

Fig. 1 shows $K_S^{(\rm I)}$ for $p = b = 1$ and $d = -e^2$. We see that there are two solutions to $K_S^{(\rm I)} = 0$ (except the runaway one); one corresponds to the solution with $K_{S\bar{S}}^{(\rm I)} > 0$ while the other gives $K_{S\bar{S}}^{(\rm I)} < 0$. Thus the physical solution is given by ${\rm Re}(S) = 2$ as mentioned above. We also show in Fig. 2 how the stabilized dilaton VEV depends on the parameter $d < 0$. As $|d|$ becomes large, the stabilized value becomes small. In the limit $|d| \to \infty$, the stabilized value ${\rm Re}(S)$ comes close to $1/2$. On the other hand, as $|d|$ becomes small,
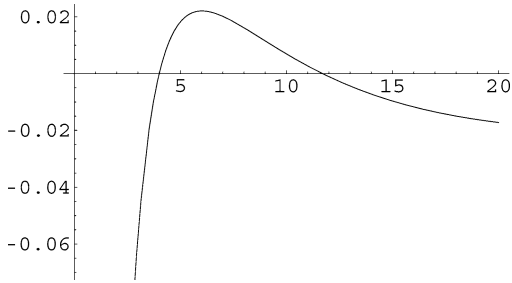
Fig. 1. $K_S^{(I)}$ as a function of $s = 2\,\mathrm{Re}(S)$. The parameters are $p = b = 1$ and $d = -e^2$.
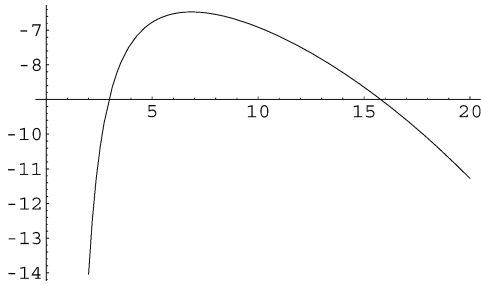

Fig. 2. The curve of d (the vertical axis) against $s = 2\,\mathrm{Re}(S)$ (the horizontal axis) which satisfy $K_S^{(I)} = 0$ for $p = b = 1$. For $s > 7.8$, we have $K_{S\bar{S}} < 0$ and such part of this curve does not correspond to a physical solution.

the stabilized value $\mathrm{Re}(S)$ becomes large. However, for $d > -6.5$, we have no solution to $K_S^{(I)} = 0$. The maximum value of the dilaton VEV is $\mathrm{Re}(S) \approx 3.4$ for $d \approx -6.5$. We note that in general the second derivative $K_{S\bar{S}}^{(I)}$ is suppressed slightly. For example, we have $K_{S\bar{S}} = 1/32$ for $d = -e^2$.

For other values of p and b, we obtain qualitatively the same results. The limit $|d| \to \infty$ corresponds to the minimum of $\mathrm{Re}(S)$, which is obtained as $\mathrm{Re}(S) = p^2/(2b^2)$. As d decreases, the stabilized value increases.

Here we give a comment on the case with $d > 0$. For p and b fixed positively, as d decreases, the stabilized value of $\mathrm{Re}(S)$ increases, but it cannot be larger than $p^2/(8b^2)$. Thus, for $d > 0$ we have $\mathrm{Re}(S) = O(1)$ for a large ratio of $p^2/(8b^2)$ and a small value of d.

Similarly we can discuss the dilaton stabilization for $K^{(I)}$. Its first derivative $K_S^{(I)}$ is calculated to be

$$K_S^{(I)} = \frac{1}{1 + s\exp(ds^{p/2}e^{-bs^{1/2}})}$$
$$\times \left[-\frac{1}{s} \right.$$
$$+ \frac{d}{2}s^{p/2}e^{-bs^{1/2}}$$
$$\left. \times \left(p - bs^{1/2}\right)\exp\!\left(ds^{p/2}e^{-bs^{1/2}}\right) \right]. \quad (12)$$

For example, when $p = b = -d = 1$, the equation $K_S^{(I)} = 0$ is satisfied by $\mathrm{Re}(S) = 3.9$, where we have $K_{S\bar{S}} = 0.13$.

So far, we have considered the model without dilaton-dependent superpotential. In that case, the minimum of the scalar potential is determined by $K_S = 0$ corresponding to vanishing FI term. On the other hand, if a dilaton-dependent term is generated non-perturbatively in the superpotential, one may expect that such term would drastically change the situation, that is, the dilaton VEV would no longer be determined by the anomalous $U(1)$ D-term. This is not necessarily the case, however. We now present a class of models in which the superpotential contains a dilaton-dependent term, but the dilaton VEV is dominantly determined by the anomalous $U(1)$ D-term. In fact, a sub-dominant effect from the superpotential slightly shifts the minimum from the point $K_S = 0$, as we shall see shortly.

Here we consider a toy model with $SU(2) \times U(1)_A$ gauge group. The model has four $SU(2)$ doublet chiral superfields Q_i^a ($i = 1, \ldots, 4$; $a = 1, 2$) which have anomalous $U(1)_A$ charges q_i with $\sum_i q_i \neq 0$. In this case, the $SU(2)$ strong dynamics deforms the moduli space of vacua into [27]

$$\mathrm{Pf}(M_{ij}) = \exp\!\left(-8\pi^2 S\right), \quad (13)$$

where M_{ij} is the meson operator corresponding to $Q_i Q_j$. The right-hand side corresponds to Λ^4, where $\Lambda = \exp[-2\pi^2 S]$ is the dynamical scale (in the $M = 1$ unit). Suppose that the superpotential includes only the term with a Lagrange multiplier that enforces the above constraint (13). Furthermore, we assume Kähler potentials of M_{ij} to be $K(M_{ij}, \bar{M}_{ij}) = (M_{ij}\bar{M}_{ij})^{1/2}$ for simplicity. Then, the anomalous $U(1)_A$ D-term takes the form

$$D = \delta_{\mathrm{GS}} K_S - \sum \frac{q_{ij}}{2}(M_{ij}\bar{M}_{ij})^{1/2}, \quad (14)$$

where $q_{ij} = q_i + q_j$.

Now, we may estimate the minimum of the scalar potential by solving

$$\delta_{\text{GS}} K_S = \sum \frac{q_{ij}}{2} (M_{ij} \bar{M}_{ij})^{1/2}. \tag{15}$$

Combining Eq. (15) with the quantum constraint (13), we obtain

$$K_S = O\big(\exp[\beta - 4\pi^2 \operatorname{Re}(S)]\big), \tag{16}$$

where we have defined $\delta_{\text{GS}} \equiv e^{-\beta}$ and assumed that $q_{ij} = O(1)$. Normally we have $\beta = O(1)$ since $\delta_{\text{GS}} = 10^{-1}$–$10^{-2}$ in the unit $M = 1$. If the stabilized value before adding the superpotential is given by $\operatorname{Re}(S) = O(1)$, the right-hand side in Eq. (16) is sufficiently suppressed as long as $\beta = O(1)$. If this is the case, we may consistently approximate the minimum condition by $K_S \approx 0$ as before. This situation does not change even for $\beta = O(10)$ because $4\pi^2 \operatorname{Re}(S) \gg \beta$.

It is important, however, to notice that the FI term ξ does not vanish exactly. In the above toy model, it is estimated as

$$\begin{aligned}
|\xi| &= |\delta_{\text{GS}} K_S| M^2 \\
&= O\big(M^2 \exp(-8\pi^2)\big) \sim O(10^2) \text{ GeV}^2,
\end{aligned} \tag{17}$$

when $\operatorname{Re}(S) = 2$. Thus the FI term is non-vanishing, but quite suppressed in this model. If we consider a model with larger rank of gauge group, the dynamical scale Λ can be larger. Accordingly a larger FI term $\xi = O(\Lambda^2)$ can be generated. For example, in the model which has $SU(7)$ gauge group with seven flavors and $\operatorname{Re}(S) = 2$, we obtain the dynamical scale $|\Lambda| \approx 10^{13}$ GeV. In general, this type of models lead, up to $U(1)$ charges, to

$$\frac{|\xi|}{M^2} = |\delta_{\text{GS}} K_S| = \exp\left[-\frac{8\pi^2}{b'} 2 \operatorname{Re}(S) \right], \tag{18}$$

where b' is the one-loop gauge beta-function coefficient in the model with quantum moduli space. We also note that the stabilized VEV of $2 \operatorname{Re}(S)$ is slightly shifted from the value s_0 of previous case satisfying $K_S(s_0) = 0$ exactly. Such shift δs is negligible as long as

$$\frac{8\pi^2}{b' \delta_{\text{GS}} K_{S\bar{S}}(s_0)} \exp\left(-\frac{8\pi^2}{b'} s_0 \right) \ll 1. \tag{19}$$

Otherwise, the shift is not small, and we have to fully solve the stationary condition of the scalar potential.

To summarize, we have studied the dilaton stabilization in the model with the non-perturbative dilaton Kähler potential and anomalous $U(1)$ gauge symmetry. It is found that non-perturbative effects can stabilize the dilaton at a finite value of $O(1)$. Another interesting property of this stabilization mechanism is that one can reduce the order of magnitude of FI term. We give a toy model in which small dynamical scale and FI term are generated. If gauge group is larger, they can become larger. That would have interesting applications, e.g., for the D-term inflation scenario. Finally we add that in the models discussed here, SUSY is not broken in the dilaton sector, and the tree-level vacuum energy contribution from this sector vanishes. In order to break SUSY, we must take into account effects from other moduli fields or tree-level superpotential.

Acknowledgements

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