The fundamental solution of Mindlin plates resting on an elastic foundation in the Laplace domain and its applications

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Abstract

The aim of this study is to investigate the method of fundamental solution (MFS) applied to a shear deformable plate (Reissner/Mindlin’s theories) resting on the elastic foundation under either a static or a dynamic load. The complete expressions for internal point kernels, i.e. fundamental solutions by the boundary element method, for the Mindlin plate theory are derived in the Laplace transform domain for the first time. On employing the MFS the boundary conditions are satisfied at collocation points by applying point forces at source points outside the domain. All variables in the time domain can be obtained by Durbin’s Laplace transform inversion method. Numerical examples are presented to demonstrate the accuracy of the MFS and comparisons are made with other numerical solutions. In addition, the sensitivity and convergence of the method are discussed for a static problem. The proposed MFS is shown to be simple to implement and gives satisfactory results for shear deformable plates under static and dynamic loads.

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1. Introduction

The problem of plate resting on elastic foundations is important in analysing structural problems. Two main theories are available to model plate structures: classical thin plate theory (Kirchhoff’s theory) and the shear deformable plate theory (Mindlin–Reissner’s theory). Kirchhoff’s theory ignores the effect of shear deformation through the thickness. Numerical methods to solve plate problems, such as the finite element and boundary element methods, are well developed. To reduce the model’s dimension, the application of the boundary integral equation method to the classical plate bending problem was presented by Jaswon and Maiti (1968). Later, the indirect boundary integral equation solutions of the Kirchhoff plate bending problems were presented by Altiero and Sikarskie (1978) and Tottenham (1979). For the shear deformable Reissner/Mindlin plates, the boundary integral equation method was reported by Vander Weeën (1982) for static problem.

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Later, the direct boundary element formulation for Reissner/Mindlin’s plate bending was presented by Wen and Aliabadi (2006), in which the fundamental solutions of the displacement and traction in the Laplace transform domain were derived. In their work, the behaviour of the three flexural waves, i.e. slow flexural, fast flexural and thickness shear waves, was studied comprehensively.

In recent years, there has been a growing interest in meshless method for the numerical solutions, see Atluri (2004). The method of fundamental solution (MFS) is regarded as one of the mesh free methods, since it does not require an elaborate discretisation of the boundary. Integrations over the boundary and domain are avoided and the solutions in the interior of the domain are evaluated without extra treatments. The main idea of the MFS consists of approximating the exact solution by a linear combination of fundamental solutions with respect to some source points, which are located outside the domain. This method was originally introduced by Kupradze and Aleksidze (1964) and was successfully applied for solving a wide range of boundary value problems, when the fundamental solution is available for the governing partial differential equations. The MFS applied to the Kirchhoff plate on a Winkler’s foundation was found by Wen (1987, 1988, 1989). A review of the MFS for two- and three-dimensional elasticity was given by Marin and Lesnic (2004) and for diffusion equations by Chen et al. (1998).

One- and two-parameter models for the soil underneath the plate are introduced to model the foundation. The one-parameter model (Winkler’s model) can be represented by continuous springs whilst the two-parameter model (Pasternak’s model) considers the shear deformation between the springs over the one-parameter model. Therefore, Winkler’s model can be considered as a special case of Pasternak’s model by setting the shear modulus to zero. However, to use the boundary element method, the fundamental solutions must be known. For the static problem, Yu (1957) derived an analytical solution for a circular thin plate resting on both Winkler and Pasternak foundations and Balas et al. (1984) derived the fundamental solutions for a thin plate on a Pasternak foundation. Fundamental solutions for a thick plate on a two-parameter foundation were derived by Wang et al. (1992), Fadhil and El-Zafrany (1994) and Rashed et al. (1999). For the dynamic case, the fundamental solution of a Kirchhoff plate resting on Winkler’s foundation was derived by Wen et al. (1999), by the use of the Laplace transform technique.

In this paper, fundamental solutions in the Laplace domain for a moderate thick plate (Mindlin’s theory) resting on the two-parameter model (Pasternak’s theory) are derived by using three potential functions. These fundamental solutions can be used to derive the boundary element formulations directly. However, the applications of these fundamental solutions in this paper are demonstrated for the MFS in the case of static and dynamic problems. Furthermore, the behaviour of the elastic waves, i.e. two slow flexural waves, one fast flexural wave and one thickness shear wave, are studied. For general plate geometry, a set of linear algebraic equations with unknown coefficients (the intensity of point force located outside of the domain) are established by considering boundary conditions at specified boundary collocations. The optimised selection of the distance between the source and the collocation points has been discussed. Comparisons are made with available numerical solutions to show the accuracy and convergence of the MFS. It is well known that there are problems of stability and convergence for the MFS and the inversion of the Laplace transformation. However, we are still able to obtain stable solutions in the large range selection of the free parameter, particularly for the solid elastic wave propagations. There is also a large number of articles dealing with the stability of the MFS and the Laplace inversion, see Cheng and Cheng (2005), Lee and Yoon (2004).

2. Fundamental solution for Mindlin plates on Pasternak foundation

With the small motion assumption, the governing equations for the deflection and rotations of a moderate thick plate resting on a Pasternak foundation can be expressed as, see Rashed et al. (1999)

$$\frac{D}{2} \left[ (1 - \nu)\nabla^2 w_z + (1 + \nu)w_{\beta,\beta} \right] - \kappa \mu h (w_{3,z} + w_z) = \frac{\rho h^3}{12} \frac{\partial^2 w_z}{\partial t^2}$$

$$\kappa \mu h (\nabla^2 w_z + w_{z,z}) + G_f \nabla^2 w_3 - k_f w_3 + q(t) = \frac{\rho h}{\partial t} \frac{\partial^2 w_3}{\partial t^2}$$

(1)
where \( q(t) \) is the pressure load in the domain and \( w_z \) (see Fig. 1) denotes rotations respecting to axis \( x_z \) and \( w_3 \) is the out-of-plane deflection. Here, Greek indices vary from 1 to 2. The parameter \( \kappa \) denotes the shear coefficient \( (\kappa = \pi^2/12 \) for Mindlin’s theory and \( \kappa = 5/6 \) for Reissner’s theory), the bending stiffness of the plate \( D = Eh^3/12(1 - \nu^2) \), the shear modulus \( G = E/2(1 + \nu) \) and \( h \) denotes the thickness of the plate, \( \rho \) is the density of the plate, \( E \) and \( \nu \) are elastic constants. For the soil foundation, the constants \( k_f \) and \( G_f \) are the modulus of sub-grade reaction and shear modulus, respectively. The resultants of moment \( M_{z\beta} \) and shear force resultant \( Q_z \) can be written, in terms of displacements, as

\[
M_{z\beta} = \frac{1 - \nu}{2} D \left( w_{z,\beta} + w_{\beta,3} + \frac{2v}{1 - \nu} w_{3,\beta} \delta_{z\beta} \right)
\]

\[
Q_z = \kappa \mu h (w_z + w_{3,3})
\]

in which \( \delta_{z\beta} \) denotes the Kronecker delta function. Considering the Laplace transform of the function \( f(x, t) \)

\[
L[f(x, t)] = \tilde{f}(x, p) = \int_0^\infty f(x, t)e^{-pt} dt
\]

where \( p \) is the parameter of the Laplace transformation, and applying the Laplace transform to the governing equations (1) yields

\[
6D \left[ (1 - \nu) \nabla^2 \tilde{w}_z + (1 + \nu) \nabla^2 \tilde{w}_{\beta,\alpha} \right] - 12\kappa \mu h (\tilde{w}_{1,3} + \tilde{w}_3) = \rho h^2 p^2 \tilde{w}_z
\]

\[
k\mu h (\nabla^2 \tilde{w}_3 + \tilde{w}_{2,3}) + G_f \nabla^2 \tilde{w}_3 - k_f \tilde{w}_3 + \tilde{q} = \rho h^2 p \tilde{w}_3
\]

It is apparent that the rotations and deflection can be written in terms of three potentials \( \phi_k (k = 1, 2, 3) \) as, see Sih and Hagendorf (1978) and Wen and Aliabadi (2006)

\[
\tilde{w}_1 = (\xi_1 - 1) \phi_{1,1} + (\xi_2 - 1) \phi_{2,1} + \phi_{3,1}
\]

\[
\tilde{w}_2 = (\xi_1 - 1) \phi_{1,1} + (\xi_2 - 1) \phi_{2,2} + \phi_{3,1}
\]

\[
\tilde{w}_3 = \phi_1 + \phi_2
\]

where the non-dimensional parameters \( \xi_\beta \) are two roots in the following equation, for the Mindlin plate (i.e. \( \kappa = \pi^2/12 \)), as

\[
\left( 1 + \left( \frac{p}{\omega_0} \right)^2 \right) \xi^2 + \left\{ \frac{12G_f}{\mu h^2} \left[ 1 + \left( \frac{p}{\omega_0} \right)^2 \right] - \left( \frac{p}{\omega_0} \right)^2 - \frac{24}{(1 - \nu) \pi^2} \left[ k_f h \mu + \left( \frac{p}{\omega_0} \right)^2 \right] \left( \frac{p}{\omega_0} \right)^2 \right\} \xi - \frac{12G_f}{\mu h^2} \left( \frac{p}{\omega_0} \right)^2 = 0
\]

\( \omega_0 \) denotes the cut-off frequency \( \pi c_2/h \) and \( c_2 = \sqrt{\mu/\rho} \) is the velocity of shear wave.

As in the Laplace domain, the parameter \( p \) is complex, thus the two roots of the above equation \( \xi_\beta \) are complex values in general. Substituting Eq. (5) into the governing equations (4) gives the Helmholz type differential equations for the displacement potential, namely

\[
(\nabla^2 - \xi_\beta^2) \phi_k = 0, \quad k = 1, 2, 3
\]
where
\[
\alpha_\beta^2 = \frac{12}{h^2} \left[ \frac{k}{\mu} + \left( \frac{p}{\omega_0} \right)^2 \right] \left( \frac{2}{\mu \rho \pi^2} \right), \quad \beta = 1, 2
\]  
(8)
and
\[
\alpha_3^2 = \frac{\pi^2}{h^2} \left[ 1 + \left( \frac{p}{\omega_0} \right)^2 \right]
\]  
(9)

For a symmetric problem with respect to the \(x_1\)-axis, the general solutions of the above equations can be obtained by using the Fourier transform (see Wen and Aliabadi, 2006 for plate bending)

\[
\phi_1 = \frac{2}{\pi} \int_0^\infty A_1(s) e^{-\beta_1 x_2} \cos(s x_1) ds
\]
\[
\phi_2 = \frac{2}{\pi} \int_0^\infty A_2(s) e^{-\beta_2 x_2} \cos(s x_1) ds
\]
\[
\phi_3 = \frac{2}{\pi} \int_0^\infty A_3(s) e^{-\beta_3 x_2} \sin(s x_1) ds, \quad x_2 \geq 0
\]  
(10)

where \(\beta_1^2 = s^2 + \alpha_1^2\) and \(A_k(s)\) are unknown coefficients which are to be determined by considering the boundary conditions for each point force. Therefore, the rotations and deflection are given by

\[
\tilde{\omega}_1 = \frac{2}{\pi} \int_0^\infty \left[ (1 - \xi_1) A_1(s) e^{-\beta_1 x_2} + (1 - \xi_2) A_2(s) e^{-\beta_2 x_2} - \beta_3 A_3(s) e^{-\beta_3 x_2} \right] \sin(s x_1) ds
\]
\[
\tilde{\omega}_2 = \frac{2}{\pi} \int_0^\infty \left[ (1 - \xi_1) \beta_1 A_1(s) e^{-\beta_1 x_2} + (1 - \xi_2) \beta_2 A_2(s) e^{-\beta_2 x_2} - s A_3(s) e^{-\beta_3 x_2} \right] \cos(s x_1) ds
\]
\[
\tilde{\omega}_3 = \frac{2}{\pi} \int_0^\infty \left[ A_1(s) e^{-\beta_1 x_2} + A_2(s) e^{-\beta_2 x_2} \right] \cos(s x_1) ds, \quad x_2 \geq 0
\]  
(11)

The resultants of moment \(\tilde{M}_{2\theta}\) and shear stress resultants \(\tilde{Q}_2\) in the transformed domain can then be obtained as

\[
\tilde{M}_{11} = \frac{2D}{\pi} \int_0^\infty \left[ (1 - \xi_1)(s^2 - v\beta_1^2) A_1(s) e^{-\beta_1 x_2} + (1 - \xi_2)(s^2 - v\beta_2^2) A_2(s) e^{-\beta_2 x_2} - s A_3(s) e^{-\beta_3 x_2} \right] \cos(s x_1) ds
\]
\[
\tilde{M}_{22} = \frac{2D}{\pi} \int_0^\infty \left[ (1 - \xi_1)(v^2 - \beta_1^2) A_1(s) e^{-\beta_1 x_2} + (1 - \xi_2)(v^2 - \beta_2^2) A_2(s) e^{-\beta_2 x_2} + s A_3(s) e^{-\beta_3 x_2} \right] \cos(s x_1) ds
\]
\[
\tilde{M}_{12} = -\frac{(1 - v)D}{\pi} \int_0^\infty \left[ 2s \beta_1 (1 - \xi_1) A_1(s) e^{-\beta_1 x_2} + 2s \beta_2 (1 - \xi_2) A_2(s) e^{-\beta_2 x_2} + (\beta_3^2 + s^2) A_3(s) e^{-\beta_3 x_2} \right] \sin(s x_1) ds
\]  
(12)

and

\[
\tilde{Q}_1 = -\frac{2k \mu h}{\pi} \int_0^\infty \left[ s \xi_1 A_1(s) e^{-\beta_1 x_2} + s \xi_2 A_2(s) e^{-\beta_2 x_2} + \beta_3 A_3(s) e^{-\beta_3 x_2} \right] \sin(s x_1) ds
\]
\[
\tilde{Q}_2 = -\frac{2k \mu h}{\pi} \int_0^\infty \left[ \beta_1 \xi_1 A_1(s) e^{-\beta_1 x_2} + \beta_2 \xi_2 A_2(s) e^{-\beta_2 x_2} + s A_3(s) e^{-\beta_3 x_2} \right] \cos(s x_1) ds
\]  
(13)

Considering a quarter of the infinite domain and one point moment acting at the origin, the conditions along axis \(x_1\) are described by
Substituting Eqs. (11) and (12) into (14) results
\[
(1 - \tilde{\zeta}_1)(v s^2 - \beta_1^2)A_1(s) + (1 - \tilde{\zeta}_2)(v s^2 - \beta_2^2)A_2(s) + s\beta_3(1 - v)A_3(s) = -\frac{1}{4D} s(1 - \tilde{\zeta}_1)A_1(s) + s(1 - \tilde{\zeta}_2)A_2(s) - \beta_3 A_3(s) = 0
\]
\[
A_1(s) + A_2(s) = 0
\]
Therefore, three coefficients \(A_k(s)\) are obtained as
\[
A_1(s) = -\frac{1}{4D(x_1^2 - x_2^2)(1 + G)}
\]
\[
A_2(s) = \frac{1}{4D(x_1^2 - x_2^2)(1 + G)}
\]
\[
A_3(s) = \frac{s(\tilde{\zeta}_2 - \tilde{\zeta}_1)}{4D(x_1^2 - x_2^2)(1 + G)}
\]
where \(G = 12G_1/\mu h^2\). Considering following identity
\[
K_0(z_1) = \int_0^\infty \frac{\cos(x_1 x_2)}{\sqrt{s^2 + x_2^2}} e^{-\sqrt{x_1^2 + x_2^2} s} ds
\]
where \(K_0(z_1)\) represents the zero order Bessel modified function and \(r = \sqrt{x_1^2 + x_2^2}\), thus the displacements and rotations can be re-arranged as
\[
\tilde{w}_1 = \tilde{U}_{21} = -\frac{1}{2\pi D(x_1^2 - x_2^2)(1 + G)} \frac{\partial^2}{\partial x_1 \partial x_2} [(1 - \tilde{\zeta}_1)K_0(z_1) - (1 - \tilde{\zeta}_2)K_0(z_2) + (\tilde{\zeta}_1 - \tilde{\zeta}_2)K_0(z_3)]
\]
\[
\tilde{w}_2 = \tilde{U}_{22} = -\frac{1}{2\pi D(x_1^2 - x_2^2)(1 + G)} \frac{\partial^2}{\partial x_2^2} [(1 - \tilde{\zeta}_1)K_0(z_1) - (1 - \tilde{\zeta}_2)K_0(z_2) + (\tilde{\zeta}_1 - \tilde{\zeta}_2)K_0(z_3)] - \frac{(\tilde{\zeta}_2 - \tilde{\zeta}_1)x_2}{2\pi D(x_1^2 - x_2^2)(1 + G)} K_0(z_3)
\]
\[
\tilde{w}_3 = \tilde{U}_{23} = -\frac{1}{2\pi D(x_1^2 - x_2^2)(1 + G)} \frac{\partial}{\partial x_2} [K_0(z_1) - K_0(z_2)]
\]
where \(z_i = x_i r\) and \(\tilde{U}_{ik}\) are the displacement fundamental solutions using BEM in the Laplace domain (see Aliabadi, 2002). The first subscript \(i\) in the fundamental solutions \(\tilde{U}_{ik}\) denotes the direction of the point force and \(k\) the direction of displacement. In the same way, we can derive the other two sets of fundamental solutions.
For an anti-symmetric problem with respect the \(x_1\)-axis, the displacement potential functions can be written as
\[
\phi_1 = \frac{2}{\pi} \int_0^\infty A_1(s)e^{-\beta_1 x_2} \sin(x_1 x_2) ds
\]
\[
\phi_2 = \frac{2}{\pi} \int_0^\infty A_2(s)e^{-\beta_2 x_2} \sin(x_1 x_2) ds
\]
\[
\phi_3 = \frac{2}{\pi} \int_0^\infty A_3(s)e^{-\beta_3 x_2} \cos(x_1 x_2) ds, \quad x_2 \geq 0
\]
\[ \tilde{w}_1 = -\frac{2}{\pi} \int_0^\infty \left[ (1 - \xi_1) s A_1(s) e^{-\beta_1 s x} + (1 - \xi_2) s A_2(s) e^{-\beta_2 s x} + \beta_3 A_3(s) e^{-\beta_3 s x} \right] \cos(sx_1) ds \]
\[ \tilde{w}_2 = \frac{2}{\pi} \int_0^\infty \left[ (1 - \xi_1) \beta_1 A_1(s) e^{-\beta_1 s x} + (1 - \xi_2) \beta_1 A_2(s) e^{-\beta_2 s x} + s A_3(s) e^{-\beta_3 s x} \right] \sin(sx_1) ds \]
\[ \tilde{w}_3 = \frac{2}{\pi} \int_0^\infty \left[ A_1(s) e^{-\beta_1 s x} + A_2(s) e^{-\beta_2 s x} \right] \sin(sx_1) ds, \quad x_2 \geq 0 \]

The resultants of moment \( \tilde{M}_{s\beta} \) and shear stress resultants \( \tilde{Q}_s \) in the transformed domain can be obtained as
\[ \tilde{M}_{11} = \frac{2D}{\pi} \int_0^\infty \left[ (1 - \xi_1) (s^2 - v \beta_1^2) A_1(s) e^{-\beta_1 s x} + (1 - \xi_2) (s^2 - v \beta_2^2) A_2(s) e^{-\beta_2 s x} + s \beta_3 A_3(s) e^{-\beta_3 s x} \right] \sin(sx_1) ds \]
\[ \tilde{M}_{12} = \frac{2}{\pi} \int_0^\infty \left[ (1 - \xi_1) (s^2 - \beta_1^2) A_1(s) e^{-\beta_1 s x} + (1 - \xi_2) (s^2 - \beta_2^2) A_2(s) e^{-\beta_2 s x} \right] \sin(sx_1) ds \]
\[ \tilde{M}_{13} = \frac{(1 - v)D}{\pi} \int_0^\infty \left[ 2s \beta_1 (1 - \xi_1) A_1(s) e^{-\beta_1 s x} + 2s \beta_2 (1 - \xi_2) A_2(s) e^{-\beta_2 s x} + (\beta_3^2 + s^2) A_3(s) e^{-\beta_3 s x} \right] \cos(sx_1) ds \]

and
\[ \tilde{Q}_1 = \frac{2\kappa \mu h}{\pi} \int_0^\infty \left[ s \xi_1 A_1(s) e^{-\beta_1 s x} + s \xi_2 A_2(s) e^{-\beta_2 s x} - \beta_3 A_3(s) e^{-\beta_3 s x} \right] \cos(sx_1) ds \]
\[ \tilde{Q}_2 = -\frac{2\kappa \mu h}{\pi} \int_0^\infty \left[ \beta_1 \xi_1 A_1(s) e^{-\beta_1 s x} + \beta_2 \xi_2 A_2(s) e^{-\beta_2 s x} - s A_3(s) e^{-\beta_3 s x} \right] \sin(sx_1) ds \]

The conditions along the \( x_1 \)-axis in this case are given by
\[ \tilde{M}_{12} = -\frac{1}{4} \delta(t) \delta(x_1), \quad \tilde{w}_2 = \tilde{Q}_2 = G_1 \tilde{w}_{3,2} = 0 \]

where \( \tilde{Q}_{s\alpha} = G_1 \tilde{w}_{3,\alpha} \) represents the contribution to the shear force from the foundation. Thus, the coefficients \( A_\alpha(s) \) can be obtained as
\[ A_1(s) = -\frac{s}{2(1 - v)D (\xi_2 - \xi_1) \xi_1^2 \beta_1} \]
\[ A_2(s) = \frac{s}{2(1 - v)D (\xi_2 - \xi_1) \xi_2^2 \beta_2} \]
\[ A_3(s) = \frac{1}{2(1 - v)D \xi_3^2} \]

Finally, for a unit concentrated shear force, the conditions below are used to determine the three unknown coefficients \( A_\alpha(s) \) in Eq. (10)
\[ \tilde{Q}_2 + \tilde{Q}_{f2} = -\frac{1}{4} \delta(t) \delta(x_1), \quad \tilde{M}_{12} = \tilde{w}_2 = 0 \]

Solving the above equations, we obtain
\[ A_1(s) = -\frac{(1 - \xi_2)}{4\kappa \mu h (\xi_2 - \xi_1)(1 + G_1) \beta_1} \]
\[ A_2(s) = -\frac{(1 - \xi_1)}{4\kappa \mu h (\xi_2 - \xi_1)(1 + G_1) \beta_2} \]

Considering the relationship between the roots \( \xi_\beta \) in Eq. (5) and \( z_\beta \) in Eq. (8), we can prove
\[
(z_2^2 - z_1^2)(1 + G_t) = -\frac{1-v}{2} x_1^2 (\xi_2 - \xi_1)
\]  
(27)

Therefore, the fundamental solutions for rotations and deflection \(\hat{U}_{ik}\) in the Laplace domain for three unit concentrated force (point forces) can be arranged as

\[
\hat{U}_{\alpha\beta}(x,p) = \frac{1}{\pi(1-v)Dz_1^2(\xi_2 - \xi_1)} (f_{\alpha\beta} + g\delta_{\alpha\beta})
\]
\[
\hat{U}_{\alpha 3}(x,p) = \frac{1}{\pi(1-v)Dz_1^2(\xi_2 - \xi_1)} h_{\alpha}
\]

and

\[
\hat{U}_{3\alpha}(x,p) = \frac{(1 - \xi_1^2)(1 - \xi_1)}{2\pi\kappa\mu h(\xi_2 - \xi_1)(1 + G_t)} h_{\alpha}
\]
\[
\hat{U}_{33}(x,p) = -\frac{1}{2\pi\kappa\mu h(\xi_2 - \xi_1)(1 + G_t)} \left[ (1 - \xi_1^2)K_0(z_1) - (1 - \xi_1^2)K_0(z_2) \right]
\]

where \(z_j = x, r\) and

\[
f = (1 - \xi_1)K_0(z_1) - (1 - \xi_2)K_0(z_2) - (\xi_2 - \xi_1)K_0(z_3)
\]
\[
g = (\xi_2 - \xi_1)x_1^2K_0(z_3)
\]
\[
h = K_0(z_1) - K_0(z_2)
\]

By using the property of the modified Bessel functions, the displacement fundamental solutions can be rewritten as

\[
\hat{U}_{\alpha\beta} = \frac{1}{\pi D(1-v)x_1^3(\xi_2 - \xi_1)} \left[ (1 - \xi_1)x_1K_1(z_1) - (1 - \xi_2)x_1K_1(z_2) - (\xi_2 - \xi_1)x_3K_1(z_3) \right] \left( 2r_{\alpha\beta} - \delta_{\alpha\beta} \right) / r
\]
\[
+ \left[ (1 - \xi_1)x_2^2K_0(z_1) - (1 - \xi_2)x_2^2K_0(z_2) - (\xi_2 - \xi_1)x_3^2K_0(z_3) \right] r_{\alpha\beta} + (\xi_2 - \xi_1)x_3^2K_0(z_3) \delta_{\alpha\beta}
\]
\[
\hat{U}_{\alpha 3} = \frac{1}{\pi D(1-v)x_1^3(\xi_2 - \xi_1)} \left[ x_1K_1(z_1) - x_2K_1(z_2) \right] r_{\alpha}
\]
\[
\hat{U}_{3\alpha} = -\frac{(1 - \xi_2)(1 - \xi_1)}{2\pi\kappa\mu h(\xi_2 - \xi_1)(1 + G_t)} \left[ x_1K_1(z_1) - x_2K_1(z_2) \right] r_{\alpha}
\]
\[
\hat{U}_{33} = -\frac{(1 - \xi_2)(1 - \xi_1)}{2\pi\kappa\mu h(\xi_2 - \xi_1)(1 + G_t)} \left[ \frac{K_0(z_1)}{1 - \xi_1} - \frac{K_0(z_2)}{1 - \xi_2} \right]
\]

Then, the traction fundamental solutions are obtained from the relationships in Eq. (2)

\[
\tilde{T}_{\alpha\beta} = \frac{1}{2\pi z_1^2(\xi_2 - \xi_1)} \left[ \left\{ (1 - \xi_1)x_1[2K_1(z_1) + z_1K_0(z_1)] - (1 - \xi_2)x_1[2K_1(z_2) + z_1K_0(z_1)] \right\} 
\]
\[
- (\xi_2 - \xi_1)x_3[2K_1(z_3) + z_3K_0(z_3)] \right\} \times \frac{2}{r^2} \left[ 4r_{\alpha\beta}r_{\gamma\beta} - (r_{\alpha\beta}n_\beta + r_{\beta\gamma}n_\alpha + r_{\alpha\beta}n_{\beta\gamma}) \right]
\]
\[
- 2\left[ (1 - \xi_1)x_1^2K_1(z_1) - (1 - \xi_2)x_2^2K_1(z_2) - (\xi_2 - \xi_1)x_3^2K_1(z_3) \right] r_{\alpha\beta} + (\xi_2 - \xi_1)x_3^2K_0(z_3) \delta_{\alpha\beta}
\]
\[
+ \frac{2v}{1-v} \left\{ (1 - \xi_1)x_1^2K_1(z_1) + (1 - \xi_2)x_2^2K_1(z_2) \right\} r_{\alpha\beta}
\]
\[
T_{\alpha 3} = -\frac{\kappa\mu h}{\pi D(1-v)x_1^3(\xi_2 - \xi_1)} \left\{ x_1[2K_1(z_1) - \xi_2] - x_3[2K_1(z_3) - \xi_2] \right\} \times \frac{1}{r} (2r_{\alpha\beta}n_\beta - n_\beta)
\]
\[
+ \left\{ x_1^2K_0(z_1) - \xi_2^2K_0(z_2) + (\xi_2 - \xi_1)x_3^2K_0(z_3) \right\} r_{\alpha\beta} - (\xi_2 - \xi_1)x_3^2K_0(z_3) n_\beta
\]  
(35)
the Helmholtz equation (7) becomes the frequency domain by Wen and Aliabadi (2006). Considering Pasternak foundation and letting the fundamental solutions given by Eqs. (31)–(34) become 

$$ T_{33} = \frac{(1 - \mu)(1 - \xi_2)(1 - \xi_1)D}{2\pi K\mu h(\xi - \xi_1)(1 + G)} \left\{ x_1K_1(z_1) - x_2K_1(z_2) \right\} \frac{1}{r} (2r_n r_m - n_s) + \left\{ x_1^2K_0(z_1) - x_2^2K_0(z_2) \right\} r_m n_s $$

$$ + \frac{v}{1 - v} \left\{ x_1^2K_0(z_1) - x_2^2K_0(z_2) \right\} n_s $$

(37)

$$ T_{33} = \frac{(1 - \mu)(1 - \xi_1)}{2\pi(\xi - \xi_1)(1 + G)} \left\{ x_1K_1(z_1) - x_2K_1(z_2) \right\} \frac{1}{r} - \frac{1}{1 - \xi_2} - \left\{ x_1K_1(z_1) - x_2K_1(z_2) \right\} r_m $$

(38)

where $n_\theta$ denotes the component of the outward normal vector to the boundary of the plate ($\Gamma$) and $r_m = r_n n_s$.

As the above solutions are general, the static fundamental solutions by Rashed et al. (1999) for a moderate thick plate resting on the Pasternak foundation and the fundamental solutions in the Laplace domain for a moderate plate without foundation by Wen and Aliabadi (2006) are all special cases of the solutions given by Eqs. (31)–(38). It is worth observing notice that if there are two double roots in Eq. (5), the fundamental solutions can be obtained from the above equations directly. In this case, $\xi_2 = \xi_1$ and the displacement fundamental solutions given by Eqs. (31)–(34) become

$$ U_{x \beta}(x, p) = \frac{1}{\pi(1 - \mu)dx^2} \frac{\delta}{\delta \xi_2} \left[ f_{x \beta} + g_{x \beta} \right] \big|_{\xi_2 = \xi_1} $$

$$ U_{z \beta}(x, p) = \frac{1}{\pi(1 - \mu)dx^2} \frac{\delta}{\delta \xi_2} h_{, \beta} \big|_{\xi_2 = \xi_1} $$

(39)

and

$$ U_{x \alpha}(x, p) = \frac{(1 - \xi_1)^2}{2\pi K\mu h(1 + G)} \frac{\delta}{\delta \xi_2} h_{, \alpha} \big|_{\xi_2 = \xi_1} $$

$$ U_{z \alpha}(x, p) = -\frac{(1 - \xi_1)^2}{2\pi K\mu h(1 + G)} \frac{\delta}{\delta \xi_2} \left[ \frac{K_0(z_2)}{(1 - \xi_2)} \right] \big|_{\xi_2 = \xi_1} $$

(40)

The traction fundamental solutions can be derived in the same way.

For the static case, letting $p = 0$ in Eq. (5) gives

$$ \xi^2 + \frac{G_i C - Dk_i}{C^2} \xi + \frac{Dk_i}{C^2} = 0 $$

(41)

where $C = K\mu h$. For two double roots in (41), the parameters of material and foundation are related by

$$ (G_i C - Dk_i)^2 - 4C^2 Dk_i = 0 $$

(42)

In this particular case, the fundamental solutions were discussed in detail by Rashed et al. (1999).

3. Velocities of flexural waves

For the Mindlin plate without foundation, there are three types of flexural waves which were discussed in the frequency domain by Wen and Aliabadi (2006). Considering Pasternak foundation and letting $p = -i\omega$, the Helmholtz equation (7) becomes

$$ (\nabla^2 + \xi^2) \phi_k = 0, \quad k = 1, 2, 3 $$

(43)

where

$$ \xi^2 = \frac{12}{h^2} \left[ \frac{k_i h}{\mu \pi^2} - \left( \frac{\omega}{\omega_0} \right)^2 \right] \left( \frac{\tilde{z}_\beta}{\tilde{z}_\beta} + \frac{12G_i}{\mu h \pi^2} \right), \quad \beta = 1, 2 $$

(44)

and
\[ \tilde{x}_i^2 = \frac{\pi^2}{h^2} \left[ 1 - \left( \frac{\omega}{\omega_0} \right)^2 \right] \]  

(45)

The non-dimensional parameters \( \xi_\beta \) are the two roots of the following equation:

\[
\left[ 1 - \left( \frac{\omega}{\omega_0} \right)^2 \right] \tilde{x}_i^2 + \frac{12G_f}{\mu h^2} \left[ 1 - \left( \frac{\omega}{\omega_0} \right)^2 \right] + \frac{24}{(1-v)\pi^2} \left[ \frac{k_l h}{\mu h^2 - \left( \frac{\omega}{\omega_0} \right)^2} \right] \tilde{x} + \frac{12G_f}{\mu h^2} \left( \frac{\omega}{\omega_0} \right)^2 = 0
\]  

(46)

Considering a flexural wave

\[ \phi_j = A_j e^{-i(kx - \omega t)} \]  

(47)

where \( k \) is the wave number and \( A_j \) are arbitrary constants, we obtain the following relation:

\[ k^2 - \tilde{x}_j^2 = 0 \]  

(48)

By the definition of velocity of the flexural wave and Eqs. (44), (46) and (48), we obtain the velocity

\[ c = \frac{\omega}{k} = \sqrt{\lambda} c_2 \]  

(49)

where the non-dimensional factor \( \lambda \) is the root of the following equation:

\[ \lambda^3 + \varepsilon_2 \lambda^2 + \varepsilon_1 \lambda + \varepsilon_0 = 0 \]  

(50)

with the coefficients

\[ \varepsilon_2 = - \frac{1}{h^2 k^2} \left[ \frac{1}{12} \left( \tilde{G}_f + 1 + \frac{24}{(1-v)\pi^2} \right) \right] \]

\[ \varepsilon_1 = \frac{1}{12} \left[ \frac{\tilde{G}_f}{h^2 k^2} + \frac{4\tilde{k}_f}{h^2 k^2(1-v)\pi^2} + \frac{2(1+\tilde{G}_f)}{(1-v)\pi^2} + \frac{\tilde{k}_f}{12(1-v)h^2 k^2} \right] \]

\[ \varepsilon_0 = - \frac{\tilde{k}_f}{144h^2 k^2} \left[ \frac{2(1+\tilde{G}_f)}{(1-v)\pi^2} + \frac{1}{h^2 k^2} \left( \frac{2\tilde{k}_f}{(1-v)\pi^2} - \tilde{G}_f \right) \right] \]  

(51)

and \( \tilde{k}_f = 12k_f h/\mu h^2 \). There are three non-zero real roots, which can be obtained analytically if the stiffness of the foundation \( k_f \) is not zero. Furthermore, when \( j = 3 \) in Eq. (43), we have the fourth velocity of the flexural wave, defined as the thickness shear wave

\[ \lambda = 1 + \frac{\pi^2}{h^2 k^2} \]  

(52)

It is evident that all four velocities of flexural waves depend on the wave number \( k \) and all four flexural waves are dispersive without being dissipative. If the wave number \( k \) tends to a large value (i.e. \( hk \to \infty \)), the velocities for a high frequency are solved from Eq. (50), as

\[ c_{f}^\infty = \sqrt{\frac{2}{1-v} c_2}, \quad c_3^\infty = \sqrt{\frac{\pi^2}{12} (1+\tilde{G}_f)c_2}, \quad c_{s}^\infty = 0, \quad c_{t}^\infty = c_2 \]  

(53)

where \( c_f, c_3, c_s \) and \( c_t \) are the velocities for a fast wave, two slow waves and a thickness shear flexural wave, respectively. The variations of the four normalised velocities against wave number \( hk \) are shown in Fig. 2 for different parameters of the foundation \( \tilde{G}_f \) and \( k_f \), where \( v = 0.3 \). Fig. 2 shows that the parameters of the foundation \( \tilde{G}_f \) and \( k_f \) have no effect on the velocities of fast flexural and thickness shear flexural waves. However, the effect of these parameters of the foundation on the slow waves \( c_s \) is significant. When the wave number \( hk \) is taken to be small, all velocities for these four flexural waves tend to infinity. To demonstrate the existence of high frequency flexural waves in the time domain, Figs. 3 and 4 show the variation of the resultants \( M_{22}(t)/M_0 \)
and \( Q_2(t)/P \) at point \((0, a)\) due to two point loads \( M_0 H(t) \) and \( PH(t) \) (where \( H(t) \) is the Heaviside function) at the origin for different selections of foundation \( \tilde{G}_f \) and \( \tilde{k}_f \), where \( \nu = 0.33 \) and \( h/a = 0.2 \) (\( a \) is a unit of length).

4. Method fundamental solution in the Laplace domain

In the MFS, the source points are located outside the physical domain in order to avoid the singularities of the fundamental solution. By using the superposition principle for linear elasticity, Wen (1987, 1988, 1989) studied the MFS to the Kirchhoff plate resting on the Winkler foundation subjected to static loads. As the superposition principle is still valid in the transformed domain, the approximate solutions of the displacement (deflection and rotations) and the resultants of moment and shear force in the Laplace domain at the boundary collocation point can be expressed as
\[
\tilde{w}_i(P) = \tilde{w}_i^*(P) + \sum_{n=1}^{N} \tilde{U}_{ik}(P,Q)c_{kn}^*(Q)
\]
\[
\tilde{p}_i(P) = \tilde{p}_i^*(P) + \sum_{n=1}^{N} \tilde{T}_{ik}(P,Q)c_{kn}^*(Q)
\]

where \(\tilde{p}_i = \tilde{M}_{ik}n_{ik}\), \(\tilde{p}_3 = \tilde{Q}_{ik}n_{ik}\), \(\tilde{w}_i^*(P)\) and \(\tilde{p}_i^*(P)\) represent the particular solutions in governing equation (4), \(P\) and \(Q\) denote collocation and source points in Fig. 5, respectively, and \(c_{kn}\) are unknown densities of concentrated forces at the source point \(n\). In general, the number of collocation points on the real boundary \(N_c\) should be larger than or equal to the number of source points outside of domain \(N_s\). The least-squares method should be utilised to determine the unknown densities if \(N_c > N_s\). In this paper, the case \(N_c = N_s\) is considered only.

If uniform and linear distributed loads are considered in the domain, i.e.
\[
q(P,t) = q_0(t) + q_1(t)x_1 + q_2(t)x_2
\]

where \(q_0(t)\), \(q_1(t)\) and \(q_2(t)\) are time-dependent functions only. Thus, the transformation of the applied load in the Laplace domain is given by
\[
\bar{q}(P,p) = \bar{q}_0(p) + \bar{q}_1(p)x_1 + \bar{q}_2(p)x_2
\]

Therefore, particular solutions for the displacement can be obtained from the governing equations (4) and are given by
\[ \tilde{w}_1 = -\frac{\kappa \mu \tilde{q}_1}{(\kappa \mu + \rho h^2 p^2)(k_i + \rho hp^2)} \]

\[ \tilde{w}_2 = -\frac{\kappa \mu \tilde{q}_2}{(\kappa \mu + \rho h^2 p^2)(k_i + \rho hp^2)} \]

\[ \tilde{w}_3 = \frac{1}{k_i + \rho hp^2} [\tilde{q}_0(p) + \tilde{q}_1(p)x_1 + \tilde{q}_2(p)x_2] \]

while the particular solutions for the moment and the shear forces are obtained as

\[ \tilde{M}_{11} = \tilde{M}_{12} = \tilde{M}_{22} = 0 \]

\[ \tilde{Q}_i = \frac{\kappa \mu \tilde{q}_1}{k_i + \rho hp^2} \left( 1 - \frac{\kappa \mu}{\kappa \mu + \rho h^2 p^2} \right) \]

\[ \tilde{Q}_3 = \frac{\kappa \mu \tilde{q}_2}{k_i + \rho hp^2} \left( 1 - \frac{\kappa \mu}{\kappa \mu + \rho h^2 p^2} \right) \]

Considering the displacement and traction boundary conditions for the plate, we arrive at the following system of linear algebraic equations:

\[ \sum_{n=1}^{N} \tilde{u}_{ik}(P, Q) c_n^k(Q) = \tilde{w}_i(P) - \tilde{w}_i^*(P) \quad \text{for} \quad P \in \Gamma_u \]

\[ \sum_{n=1}^{N} \tilde{T}_{ik}(P, Q) c_n^k(Q) = \tilde{p}_i(P) - \tilde{p}_i^*(P) \quad \text{for} \quad P \in \Gamma_t \]

where \( \tilde{w}_i \) and \( \tilde{p}_i \) are specified displacement and traction boundary values in the Laplace domain, respectively, whilst \( \Gamma_u \) and \( \Gamma_t \) represent the displacement and traction boundaries (\( F = \Gamma_u + \Gamma_t \)). Thus a set of unknown intensity of point force can be obtained by solving system of linear algebraic equations (54) in the Laplace domain. Although the MFS-matrix for the system of linear algebraic equations is ill-conditioned, stable and convergent solutions can be obtained by selecting properly the gap between collocation and source points. The accuracy and sensitivity with respect to this gap are examined in Example 5.1.

In the Laplace domain, a total number of samples (\( L + 1 \)) in the transformation space \( p_k, k = 0, 1, 2, \ldots, L \), are selected. The transformed variables are evaluated for these specified transform parameters. Then the variables in the time domain can be determined by the Laplace inversion technique. Here, the method proposed by Durbin (1974) is adopted. The application of the Durbin’s Laplace transform inverse method was performed by Wen et al. (1996) in the case of the elasticity wave propagations in two-dimensions. The inversion formula is written as

\[ f(t) = \frac{2e^{\eta T}}{T} \left[ -\frac{1}{2} \bar{f}(\eta) + \sum_{k=0}^{L} \text{Re}\{\bar{f}(\eta + 2k\pi i/T)e^{2k\pi i/T}\} \right] \]

where \( \bar{f}(p_k) \) denotes the transformed variable in the Laplace domain and \( p_k = \eta + 2k\pi i/T(i = \sqrt{-1}) \). The selection of the free parameters \( \eta \) and \( T \) affects the accuracy of inversion slightly.

5. Numerical examples

5.1. A simply supported square plate

In this example, a simply supported square plate of length \( a \) and thickness \( h \) subjected to a uniform static load \( q_0 \), as shown in Fig. 6(a), is analysed. The aim of this example is to demonstrate the accuracy and stability of the MFS for a moderate thick plate. The collocation and source points are distributed uniformly along each edge of the plate, as shown in Fig. 6(b), where \( \Delta \) presents the distance between the collocation and source points, and \( d \) is the distance between two collocation points.

Comparison has been made between the MFS and the boundary element method by Wang et al. (1992). In this case, we take \( v = 0.25, k_i = 200D/a^4 \) and \( h/a = 0.1 \). The total number of
collocation points is taken to be 84. The variations of the normalised deflection \( w_3 D/q_0 a^4 \) and the resultant moment \( M_{22} D/q_0 a^2 \) at the centre of the plate are plotted in Fig. 7 against the normalised gap \( \Delta/d \) for convenience. In addition, the free parameter \( \Delta \) can be normalised with respect to the width of the plate \( a \) for a rectangular sheet or other unit of length in different cases. Apparently, the relative errors are less than 3% for the deflection and moment when \( \Delta/d \geq 1 \). In addition, these results indicate that the selection of the source points is not sensitive provided that the source points are not too close to the boundary. It is worth noting that choosing a set of uniformly distributed boundary collocation points is not essential for the MFS. It has been found that the similar degree of accuracy for the numerical solutions can be obtained by an irregular distribution of collocation points.

### 5.2. Rectangular plate resting freely on the Winkler foundation

In the case of the free boundary conditions for the two-parameter foundation problems, an additional unknown traction appears along the free edge due to the discontinuity of the normal slopes between the
plate and the foundation. However, for the one-parameter foundation (Winkler), such discontinuity of normal slopes is ignored. In this example, a rectangular plate of width \( a \) and height \( b \) on the Winkler foundation, as shown in Fig. 8(a), subjected to a centric/eccentric concentrated forces \( P \) is analysed. Here we have taken \( b/a = 0.5, \quad v = 0.167, \quad k_f = D/l^4, \quad l = 0.5a \) (where \( l \) is defined as a characteristic length of Winkler’s foundation), \( h = 0.05a \) and \( A/d = 3 \). Two positions of concentrated force \( P \) are considered. In case one, a point force acts at the centre of plate \((0.5a, 0.25a)\) and in case two, a point force acts at location \((0.3a, 0.25a)\). The total number of collocation points is taken to be 64 and these are uniformly distributed along the edges. Fig. 8(b) shows the variations of the normalised deflection \( w_3D/Pa^2 \) and normalised moment \( M_{11}/P \) along the axis \( x_1 \). The results for Kirchhoff plate given by Wen (1989) are plotted in the same figure for comparison. It is obvious that there is a singularity of order \( \ln r \) in the deflection fundamental solution and order \( 1/r \) in the moment fundamental solution for the Mindlin plate at the location of the point force (source point). However, the deflection by the Kirchhoff’s theory is normal in the domain.

5.3. A square plate containing a circular hole under uniform moment

A square plate of width \( a \) containing a circular hole of radius \( c \) with free edges is shown in Fig. 9. The uniformly distributed moment \( M_0 \) is applied along two edges in this example and the Poisson’s ratio is taken as 1/3. To demonstrate the convergence of the MFS with respect to the number of source (collocation) points, the concentration factors at the point A on the circular hole in an infinite plate are
presented in Fig. 10, where the distance between collocation and source points $\Delta/c = 0.4$. Also a comparison of the concentration factors with the analytical solution given by Young and Budynas (2002) has been made in Fig. 11, i.e.

$$K\left(\frac{c}{h}\right) = \frac{M_{11}(A)}{M_0} = 1.79 + \frac{0.25}{0.39 + (2c/h)} + \frac{0.81}{1 + (2c/h)^2} - \frac{0.26}{1 + (2c/h)^3}$$

For the MFS adopted here, the total number of collocation points is taken as 32, whilst $\Delta/c = 0.4$. Apparently, an excellent agreement with the analytical solution has been achieved. In addition, the concentration factors $K(c/h)$ for different foundations (Winkler’s $G_f = 0$) are presented in Fig. 12 for $alc = 2$. The number of collocation points on the outer boundary has been taken as 64 and the number of collocation points on the circular hole has been chosen to be 32. All the collocation points are uniformly distributed with the normalised distance $\Delta/c = 0.4$. In the case of $k_f = 10D/c^4$, the concentration factor is less than one and tends to zero rapidly when the ratio $c/h > 0.5$.
5.4. Simply supported square plate subjected to uniform and concentrated loads

A simply supported square plate of width $a$ with a uniformly distributed pressure $q_0 H(t)$ in the domain, as shown in Fig. 6(a), is considered first in this example. The particular solution for uniform loads is $\tilde{w}_1 = 0, \tilde{w}_2 = 0, \tilde{w}_3 = \tilde{q}_0 / (k_t + \rho p^2)$ (where $\tilde{q}_0 = q_0 / p$) and the particular solutions for the moment and shear forces are zero. Here $\nu = 0.25, hla = 0.1, \Delta/d = 2$, whilst $\eta = 5/t_0$ and $T = 20t_0$, where $t_0 = a/c_2$ (unit of time).
The total number of collocation points is taken as 84 and the number of samples in the Laplace domain is given by \( L = 100 \). The normalised deflection \( w_3(t)/q_0a^4 \) and the resultant of moment \( M_{11}(t)/q_0a^2 \) at the centre of the plate against the normalised time \( c_2t/a \) are plotted in Figs. 13(a) and (b), respectively. Two foundations are considered: (1) \( k_f = 200D/a^4, G_f = 20D/a^2 \) and (2) \( k_f = 0, G_f = 0 \). The horizontal dash lines in Figs. 13(a) and (b) denote the solutions for the static uniform load \( q_0 \) as obtained by Wang et al. (1992), i.e. \( w_{3\text{static}}^t = 1.58 \times 10^3 \) and \( M_{11\text{static}}^t = 1.563 \times 10^2 \), respectively. We can see that the plate starts to vibrate about the static equilibrium position with different frequencies, which depend on the Pasternak foundation parameters. In addition, the normalised particular solution for the deflection in the time domain can be solved analytically, from the solution in (57), namely

\[
\dot{w}_3(t) = \frac{w_3^t D}{q_0a^4} = \frac{D}{k_f a^4} \left( 1 - \cos \left( \sqrt{\frac{k_f}{\phi h}} t \right) \right) \tag{62}
\]

Before the arrival of high frequency flexural waves travelling from the edge (simply supported) to the centre of plate, the deflection should be equal to the particular solution in (62) (see the dash curve in Fig. 13(a) when \( k_f = 0 \)).

Finally, a simply supported square plate subjected to a concentrated shear force \( PH(t) \) at the centre of the plate is investigated. The normalised deflection \( w_3D/P\alpha^2 \), moment \( M_{11}/P \) and shear force \( Q_1/P \) at point

![Fig. 13. Simply supported square plate resting on a Pasternak foundation subjected to the uniform load \( q_0H(t) \): (a) normalised deflection \( w_3D/q_0a^4 \) at the centre of plate; (b) normalised moment \( M_{11}/q_0a^2 \) at the centre of plate.](image-url)
(0.75a, 0.5a) are shown in Figs. 14(a)–(c), respectively. It can be seen from these figures that the vibration of the plate occurs at about the static equilibrium position and the frequency of vibration increases when the foundation parameter $k_f$ increases.

6. Conclusions

In this paper, the fundamental solution for the Mindlin plate resting on the Pasternak foundation was derived in the Laplace transform domain and the MFS was applied for static and dynamic problems. The accuracy and stability of the MFS was examined for the static case and the optimised distance between the
collocation and source points was analysed. Excellent agreement with the boundary element method was achieved for static problems. The MFS has most advantages of the mesh free methods and demonstrates three major features in their computations, namely simplicity, accuracy and efficiency. We can conclude with the following observations: (1) there are four flexural waves for the Mindlin plate on the Pasternak foundation in the frequency domain and only slow flexural waves are influenced significantly by the parameter of the foundation $k_f$; (2) the effect of flexural waves on the resultants of the moment and the shear force are significant; (3) the MFS is suitable for the shear deformable plate for both static and dynamic problems; (4) compared with the FEM and BEM, the MFS is more flexible and simpler to implement; (5) disadvantages of the MFS are also evident, such as the fundamental solutions must be available for the problem and the optimised source point distribution needs to be investigated. Finally, the boundary integral formulations using these fundamental solutions can be derived directly. The investigations of the Mindlin’s plate on an elastic foundation by the BEM is deferred as future work.

References


